

# Physics Notes

## Physics 1 & 2

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**Lynn English High School**  
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<http://www.mrbigler.com/documents/Physics-Notes.pdf>

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This is a set of class notes for physics. This hardcopy is provided so that you can fully participate in class discussions without having to worry about writing everything down.

While these notes are intended to be reasonably complete, you will probably want to add your own notes. If you have purchased this copy, you are encouraged to write directly in it, just as you would write in your own notebook. However, if this copy was issued to you by the school and intend to return it at the end of the year, you will need to write your supplemental notes on separate paper. If you do this, be sure to write down page numbers in your notes, to make cross-referencing easier.

You should bring these notes to class every day. Besides their obvious usefulness in knowing/remembering how to do the homework, in most instances, you will be allowed to use them during quizzes and tests, because the point of these assessments is not to see what you can *remember*, but to see how well you can *apply* what you have learned.

These notes will be used with both the Honors and CP Physics I classes. However, the CP course requires less depth, and will skip some of the topics and sections of these notes.





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## Cornell Notes

**Unit:** Introduction

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- how to take advantage of the Cornell note-taking system

**Language Objectives:**

- Understand the term Cornell Notes and be able to describe how Cornell Notes are different from ordinary note-taking.

**Notes:**

The Cornell note-taking system was developed about fifty years ago at Cornell University. I think it's a great way to get more out of your notes. I think it's an especially useful system for adding your comments to someone else's notes /(such as mine).

The main features of the Cornell Notes system are:

1. The main section of the page is for what actually gets covered in class.
2. The left section (officially 2½ inches, though I have shrunk it to 2 inches for these notes) is for "cues"—questions or comments of yours that will help you find, remember, or effectively use these notes.
3. The bottom section (2 inches) is officially for you to add a 1–2 sentence summary of the page in your own words. This is a good idea. However, because the rest of the page is my notes, not yours, you may also want to use that space for anything else you want to remember that wasn't in the pre-printed notes.

Use this space for summary and/or additional notes:

## Reading & Taking Notes from a Textbook

**Unit:** Introduction

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- pre-reading and reading a section of a textbook and taking notes

**Language Objectives:**

- understand and be able to describe the reading and note-taking strategies presented in this section

**Notes:**

If you read a textbook the way you would read a novel, you probably won't remember much of what you read. Before you can understand anything, your brain needs enough context to know how to file the information. This is what Albert Einstein was talking about when he said, "It is the theory which decides what we are able to observe."

When you read a section of a textbook, you need to create some context in your brain, and then add a few observations to solidify the context before reading in detail.

René Descartes described this process in 1644 in the preface to his *Principles of Philosophy*:

"I should also have added a word of advice regarding the manner of reading this work, which is, that I should wish the reader at first go over the whole of it, as he would a romance, without greatly straining his attention, or tarrying at the difficulties he may perhaps meet with, and that afterwards, if they seem to him to merit a more careful examination, and he feels a desire to know their causes, he may read it a second time, in order to observe the connection of my reasonings; but that he must not then give it up in despair, although he may not everywhere sufficiently discover the connection of the proof, or understand all the reasonings—it being only necessary to mark with a pen the places where the difficulties occur, and continue reading without interruption to the end; then, if he does not grudge to take up the book a third time, I am confident that he will find in a fresh perusal the solution of most of the difficulties he will have marked before; and that, if any remain, their solution will in the end be found in another reading."

Use this space for summary and/or additional notes:

The following 4-step system takes about the same amount of time you're used to spending on reading and taking notes, but it will probably make a tremendous difference in how much you understand and remember.

1. Copy the titles/headings of each section. Leave about  $\frac{1}{4}$  page of space after each one. (Don't do anything else yet.) This should take about 2–3 minutes.
2. Do not write anything yet! Look through the section for pictures, graphs, and tables. Take a minute to look at these—the author must have thought they were important. Also read over (but don't try to answer) the homework questions/problems at the end of the section. (For the visuals, the author must think these things illustrate something that is important enough to dedicate a significant amount of page real estate to it. For the homework problems, these illustrate what the author thinks you should be able to do once you know the content.) This process should take about 10–15 minutes.
3. Actually read the text, one section at a time. For each section, jot down keywords and sentence fragments that remind you of the key ideas. You are not allowed to write more than the  $\frac{1}{4}$  page allotted. (You don't need to write out the details—those are in the book, which you already have!) This process is time consuming, but shorter than what you're probably used to doing for your other teachers.
4. Read the summary at the end of the chapter or section—this is what the author thinks you should know now that you've finished the reading. If there's anything you don't recognize, go back and look it up. This process should take about 5–10 minutes.

You shouldn't need to use more than about one sheet of paper (both sides) per 10 pages of reading!

Use this space for summary and/or additional notes.

## Taking Notes on Math Problems

**Unit:** Introduction

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- taking notes on a mathematical problem

**Language Objectives:**

- understand and be able to describe the strategies presented in this section

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**Notes:**

If you were to copy down a math problem and look at it a few days or weeks later, chances are you'll recognize the problem, but you won't remember how you solved it.

Solving a math problem is a process. For notes to be useful, they need to describe the process as it happens, not just the final result.

If you want to take good notes on how to solve a problem, you need your notes to show what you did at each step.

For example, consider the following physics problem:

A 25 kg cart is accelerated from rest to a velocity of  $3.5 \frac{\text{m}}{\text{s}}$  over an interval of 1.5 s. Find the net force applied to the cart.


The process of solving this problem involves applying two equations:  
 $v = v_0 + at$  and  $F = ma$ .

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Use this space for summary and/or additional notes:

A good way to document the process is to use a two-column format, in which you show the steps of the solution in the left column, and you write an explanation of what you did and why for each step in the right column.

For this problem, a two-column description would look like the following:

Step	Description/Explanation
$m = 25 \text{ kg}$ $v_o = 0$ $v = 3.5 \frac{\text{m}}{\text{s}}$ $t = 1.5 \text{ s}$ $F = \text{quantity desired}$	Define variables.
$F = ma$ 	Choose a formula that contains $F$ . See if we have the other variables. → No. We need to find $a$ before we can solve the problem.
$v = v_o + at$	Choose a formula that contains $a$ . See if we have the other variables. → Yes.
$3.5 = 0 + a(1.5)$ $3.5 = 1.5a$ $\frac{3.5}{1.5} = \frac{1.5}{1.5}a$ $2.33 = a$	Substitute $v$ , $v_o$ and $t$ into the 1 <sup>st</sup> equation and solve for $a$ .
$F = ma$ $F = (25)(2.33)$ $F = 58.33$	Substitute $m$ and $a$ into the 2 <sup>nd</sup> equation and solve for $F$ .
$F = \boxed{58.33 \text{ N}}$	Include the units and box the final answer.

Use this space for summary and/or additional notes.





## Introduction: Laboratory & Measurement

**Unit:** Laboratory & Measurement

### Topics covered in this chapter:

The Scientific Method .....	15
Designing & Performing Experiments.....	18
Accuracy & Precision.....	23
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The purpose of this chapter is to teach skills necessary for designing and carrying out laboratory experiments, recording data, and writing summaries of the experiment in different formats.

- *Designing & Performing Experiments* discusses strategies for coming up with your own experiments and carrying them out.
- *Accuracy & Precision, Uncertainty & Error Analysis, and Recording and Analyzing Data* discuss techniques for working with the measurements taken during laboratory experiments.
- *Keeping a Laboratory Notebook* and *Formal Laboratory Reports* discuss ways in which you might communicate (write up) your laboratory experiments.

Calculating uncertainty (instead of relying on significant figures) is a new and challenging skill that will be used in lab write-ups throughout the year.

### Standards addressed in this chapter:

#### **Next Generation Science Standards (NGSS):**

No NGSS standards are addressed in this chapter.

#### **Massachusetts Curriculum Frameworks (2006):**

No MA curriculum frameworks are specifically addressed in this chapter.

Use this space for summary and/or additional notes:

**Skills learned & applied in this chapter:**

- Designing laboratory experiments
- Error analysis (calculation & propagation of uncertainty)
- Formats for writing up lab experiments

Use this space for summary and/or additional notes.

## The Scientific Method

**Unit:** Laboratory & Measurement

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- Understand the scientific method

**Language Objectives:**

- Understand and correctly use terms relating to the scientific method, such as “peer review”

**Notes:**

The scientific method is a fancy name for “figure out what happens by trying it.”

In the middle ages, “scientists” were called “philosophers.” These were church scholars who decided what was “correct” by arguing and debating with each other.

During the Renaissance, scientists like Galileo Galilei and Leonardo da Vinci started using experiments instead of argument to decide what really happens in the world.

**Steps:**

1. Observe something interesting.
2. Figure out and perform an experiment that will have different outcomes depending on the parameter(s) being tested.
3. Repeat the experiment, varying your conditions as many ways as you can.
4. If possible, come up with a model that explains and predicts the behavior you observed. This model is called a **theory**.

Use this space for summary and/or additional notes:

5. Share your theory, your experimental procedures, and your data with other scientists. Some of these scientists may:
  - a. Look at your experiments to see whether the experiments really can distinguish between the different outcomes.
  - b. Look at your data to see whether the data really do support your theory.
  - c. Try your experiments or other related experiments themselves and see if the results are consistent with your theory.
  - d. Add to, modify, limit, refute (disprove), or suggest an alternative to your theory.

This process is called “peer review.” If a significant number of scientists have reviewed your claims and agree with them, and no one has refuted your theory, your theory may gain acceptance within the scientific community.

Note that the word “theory” in science has a different meaning from the word “theory” in everyday language. In science, a theory is a model that:

- Has never failed to explain a collection of related observations
- Has never failed to successfully predict the outcomes of related experiments

For example, the theory of evolution has never failed to explain the process of changes in organisms because of factors that affect the survivability of the species.

If a repeatable experiment contradicts a theory, and the experiment passes the peer review process, the theory is deemed to be wrong. If the theory is wrong, it must be either modified to explain the new results or discarded completely.

Notice also that the word “hypothesis” does not appear anywhere in the description of the scientific method. It is possible (and sometimes useful) to have a hypothesis before performing an experiment, but an experiment is just as valid and just as useful whether or not a hypothesis was involved.

Use this space for summary and/or additional notes.

### Theories vs. Natural Laws

A theory is a model that attempts to explain *why* or *how* something happens. A law simply describes what happens without attempting to provide an explanation. Both theories and laws can be used to predict the outcomes of related experiments.

For example, the Law of Gravity states that objects attract other objects based on their mass and distance from each other. It is a law and not a theory because the Law of Gravity does not explain *why* masses attract each other.

Atomic Theory states that matter is made of atoms, and that those atoms are themselves made up of smaller particles. The interactions between the particles that make up the atoms (particularly the electrons) are used to explain certain properties of the substances. This is a theory because it gives an explanation for *why* the substances have the properties that they do.

Note that a theory cannot become a law any more than a definition can become a measurement.

Use this space for summary and/or additional notes.

## Designing & Performing Experiments

**Unit:** Laboratory & Measurement

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- determine what you are testing
- figure out how to get from what you can measure to what you want to determine

**Language Objectives:**

- Understand and correctly use the terms “dependent variable” and “independent variable.”
- Understand and be able to describe the strategies presented in this section.

**Notes:**

Most high school physics experiments are relatively simple to understand, set up and execute—much more so than in chemistry or biology. This makes physics well-suited for teaching you how to design experiments.

The education “buzzword” for this is *inquiry-based experiments*, which means you (or your lab group) will need to figure out what to do to perform an experiment that answers a question about some aspect of physics. In this course, you will usually be given only an objective or goal and a general idea of how to go about achieving it. You and your lab group (with help) will decide the specifics of what to do, what to measure (and how to measure it), and how to make sure you are getting good results. This is a form of *guided* inquiry.

Use this space for summary and/or additional notes:

## Framing Your Experiment

Experiments are motivated by something you want to find out, observe, or calculate.

### Independent vs. Dependent Variables

In an experiment, there is usually something you are doing, and something you are measuring or observing.

independent variable: the conditions you are setting up. These are the numbers you pick. Because you pick the numbers, they are *independent* of what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are choosing the heights, so height is the *independent* variable.

dependent variable: the things that happen in the experiment. These are the numbers you measure, which are *dependent* on what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are measuring the time, which depends on the height. This means time is the *dependent* variable.

If someone asks what your independent and dependent variables are, the question simply means, "What did you do (independent variable)?" and "What did you measure (dependent variable)?"

Use this space for summary and/or additional notes.

### Qualitative Experiments

If the goal of your experiment is to find out **whether or not** something happens at all, you need to set up a situation in which the phenomenon you want to observe can happen, then observe whether or not it does. The only hard part is making sure the conditions of your experiment don't bias whether the phenomenon happens or not.

If you want to find out **under what conditions** something happens, what you're really testing is whether or not it happens under different sets of conditions that you can test. In this case, you need to test three situations:

1. A situation in which you are sure the thing will happen, to make sure you can observe it. This is your **positive control**.
2. A situation in which you are sure the thing cannot happen, to make sure your experiment can produce a situation in which it doesn't happen and you can observe its absence. This is your **negative control**.
3. The condition you want to test to see whether or not the thing happens. The condition is your independent variable, and whether or not the thing happens is your dependent variable.

### Quantitative Experiments

If the goal of your experiment is to quantify (find a numerical relationship for) the extent to which something happens (the dependent variable), you need to figure out a set of conditions in which you can measure the thing that happens. Once you know that, you need to figure out how much you can change the parameter you want to test (the independent variable) and still be able to measure the result. This gives you the highest and lowest values of your independent variable. Then perform the experiment using a range of values for the independent value that cover the range from the lowest to the highest (or *vice-versa*).

Use this space for summary and/or additional notes.



## Determining What to Measure

Determining what to measure usually means determining what you *can* measure, and then figuring out how to get from there to what you want to know.

For a quantitative experiment, if you have a mathematical formula that includes the quantity you want to measure, you simply need to find the values of the other quantities in the equation. For example, if you want to calculate the force needed to move an object, the equation is:

$$F = ma$$

In order to calculate the force, you would need to know:

- **mass**: you can measure this using a balance
- **acceleration**: you could measure this with an accelerometer, but we do not have one in the lab. This means you will need to calculate acceleration from another formula.

Notice that we have figured out that we can measure mass directly, but we need some way to figure out the acceleration. This means that our experiment to determine the force consists of two steps:

1. Measure the mass.
2. Perform an experiment to determine the acceleration.
3. Calculate the force on the object from the mass and the acceleration.

So now we need a formula for acceleration. That formula turns out to be:

$$d = v_o t + \frac{1}{2} a t^2$$

This means in order to calculate acceleration, we need to know:

- **distance**: you can measure this with a meter stick or tape measure.
- **initial velocity**: you either need to measure this or set up your experiment so the initial velocity is zero (the object starts out at rest).
- **time**: you can measure this with a stopwatch.

Use this space for summary and/or additional notes.

Now, every variable we need in the equation is something we can measure, so we're all set. Our experiment is:

1. Measure the **mass** of the object.
2. Determine the **acceleration** of the object:
  - a. Set up an experiment in which the object starts out at rest (**initial velocity is zero**), and then it **accelerates**.
  - b. Measure the **distance** the object moves and the **time** it takes.
  - c. **Calculate** the **acceleration**.
3. **Calculate** the **force** on the object from the mass and the acceleration.

Use this space for summary and/or additional notes.

## Accuracy & Precision

**Unit:** Laboratory & Measurement

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Understand what accuracy and precision mean and the difference between the two.

**Language Objectives:**

- Understand and be able to differentiate between accuracy and precision.

**Notes:**

Science relies on making and interpreting measurements, and the accuracy and precision of these measurements affect what you can conclude from them.

### Random vs. Systematic Errors

Random errors are natural uncertainties in measurements because of the limits of precision of the equipment used. Random errors are assumed to be distributed around the actual value, without bias in either direction. Systematic errors occur from specific problems in your equipment or your procedure. Systematic errors are often biased in one direction more than another, and can be difficult to identify.

### Accuracy vs. Precision

The words “accuracy” and “precision” have specific meanings in science.

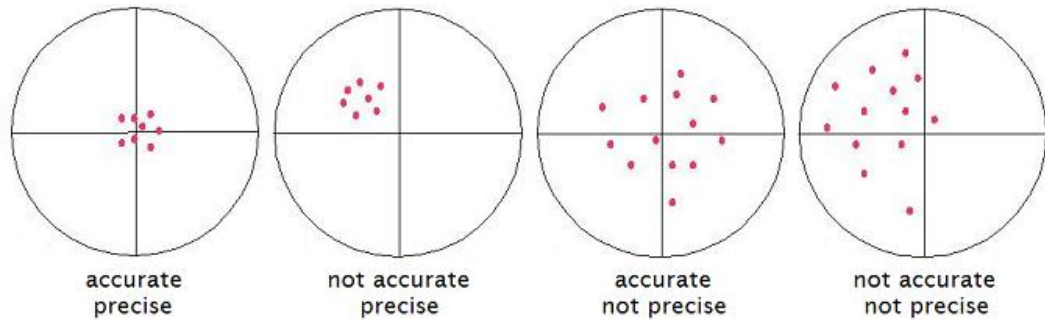
accuracy: for a single measurement, how close the measurement is to the “correct” or accepted value. For a group of measurements, how close the average is to the accepted value.

precision: for a single measurement, how finely the measurement was made. (How many decimal places it was measured to.) For a group of measurements, how close the measurements are to each other.

Use this space for summary and/or additional notes:

**Examples:**

Suppose the following drawings represent arrows shot at a target.



The first set is both accurate (the average is close to the center) and precise (the data points are all close to each other.)

The second set is precise (close to each other), but not accurate (the average is not close to the correct value). This is an example of *systemic* error—some problem with the experiment caused all of the measurements to be off in the same direction.

The third set is accurate (the average is close to the correct value), but not precise (the data points are not close to each other). This is an example of *random* error—the measurements are not biased in any particular direction, but there is a lot of scatter.

The fourth set is neither accurate nor precise, which means that there are significant random and systematic errors present.

For another example, suppose two classes estimate Mr. Bigler's age. The first class's estimates are 73, 72, 77, and 74 years old. These measurements are fairly precise (close together), but not accurate. (Mr. Bigler is actually about 49 years old.) The second class's estimates are 0, 1, 97 and 98. This set of data is accurate (because the average is 49, which is close to correct), but the set is not precise because the individual values are not close to each other.

Use this space for summary and/or additional notes.

## Uncertainty & Error Analysis

**Unit:** Laboratory & Measurement

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Understand what uncertainty of measurement means.

**Skills:**

- Read and interpret uncertainty values.
- Estimate measurement errors.
- Propagate estimate of error/uncertainty through calculations.

**Language Objectives:**

- Understand and correctly use the terms “uncertainty,” “standard uncertainty,” and “relative error.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

In science, unlike mathematics, there is no such thing as an exact answer. Ultimately, every quantity is limited by the precision and accuracy of the measurements that it came from. If you can only measure a quantity to within 10%, that means any number that is derived from that measurement can't be any better than  $\pm 10\%$ .

Error analysis is the practice of determining and communicating the causes and extents of possible errors or uncertainty in your results. Error analysis involves understanding and following the uncertainty in your data from the initial measurement to the final report.

Use this space for summary and/or additional notes:

## Uncertainty

The uncertainty of a measurement describes how different the correct value is likely to be from the measured value. For example, if a length was measured to be 22.336 cm, and the uncertainty was 0.030 cm (meaning that the measurement is only known to within  $\pm 0.030$  cm), we could represent this measurement in either of two ways:

$$(22.336 \pm 0.030) \text{ cm} \quad 22.336(30) \text{ cm}$$

The first of these states the variation ( $\pm$ ) explicitly in cm (the actual unit). The second shows the variation in the last digits shown.

### Absolute Error

Absolute error (or absolute uncertainty) refers to the uncertainty in the actual measurement, such as  $(3.64 \pm 0.22)$  cm.

### Relative Error

Relative error shows the error or uncertainty as a fraction of the measurement. (Percent error, which you used in chemistry last year, is related to relative error.)

The formula for relative error is  $R.E. = \frac{\text{uncertainty}}{\text{measured value}}$

For example, a measurement of  $(3.64 \pm 0.22)$  cm would have a relative error of 0.22 out of 3.64. Mathematically, we express this as:

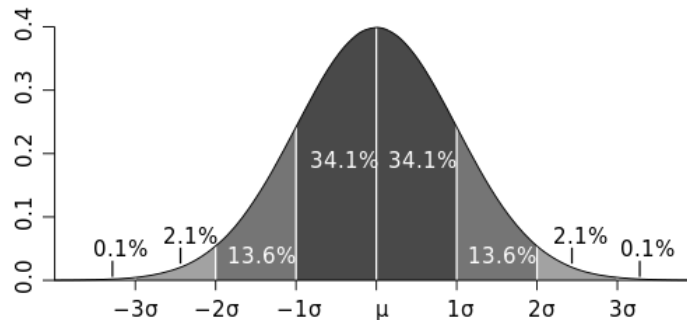
$$R.E. = \frac{0.22}{3.64} = 0.0604$$

To turn relative error into percent error, multiply by 100. A relative error of 0.0604 is the same as 6.04% error.

Use this space for summary and/or additional notes.

standard uncertainty ( $u$ ): the standard deviation of the mean of a set of measurements.

Statistically, if the measurements follow a Gaussian (normal, “bell-shaped”) distribution, one standard deviation ( $u$ ) should include 68.2% of the measurements, two standard deviations ( $2u$ ) should include 95.4% of the measurements, and three standard deviations ( $3u$ ) should include 99.6% of the measurements.



Most scientists use the standard deviation to represent uncertainty, which means roughly 68.2% of the individual measurements fall within the uncertainty.

*E.g.*, if we report a measurement of  $(22.336 \pm 0.030)$  cm, that means:

- The mean value is  $\bar{x} = 22.336$  cm
- It is 68% likely that any single measurement falls between between  $(22.336 - 0.030)$  cm and  $(22.336 + 0.030)$  cm, *i.e.*, between 22.306 cm and 22.366 cm.

However, because the reported measurement is an average of a set of measurements, if there are a lot of data points, the likelihood that the *average* of the measurements is close to the actual value is much higher than the likelihood that any individual measurement is close.

Use this space for summary and/or additional notes.

## Calculating the Uncertainty of a Set of Measurements

When you have several independent measurements of the same data point, the uncertainty is calculated using statistics, so that some specific percentage of the measurements will fall within the average plus or minus the uncertainty.

### Ten or More Data Points

If you have a large enough set of data points (at least 10), then the standard uncertainty is the standard deviation of the mean. That formula is:

$$u = \sigma_m = \frac{s}{\sqrt{n}}$$

where:

$u$  = standard uncertainty

$\sigma_m$  = standard deviation of the mean

$s$  = sample standard deviation =  $\sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$

$n$  = number of measurements

If the variable  $x$  represents the measured quantity and  $\bar{x}$  represents the arithmetic mean (average) value, you would express your result as:

$$\text{reported value} = \bar{x} \pm u = \bar{x} \pm \frac{s}{\sqrt{n}}$$

While this would be the correct formula to use when possible, often we have too few data points (small values of  $n$ ), which causes the formula to predict a much larger uncertainty than we probably actually have.

Use this space for summary and/or additional notes.



**More than One but Fewer than Ten Data Points**

If you have only a few data points (fewer than 10), then you have too few data points to accurately calculate the population standard deviation. In this case, we can estimate of the standard uncertainty by using the formula:

$$u \approx \frac{r}{\sqrt{3}}$$

where:

$u$  = uncertainty

$r$  = range (the difference between the largest and smallest measurement)

If the variable  $x$  represents the measured quantity, you would express your result as:

$$\text{reported value} = \bar{x} \pm \frac{r}{\sqrt{3}}$$

Note that we are treating  $\sqrt{3}$  as a constant. Whenever you have more than one but fewer than ten data points, find the range and divide it by  $\sqrt{3}$  to get the estimated uncertainty.

**Example:**

Suppose you measured a mass on a balance and the reading drifted between 3.46 g and 3.58 g:

$$\bar{x} = \frac{3.46 + 3.58}{2} = 3.52$$

$$r = 3.58 - 3.46 = 0.12$$

$$u \approx \frac{r}{\sqrt{3}} \approx \frac{0.12}{1.732} \approx 0.07$$

You would record the balance reading as  $(3.52 \pm 0.07)$  g.

Use this space for summary and/or additional notes.

## Uncertainty of a Single Measurement

If you have the ability to measure a quantity that is not changing (such as the mass or length of an object), you will get the same value every time you measure it. This means you have only one data point.

When you have only one data point, the standard uncertainty is the limit of how precisely you can measure it (including any estimated digits). This will be your best educated guess, based on how closely you think you actually measured the quantity.

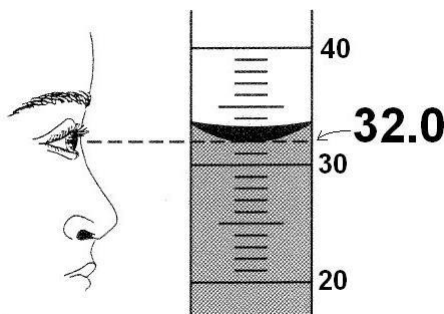
### Digital Measurements

For digital equipment, if the reading is stable (not changing), look up the published precision of the instrument in its user's manual. (For example, many balances used in high schools have a readability of 0.01 g but are only precise to within  $\pm 0.02$  g.) If there is no published value (or the manual is not available), assume the uncertainty is  $\pm 1$  in the last digit.

Use this space for summary and/or additional notes.

**Analog Measurements**

When making analog measurements, always estimate to one extra digit beyond the finest markings on the equipment. For example, in the diagram below, the graduated cylinder is marked in 1 mL increments. When measuring volume in this graduated cylinder, you would estimate and write down the volume to the nearest 0.1 mL, as shown:



In the above experiment, you should record the volume as  $32.0 \pm 0.1$  mL. It would be inadequate to write the volume as 32 mL. (Note that the zero at the end of the reading of 32.0 mL is not extra. It is necessary because you measured the volume to the nearest 0.1 mL and not to the nearest 1 mL.)

When estimating, you can generally assume that the estimated digit has an uncertainty of  $\pm 1$ . This means the uncertainty of the measurement is usually  $\pm \frac{1}{10}$  of the finest markings on the equipment.

Equipment	Markings	Estimate To	Uncertainty
triple-beam balance	0.01 g	0.001 g	$\pm 0.001$ g
ruler	1 mm	0.1 mm	$\pm 0.1$ mm
100 mL graduated cylinder	1 mL	0.1 mL	$\pm 0.1$ mL
thermometer	1°C	0.1°C	$\pm 0.1$ °C

Use this space for summary and/or additional notes.

## Propagating Uncertainty in Calculations

When you perform calculations using numbers that have uncertainty, you need to propagate the uncertainty through the calculation.

### Crank Three Times

The simplest way of doing this is the “crank three times” method. The “crank three times” method involves:

1. Perform the calculation using the actual numbers. This gives the result (the part before the  $\pm$  symbol).
2. Perform the calculation again, using the end of the range for each value that would result in the smallest value. (Note that with fractions, this means you need to subtract the uncertainty for values in the numerator and add the uncertainty for values in the denominator.) This gives the lower limit of the range.
3. Perform the calculation again using the end of the range for each value that would result in the largest value. This gives the upper limit of the range.
4. Assume you have fewer than ten data points and use the approximation that the uncertainty =  $u \approx \frac{r}{\sqrt{3}}$ , where  $r$  is the range.

The advantage to “crank three times” is that it’s easy to understand and you are therefore less likely to make a mistake. The disadvantage is that it can become unwieldy when you have multi-step calculations.

Use this space for summary and/or additional notes.

**Error Propagation**

This method requires applying a set of rules based on the formulas you use for the calculations. (For one-step calculations, the “crank three times” method is easier and faster. However, this method is often less work when you have a multi-step calculation.)

The rules are:

1. If the calculation involves **addition or subtraction**, add the absolute errors. For example,  $(8.45 \pm 0.15) \text{ cm} + (5.43 \pm 0.12) \text{ cm}$  means:

$$\begin{array}{r} (8.45 \pm 0.15) \text{ cm} \\ + (5.43 \pm 0.12) \text{ cm} \\ \hline \boxed{(13.88 \pm 0.27) \text{ cm}} \end{array}$$

(Uncertainty calculations involving addition or subtraction are intuitively easy.)

2. If the calculation involves **multiplication or division**, add the relative errors, and multiply the total relative error by the result to get the uncertainty. For example, if we have the problem  $(2.51 \pm 0.16) \text{ kg} \times (0.37 \pm 0.04) \frac{\text{m}}{\text{s}^2}$ , we would do the following:

The result is  $2.51 \text{ kg} \times 0.37 \frac{\text{m}}{\text{s}^2} = 0.9287 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$

For the error analysis:

The relative error of  $(2.51 \pm 0.16) \text{ kg}$  is  $\frac{0.16}{2.51} = 0.0637$

The relative error of  $(0.37 \pm 0.04) \frac{\text{m}}{\text{s}^2}$  is  $\frac{0.04}{0.37} = 0.10810$

The total relative error is  $0.0637 + 0.1081 = 0.1718$ .

The total relative error is what we multiply the result by to get its (absolute) uncertainty:

$0.9287 \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \times 0.1718 = \pm 0.1596 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes.

The answer with its uncertainty is  $(0.9287 \pm 0.1596) \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$ , which we can round to  $(0.93 \pm 0.16) \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$ .

In general, it's good practice to show the uncertainty to one or two significant figures. The result should be rounded to the same place value as the uncertainty.

3. For calculations that involve **exponents**, use the same rule as for multiplication and division, but multiply the relative error by the exponent. For example, consider the problem:

$$\frac{1}{2} \times (12.2 \pm 0.8) \text{ kg} \times \left( (5.8 \pm 0.2) \frac{\text{m}}{\text{s}} \right)^2$$

The result of the calculation is:  $\frac{1}{2} (12.2)(5.8)^2 = 205.2 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$

The relative errors of the two measurements are:

$$\frac{0.8}{12.2} = 0.0656 \quad \text{and} \quad \frac{0.2}{5.8} = 0.0345$$

Because the  $5.8 \frac{\text{m}}{\text{s}}$  is squared in the calculation, we need to multiply its relative error by two (the exponent). This gives a total relative error of:

$$0.0656 + (0.0345 \times 2) = 0.1345$$

Now multiply the total relative error by the result to get the uncertainty:

$$205.2 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \times 0.1345 = 27.6 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

We would round this to  $(205 \pm 28) \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$ .

Use this space for summary and/or additional notes.

### How Significant Figures Work (as an Approximation)

Note that the rules you learned in chemistry for calculations with significant figures come from these rules, with the assumption that the uncertainty is  $\pm 1$  in the last digit.

For example, consider the following problem:

<u>problem:</u>	<u>"sig figs" equivalent:</u>
$\begin{array}{r} 23000 \quad \pm 1000 \\ 0.0075 \quad \pm 0.0001 \\ + \quad 1650 \quad \pm 10 \\ \hline 24650.0075 \quad \pm 1010.0001 \end{array}$	$\begin{array}{r} 23 \text{ ???}.\text{???} ? \\ 0.0075 \\ + \quad 165?.\text{???} ? \\ \hline 24 \text{ ???}.\text{???} ? \end{array}$
<div style="text-align: center;"> <math>\uparrow</math>              -----  <math>\uparrow</math> </div>	<div style="text-align: center;"> <math>\uparrow</math>              -----  <math>\uparrow</math> </div> <p style="text-align: right;">(Check this digit for rounding)</p>

If we rounded our answer of  $24650 \pm 1010$  to  $25000 \pm 1000$ , it would agree with the number we got by using the rules for significant figures.

Note, however, that our answer of  $24650 \pm 1010$  suggests that the actual value is between 23640 and 25660, whereas the answer based on significant figures suggests that the actual value is between 24000 and 26000. This is why scientists frown upon use of "sig figs" instead of genuine error analysis. If you insist on using sig figs, it is better to report answers to *one more sig fig* than the problem calls for, in order to minimize round-off answers in your results.

Use this space for summary and/or additional notes.

As another example, consider the problem:

$$34.52 \times 1.4$$

Assume this means  $(34.52 \pm 0.01) \times (1.4 \pm 0.1)$ .

The answer (without its uncertainty) is  $34.52 \times 1.4 = 48.328$

Because the calculation involves multiplication, we need to add the relative errors, and then multiply the total relative error by the answer to get the uncertainty.

The relative uncertainties are:

$$\frac{0.01}{34.52} = 0.00029$$

$$\frac{0.1}{1.4} = 0.0714$$

$$\text{Adding them gives: } 0.00029 + 0.0714 = 0.0717$$

$$\text{Multiplying by our answer gives: } 0.0717 \times 48.328 = 3.466$$

$$\text{The uncertainty is: } \pm 3.466.$$

Our answer including the uncertainty is therefore  $48.328 \pm 3.466$ , which we can round to  $48.3 \pm 3.5$ .

Doing the calculation with sig figs would give us an answer of 48, which implies  $48 \pm 1$ . Again, this is in the ballpark of the actual uncertainty, but you should notice (and be concerned) that the sig figs method underrepresents the actual uncertainty by a factor of 3.5!

Use this space for summary and/or additional notes.



## Summary

### Uncertainty of a Single Quantity

#### Measured Once

Make your best educated guess of the uncertainty based on how precisely you were able to measure the quantity and the uncertainty of the instrument(s) that you used.

#### Measured Multiple Times (Independently)

- If you have a lot of data points, the uncertainty is the standard deviation.
- If you have few data points, use the approximation  $u \approx \frac{r}{\sqrt{3}}$ .

### Uncertainty of a Calculated Value

- For addition & subtraction, add the uncertainties of each of the measurements. The sum is the uncertainty of the result.
- For multiplication, division and exponents:
  1. Find the relative error of each measurement.
$$\text{R.E.} = \frac{\text{uncertainty}}{\text{measured value}}$$
  2. Multiply the relative error by the exponent (if any).
  3. Add each of the relative errors to find the total relative error.
  4. The uncertainty is the total R.E. times the result.

Use this space for summary and/or additional notes.

## Keeping a Laboratory Notebook

**Unit:** Laboratory & Measurement

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Understand the purpose of an informal lab write-up, such as you would write in a notebook.
- rules for recording and working with lab data

**Skills:**

- Write up an experiment in laboratory notebook format.

**Language Objectives:**

- Understand and be able to describe the sections of a laboratory notebook write-up, and which information goes in each section.

**Notes:**

A laboratory notebook serves two important purposes:

1. It is a legal record of what you did and when you did it.
2. It is a diary of what you did in case you want to remember the details later.

In a research laboratory, you would normally do a write-up in your lab notebook whenever you do a significant experiment that you believe you might want to refer back to sometime in the future.

### Your Notebook as an Official Record

Laboratory notebooks are kept by scientists in research laboratories and high tech companies. If a company or research institution needs to prove that you did a particular experiment on a particular date and got a particular set of results, your lab notebook is the primary evidence. This means you need to maintain your lab notebook in a way that gives it the best chance of being able to prove beyond a reasonable doubt exactly what you did and when you did it.

Use this space for summary and/or additional notes:

Here are some general rules for working with data. (Most of these are courtesy of Dr. John Denker, at <http://www.av8n.com/physics/uncertainty.htm>):

### Recording Data

- Write something about what you did on the same page as the data, even if it is a very rough outline. Your procedure notes should not get in the way of actually performing the experiment, but there should be enough information to corroborate the detailed summary of the procedure that you will write afterwards. (Also, for evidence's sake, the sooner after the experiment that you write the detailed summary, the more weight it will carry in court.)
- Keep all of the raw data, whether you will use it or not.
- Don't discard a measurement, even if you think it is wrong. Record it anyway and put a "?" next to it. You can always choose not to use the data point in your calculations (as long as you give an explanation).
- Never erase or delete a measurement. The only time you should ever cross out recorded data is if you accidentally wrote down the wrong number.
- Record all digits. Never round off original data measurements. If the last digit is a zero, you must record it anyway!
- For analog readings (*e.g.*, ruler, graduated cylinder, thermometer), estimate and record one extra digit.
- Always write down the units with each measurement!
- Record every quantity that will be used in a calculation, whether it is changing or not.
- Don't convert in your head before writing down a measurement. Record the original data in the units you measured it in, and convert in a separate step.

Use this space for summary and/or additional notes.

### Calculations

- Use enough digits to avoid unintended loss of significance. (Don't introduce round-off errors in the middle of a calculation.) This usually means use at least two more digits than the number of "significant figures" you expect your answer to have.
- Use few enough digits to be reasonably convenient.
- Record uncertainty separately from the measurement. (Don't rely on "sig figs" to express uncertainty.)
- Leave digits in the calculator between steps. (Don't round until the end.)
- When in doubt, keep plenty of "guard digits" (digits after the place where you think you will end up rounding).

### Integrity of Data

Your data are your data. In classroom settings, people often get the idea that the goal is to report an uncertainty that reflects the difference between the measured value and the "correct" value. That idea certainly doesn't work in real life—if you knew the "correct" value you wouldn't need to make measurements!

In all cases—in the classroom and in real life—you need to determine the uncertainty of your own measurement by scrutinizing your own measurement procedures and your own analysis. Then you judge how well they agree.

For example, we would say that the quantities  $10 \pm 2$  and  $11 \pm 2$  agree reasonably well, because there is considerable overlap between their probability distributions. However,  $10 \pm 0.2$  does not agree with  $11 \pm 0.2$ , because there is no overlap.

If your results disagree with well-established results, you should look for and comment on possible problems with your procedure and/or measurements that could have caused the differences you observed. You must *never* fudge your data to improve the agreement.

Use this space for summary and/or additional notes.

## Notebook Format

Every lab you work in, whether in high school, college, research or industry, will have its own preferred laboratory notebook format. It is much more important to understand what *kinds* of information you need to record and what you will use it for than it is to get attached to any one format.

In physics class this year, you will record data on loose paper, but in the same format as you would use for a laboratory notebook. (Otherwise, the teacher would need to take home three bushels of lab notebooks weighing more than 100 pounds to grade for every lab experiment.)

The format we will use is meant to follow an outline of the actual experiment. Don't worry too much about following the format exactly. It is much more important that the information you have is complete and recorded correctly than it is to follow the format.

### Title & Date

Each experiment should have the title and date the experiment was performed written at the top. The title should be a descriptive sentence fragment (usually without a verb) that gives some information about the purpose of the experiment.

### Objective

This should be a one or two-sentence description of what you are trying to determine or calculate by performing the experiment.

### Experimental Plan

Your experimental plan is a short description (usually a few sentences) that explains:

- what you are going to do (each of your independent variables and what you are going to do with them)
- what you are going to measure or observe (your dependent variables)
- how you are going to calculate or interpret the results.

Use this space for summary and/or additional notes.

### Notes

Your notes are your original data. In every experiment, you must write down a brief description of the procedure and all of your data (measurements and observations) *during the experiment*, and you must include *the same piece of paper that you wrote on during the experiment* as part of your notebook, no matter how messy it is! (If I catch you turning in a copied over set of notes for this section, you will lose credit.)

If you were keeping an actual notebook, your notebook would be the only place you are allowed to write anything during the experiment. However, because you are not keeping a physics notebook this year, the piece of paper with your original data serves the same purpose.

### Procedure

This is a detailed description of exactly what you did. You need to include:

- A *labeled* sketch of your experimental set-up, even if the experiment is simple. The sketch will serve to answer many questions about how you set up the experiment and most of the key equipment you used.
- A list of any equipment that you used other than what you labeled in your sketch.
- A step-by-step description of everything you did. The description needs to include the actual values of quantities you used in the experiment (your independent variables). For a repeated procedure, write the steps once, then list the differences from one trial to the next. *E.g.*, "Repeat steps 1–4 using distances of 1.5 m, 2.0 m, 2.5 m, and 3.0 m."

### Data & Observations

This is a section in which you present all of your data. This section is a clean copy of what you wrote down in your notes section during the experiment.

For a high school lab, it is usually sufficient to present a single data table that includes your measurements for each trial and the quantities you calculated from them. However, if you have other data or observations that you recorded during the lab, they must be listed here.

Use this space for summary and/or additional notes.

## Analysis

The analysis section is where you interpret your data. (Any calculated values in the table in the Data & Observations section are actually analysis, even though it is permissible to include them above.)

Your analysis needs to include:

- Any calculated values that did not appear in the data table in your Data & Observations section
- Graphs, if necessary, to convey your results. Note that any graphs you include in your write-up must be drawn accurately to scale, using graph paper, and using a ruler whenever a straight line is needed. (When a graph is required, you will lose points if you include a freehand sketch instead.)
- One (and only one) sample calculation for each separate formula that you used. For example, if you calculated acceleration for each of five data points, you would write down the formula, and then choose one set of data to plug in and show how you got the answer.
- Quantitative error analysis. You need to:
  1. Measure or estimate the uncertainty of each your measurements
  2. Calculate the relative error for each measurement.
  3. Combine your relative errors to get the total relative error for your calculated value(s)
  4. Multiply the total relative error by your calculated values to get the uncertainty ( $\pm$ ) for each one.
- Sources of uncertainty: this is a list of factors *inherent in your procedure* that limit how precise your answer can be. **Never include hypothetical human error!** A statement like “We might have written down the wrong number.” or “We might have done the calculations incorrectly.” is really saying, “We might be stupid and you shouldn’t believe anything we say in this report.” (You will lose points if you include statements like this.)

Use this space for summary and/or additional notes.

Note, however, that if a problem *actually occurred*, and if you *used that data point in your calculations anyway*, you need to explain what happened and what you think might be the effects on your results.

## Conclusions

Your conclusion should be worded the same way as your objective, this time including your calculated results with their uncertainties. You do not need to mention sources of uncertainty in your conclusions unless you believe they were significant enough to create some doubt about your results.

Use this space for summary and/or additional notes.



## Formal Laboratory Reports

**Unit:** Laboratory & Measurement

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Understand the purpose of a formal lab write-up, such as you would submit to a scientific journal

**Skills:**

- Write up an experiment formally in journal article format.

**Language Objectives:**

- Understand and correctly use the term “abstract,” as it pertains to a formal laboratory report.
- Understand and be able to describe the sections of a formal laboratory report, and which information goes in each section.

**Notes:**

A formal laboratory report serves one important purpose: to communicate the results of your experiment to other scientists outside of your laboratory or institution.

A formal report is a significant undertaking. In a research laboratory, you might submit as many as one or two articles to a scientific journal in a year. Some college professors require students to submit lab reports in journal article format.

The format of a formal journal article-style report is as follows:

Use this space for summary and/or additional notes:

### **Abstract**

This is the most important part of your report. It is a (maximum) 200-word executive summary of everything about your experiment—the procedure, results, analysis, and conclusions. In most scientific journals, the abstracts are searchable via the internet, so it needs to contain enough information to enable someone to find your abstract, and after reading it, to know enough about your experiment to determine whether or not to purchase a copy of the full article (which can sometimes cost \$100 or more). It also needs to be short enough that the person doing the search won't just say "TL; DR" ("Too Long; Didn't Read") and move on to the next abstract.

Because the abstract is a complete summary, it is always best to wait to write it until you have already written the rest of your report.

### **Introduction**

Your introduction is actually an entire research paper on its own, with citations. (For a high school lab report, it should be 1–3 pages; for scientific journals, 5–10 pages is not unheard of.) Your introduction needs to describe any general background information that another scientist might not know, plus all of the background information that specifically led up to your experiment. The introduction is usually the most time-consuming part of the report to write.

### **Materials and Methods**

This section describes your experimental procedure. Unlike a lab notebook write-up, the Materials and Methods section of a formal report is written in paragraph form, in the past tense. Again, a labeled drawing of your apparatus is a necessary part of the section, but you need to *also* describe the set-up in the text.

Also unlike the lab notebook write-up, your Materials and Methods section needs to give some explanation of your choices of the values of your independent variables (what you actually did).

Use this space for summary and/or additional notes.

### **Data and Observations**

This section is similar to the same section in the lab notebook write-up, except that:

1. You should present only data you actually recorded/measured in this section. (Calculated values are presented in the Discussion section.)
2. You need to *introduce* the data table. (This means you need to describe the important things someone should notice in the table first, and then say something like “Data are shown in Table 1.”)

Note that all figures and tables in the report need to be numbered consecutively.

### **Discussion**

This section is similar to the Analysis section in the lab notebook write-up, but with some important differences.

Your discussion is essentially a long essay discussing your results and what they mean. You need to introduce and present a table with your calculated values and your uncertainty. After presenting the table, you should discuss the results, uncertainties, and sources of uncertainty in detail. If your results relate to other experiments, you need to discuss the relationship and include citations for those other experiments.

Your discussion needs to include all of the formulas that you used as part of your discussion, but you do not need to show your work for each calculation.

### **Conclusions**

Your conclusions should start by presenting the same information that you included in the same section in your lab notebook write-up. However, in the formal report, you should mention significant sources of uncertainty, and suggest how future experiments might follow up on or expand on your experiment.

### **Works Cited**

As with a research paper, you need to include a complete list of bibliography entries for the references you cited in your introduction and/or discussion sections.

Use this space for summary and/or additional notes.



## Introduction: Mathematics

**Unit:** Mathematics

### Topics covered in this chapter:

Standard Assumptions in Physics .....	52
Assigning & Substituting Variables .....	55
The Metric System .....	62
Scientific Notation.....	67
Trigonometry .....	70
Vectors .....	74
Vectors vs. Scalars in Physics .....	79
Vector Multiplication .....	82

The purpose of this chapter is to familiarize you with mathematical concepts and skills that will be needed in physics.

- *Standard Assumptions in Physics* discusses what you can and cannot assume to be true in order to be able to solve the problems you will encounter in this class.
- *Assigning & Substituting Variables* discusses how to determine which quantity and which variable apply to a number given in a problem based on the units, and how to choose which formula applies to a problem.
- *The Metric System* and *Scientific Notation* briefly review skills that you are expected to remember from your middle school math and science classes.
- *Trigonometry, Vectors, Vectors vs. Scalars in Physics, and Vector Multiplication* discuss important mathematical concepts that are widely used in physics, but may be unfamiliar to you.

Depending on your math background, some of the topics, such as trigonometry and vectors, may be unfamiliar. These topics will be taught, but in a cursory manner.

Use this space for summary and/or additional notes:

**Standards addressed in this chapter:**

**Next Generation Science Standards (NGSS):**

No NGSS standards are addressed in this chapter.

**Massachusetts Curriculum Frameworks (2006):**

No MA curriculum frameworks are specifically addressed in this chapter. However, this chapter addresses the following mathematical understandings explicitly listed in the MA Curriculum Frameworks as prerequisites for this course:

- Construct and use tables and graphs to interpret data sets.
- Solve simple algebraic expressions.
- Perform basic statistical procedures to analyze the center and spread of data.
- Measure with accuracy and precision (*e.g.*, length, volume, mass, temperature, time)
- Convert within a unit (*e.g.*, centimeters to meters).
- Use common prefixes such as milli-, centi-, and kilo-.
- Use scientific notation, where appropriate.
- Use ratio and proportion to solve problems.

In addition, this chapter addresses the following mathematical understandings. The MA frameworks state that “the following skills are not detailed in the Mathematics Framework, but are necessary for a solid understanding in this course.”

- Determine the correct number of significant figures.
- Determine percent error from experimental and accepted values.
- Use appropriate metric/standard international (SI) units of measurement for mass (kg); length (m); time (s); force (N); speed (m/s); acceleration ( $\text{m/s}^2$ ); frequency (Hz); work and energy (J); power (W); momentum ( $\text{kg}\cdot\text{m/s}$ ); electric current (A); electric potential difference/voltage (V); and electric resistance ( $\Omega$ ).
- Use the Celsius and Kelvin scales.

Use this space for summary and/or additional notes.

**Skills learned & applied in this chapter:**

- Estimating uncertainty in measurements
- Propagating uncertainty through calculations
- Identifying quantities in word problems and assigning them to variables
- Choosing a formula based on the quantities represented in a problem
- Using trigonometry to calculate the lengths of sides and angles of triangles
- Representing quantities as vectors
- Adding and subtracting vectors
- Multiplying vectors using the dot product and cross product

Use this space for summary and/or additional notes.

## Standard Assumptions in Physics

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- what are the usual assumptions in physics problems

**Language Objectives:**

- understand and correctly use the term “assumption,” as it pertains to setting up and solving physics problems.

**Notes:**

Many of us have been told not to make assumptions. There is a popular expression that states that “when you assume, you make an ass of you and me”:

ass|u|me

In science, particularly in physics, this adage is crippling. Assumptions are part of everyday life. When you cross the street, you assume that the speed of cars far away is slow enough for you to walk across without getting hit. When you eat your lunch, you assume that the food won't cause an allergic reaction. When you run down the hall and slide across the floor, you assume that the friction between your shoes and the floor will be enough to stop you before you crash into your friend.

assumption: something that is unstated but considered to be fact for the purpose of making a decision or solving a problem. Because it is impossible to measure and/or calculate everything that is going on in a typical physics or engineering problem, it is almost always necessary to make assumptions.

Use this space for summary and/or additional notes:



In a first-year physics course, in order to make problems and equations easier to understand and solve, we will often assume that certain quantities have a minimal effect on the problem, even in cases where this would not actually be true. The term used for these kinds of assumptions is “ideal”. Some of the ideal physics assumptions we will use include the following. Over the course of the year, you can make each of these assumptions unless you are explicitly told otherwise.

- Constants (such as acceleration due to gravity) have the same value in all parts of the problem.
- Variables change in the manner described by the relevant equation(s).
- Ideal machines (such as pulleys) and other objects that are not directly considered in the problem have negligible mass, inertia, and friction.
- If the problem does not mention air resistance and air resistance is not a central part of the problem, friction due to air resistance is negligible.
- The mass of an object can be assumed to exist at a single point in 3-dimensional space.
- Sliding (kinetic) friction between surfaces is negligible. (This will not be the case in problems involving friction, though even in friction problems, ice is assumed to be frictionless unless you are explicitly told otherwise.)
- Force can be applied in any direction using an ideal rope. (You can even push on it!)
- Collisions between objects are perfectly elastic.
- No energy is lost when energy is converted from one form to another.
- The amount that solids and liquids expand or contract due to temperature differences is negligible. (This will not be the case in problems involving thermal expansion.)
- The degree to which solids and liquids can be compressed or expanded due to changes in pressure is negligible.
- Gas molecules do not interact when they collide or are forced together from pressure.
- Electrical wires have negligible resistance.
- All physics students do all of their homework. 😊

Use this space for summary and/or additional notes.

In some topics, a particular assumption may apply to some problems but not others. In these cases, the problem needs to make it clear whether or not you can make the relevant assumption. (For example, in the “forces” topic, some problems involve friction and others do not. A problem that does not involve friction might state that “a block slides across a frictionless surface.”)

If you are not sure whether you can make a particular assumption, you should ask the teacher. If this is not practical (such as an open response problem on a standardized test), you should decide for yourself whether or not to make the assumption, and explicitly state what you are assuming as part of your answer.

Use this space for summary and/or additional notes.

## Assigning & Substituting Variables

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- be able to declare (assign) variables from a word problem
- be able to substitute values for variables in an equation

**Language Objectives:**

- understand and correctly use the terms “variable” and “subscript.”
- accurately describe and apply the concepts described in this section, using appropriate academic language

**Notes:**

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

**Mathematical Operations**

You have probably been taught translations for most of the common math operations:

word	meaning	word	meaning	word	meaning
and, more than (but not “is more than”)	+	percent (“per” + “cent”)	÷ 100	is at least	≥
less than (but not “is less than”)	-	change in $x$ , difference in $x$	$\Delta x$	is more than	>
of	×	is	=	is at most	≤
per, out of	÷			is less than	<

Use this space for summary and/or additional notes:

**Identifying Variables**

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable
mass	kg	$m$
distance, length	m	$d, \ell$
area	$m^2$	$A$
acceleration	$m/s^2$	$a$
volume	$m^3$	$V$
velocity (speed)	m/s	$v$
pressure	Pa	$P^*$
momentum	N·s	$p^*$
density	$kg/m^3$	$\rho^*$
moles	mol	$n$
time	s	$t$
temperature	K	$T$
heat	J	$Q$
electric charge	C	$q$

\*Note the subtle differences between uppercase “P”, lowercase “p”, and the Greek letter  $\rho$  (“rho”).

Any time you see a number in a word problem that has a unit you recognize (such as one listed in this table), notice which quantity the unit is measuring and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes.

### Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$$\rho = \frac{m}{V} \quad \text{and you're given: } m = 12.3 \text{ g} \quad \text{and} \quad V = 2.8 \text{ cm}^3$$

simply substitute 12.3 g for  $m$ , and  $2.8 \text{ cm}^3$  for  $V$ , giving:

$$\rho = \frac{12.3 \text{ g}}{2.8 \text{ cm}^3} = 4.4 \frac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often easier to leave the units out when you substitute numbers for variables and then add them back in at the end:

$$\rho = \frac{12.3}{2.8} = 4.4 \frac{\text{g}}{\text{cm}^3}$$

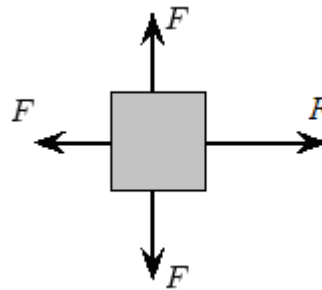
Use this space for summary and/or additional notes.

### Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

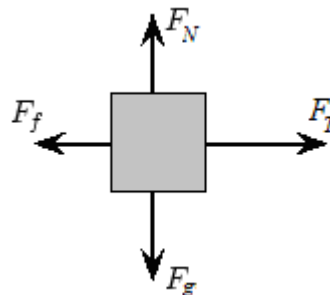
1. The force of gravity, pulling downward.
2. The “normal” force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

The variable for force is “F”, so the diagram would look like this:



In order to distinguish between the forces and make the diagram easier to understand, we add subscripts to the variables:

1.  $F_g$  is the force of gravity.
2.  $F_N$  is the normal force.
3.  $F_T$  is the tension in the rope.
4.  $F_f$  is friction.



Use this space for summary and/or additional notes.

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—it needs to be smaller than the other letters and lowered slightly. For example, when we write  $F_g$ , the variable is  $F$  (force) and the subscript  $_g$  attached to it tells which kind of force it is (gravity). This might occur in the following equation:

$$F_g = mg \quad \leftarrow \quad \text{right } \text{☺}$$

It is important that the subscript  $_g$  on the left does not get confused with the variable  $g$  on the right. Otherwise, the following error might occur:

$$Fg = mg$$

$$Fg = mg \quad \leftarrow \quad \text{wrong! } \text{☹}$$

$$F = m$$

Another common use of subscripts is the subscript “0” to mean “initial”. For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity “ $v_0$ ” where the subscript “0” means “at time zero”, *i.e.*, at the beginning of the problem, when the “time” on the “problem clock” would be zero. The final velocity is simply “ $v$ ” without the zero.

Use this space for summary and/or additional notes.

### The Problem-Solving Process

1. Identify the quantities in the problem, based on the units and any other information in the problem.
2. Assign the appropriate variables to those quantities.
3. Find an equation that relates all of the variables.
4. Substitute the values of the variables into the equation.
  - a. If you have only one variable left, it should be the one you're looking for.
  - b. If you have more than one variable left, find another equation that uses one of the variables you have left, plus other quantities that you know.
5. Solve the equation(s), using basic algebra.
6. Apply the appropriate unit(s) to the result.

Use this space for summary and/or additional notes.



**Sample Problem**

A force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object?

We have units of N and kg, and we're looking for acceleration. We need to look these up in our reference tables.

From Table D ("Quantities, Variables and Units") on page 3 of our reference tables, we find:

Symbol	Unit	Quantity	Variable
N	newton	force	$\vec{F}$
kg	kilogram	mass	$m$
		acceleration	$\vec{a}$

Now we know that we need an equation that relates the variables  $\vec{F}$ ,  $m$ , and  $a$ . ( $\vec{F}$  and  $\vec{a}$  are in boldface with an arrow above them because they are vectors. We'll discuss vectors a little later in the course.)

Now that we have the variables, we find a formula that relates them. From the second formula box in Table E ("Mechanics Formulas and Equations") on page 4 of our reference tables, we find that:

$$\vec{F} = m\vec{a}$$

So we substitute:

$$30 = 1.5a$$

$$20 = a$$

Again from Table D, we find that acceleration has units of meters per second squared, so our final answer is  $20 \frac{\text{m}}{\text{s}^2}$ .

Use this space for summary and/or additional notes.

## The Metric System

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Understand how units behave and combine algebraically.
- Know the 4 common prefixes and their numeric meanings.

**Skills:**

- Be able to describe quantities using metric units with & without prefixes.

**Language Objectives:**

- Understand and correctly use the terms “unit” and “prefix.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

**Notes:**

A unit is a specifically defined measurement. Units describe both the type of measurement, and a base amount.

For example, 1 cm and 1 inch are both lengths. They are used to measure the same dimension, but the specific amounts are different. (In fact, 1 inch is exactly 2.54 cm.)

Every measurement is a number multiplied by its units. In algebra, the term “ $3x$ ” means “3 times  $x$ ”. Similarly, the distance “75 m” means “75 times the distance 1 meter”.

*The number and the units are both necessary to describe any measurement. You always need to write the units. Saying that “12 is the same as 12 g” would be as ridiculous as saying “12 is the same as  $12 \times 3$ ”.*

The metric system is a set of units of measurement that is based on natural quantities (on Earth) and powers of 10.

Use this space for summary and/or additional notes:

The metric system has 7 fundamental “base” units:

- meter (m): length
- kilogram (kg): mass (even though “kilo” is actually a prefix, mass is defined based on the kilogram, not the gram)
- second (s): time
- Kelvin (K): temperature
- mole (mol): amount of substance
- ampere (A): electric current
- candela (cd): intensity of light

Each of these base units is defined in some way that could be duplicated in a laboratory anywhere on Earth (except for the kilogram, which is defined by a physical object that is locked in a vault in the village of Sevres, France). All other metric units are combinations of one or more of these seven.

For example:

Velocity (speed) is a change in distance over a period of time, which would have units of distance/time (m/s).

Force is a mass subjected to an acceleration. Acceleration has units of distance/time<sup>2</sup> (m/s<sup>2</sup>), and force has units of mass × acceleration. In the metric system this combination of units (kg·m/s<sup>2</sup>) is called a newton (symbol “N”), which means:  $1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2$

Use this space for summary and/or additional notes.

The metric system uses prefixes to indicate multiplying a unit by a power of ten. There are prefixes for powers of ten from  $10^{-18}$  to  $10^{18}$ . The most commonly used prefixes are:

- mega (M) =  $10^6 = 1\,000\,000$
- kilo (k) =  $10^3 = 1\,000$
- centi (c) =  $10^{-2} = \frac{1}{100} = 0.01$
- milli (m) =  $10^{-3} = \frac{1}{1,000} = 0.001$
- micro ( $\mu$ ) =  $10^{-6} = \frac{1}{1\,000\,000} = 0.000\,001$

These prefixes can be used in combination with any metric unit, and they work just like units. “35 cm” means “35 times c times m” or “ $(35)(\frac{1}{100})(m)$ ”. If you multiply this out, you get 0.35 m.

Any metric prefix is allowed with any metric unit.

For example, standard atmospheric pressure is 101 325 Pa. This same number could be written as 101.325 kPa or 0.101 325 MPa.

There is a popular geek joke based on the ancient Greek heroine Helen of Troy. She was said to have been the most beautiful woman in the world, and she was an inspiration to the entire Trojan fleet. She was described as having “the face that launched a thousand ships.” Therefore a milliHelen must be the amount of beauty required to launch one ship.

Use this space for summary and/or additional notes.

### Conversions

If you need to convert from one prefix to another, the rule of thumb is that if the prefix gets larger, then the number needs to get smaller and vice-versa.

For example, suppose we need to convert 0.25 mg to  $\mu\text{g}$ .

The prefix "m" means  $10^{-3}$  and " $\mu$ " means  $10^{-6}$ . The prefix is getting smaller by 3 decimal places, so the number needs to get bigger by 3 decimal places. The answer is therefore 250  $\mu\text{g}$ .

You can think of the rule in the following way:

$$0.25 \text{ mg} = (0.25) (0.001) \text{ g} = 0.00025 \text{ g}$$

$$250 \mu\text{g} = (250) (0.000001) \text{ g} = 0.00025 \text{ g}$$

As you can see, when the prefix got smaller, the number had to get bigger in order for the value to remain equal to 0.00025 g.

Use this space for summary and/or additional notes.

### The MKS vs. cgs Systems

Because physics heavily involves units that are derived from other units, it is important that you make sure all quantities are expressed in the appropriate units before applying formulas. (This is how we get around having to do factor-label unit-cancelling conversions—like you learned in chemistry—for every single physics problem.)

There are two measurement systems used in physics. In the MKS, or “meter-kilogram-second” system, units are derived from the S.I. units of meters, kilograms, seconds, moles, Kelvins, amperes, and candelas. In the cgs, or “centimeter-gram-second” system, units are derived from the units of centimeters, grams, seconds, moles, Kelvins, amperes, and candelas. The following table shows some examples:

Quantity	MKS Unit	S.I. Equivalent	cgs Unit	S.I. Equivalent
force	newton (N)	$\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	dyne (dyn)	$\frac{\text{g}\cdot\text{cm}}{\text{s}^2}$
energy	joule (J)	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$	erg	$\frac{\text{g}\cdot\text{cm}^2}{\text{s}^2}$
magnetic flux density	tesla (T)	$\frac{\text{N}}{\text{A}}, \frac{\text{kg}\cdot\text{m}}{\text{A}\cdot\text{s}^2}$	gauss (G)	$\frac{0.1 \text{ dyn}}{\text{A}}, \frac{0.1 \text{ g}\cdot\text{cm}}{\text{A}\cdot\text{s}^2}$

In this class, we will use exclusively MKS units. This means you only have to learn one set of derived units. However, you can see the importance, when you solve physics problems, of making sure all of the quantities are in MKS units before you plug them into a formula!

Use this space for summary and/or additional notes.

## Scientific Notation

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- Be able to convert numbers to and from scientific notation.
- Be able to enter numbers in scientific notation correctly on your calculator.

**Language Objectives:**

- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

Scientific notation is a way of writing a very large or very small number in compact form. The value is always written as a number between 1 and 10 multiplied by a power of ten.

For example, the number 1 000 would be written as  $1 \times 10^3$ . The number 0.000 075 would be written as  $7.5 \times 10^{-5}$ . The number 602 000 000 000 000 000 000 000 would be written as  $6.02 \times 10^{23}$ . The number 0.000 000 000 000 000 000 000 000 000 000 000 000 663 would be written as  $6.6 \times 10^{-34}$ .

(Note: in science, large numbers are typeset with a space after every three digits, both before and after the decimal point. To avoid confusion, commas are not used because in some countries, the comma is used as a decimal point.)

Scientific notation is really just math with exponents, as shown by the following examples:

$$5.6 \times 10^3 = 5.6 \times 1000 = 5600$$

$$2.17 \times 10^{-2} = 2.17 \times \frac{1}{10^2} = 2.17 \times \frac{1}{100} = \frac{2.17}{100} = 0.0217$$

Use this space for summary and/or additional notes:

Notice that if 10 is raised to a positive exponent means you're multiplying by a power of 10. This makes the number larger, and the decimal point moves to the right. If 10 is raised to a negative exponent, you're actually dividing by a power of 10. This makes the number smaller, and the decimal point moves to the left.

Significant figures are easy to use with scientific notation: all of the digits before the "x" sign are significant. The power of ten after the "x" sign represents the (insignificant) zeroes, which would be the rounded-off portion of the number. In fact, the mathematical term for the part of the number before the "x" sign is the significand.

### Math with Scientific Notation

Because scientific notation is just a way of rewriting a number as a mathematical expression, all of the rules about how exponents work apply to scientific notation.

Adding & Subtracting: adjust one or both numbers so that the power of ten is the same, then add or subtract the significands.

$$\begin{aligned} (3.50 \times 10^{-6}) + (2.7 \times 10^{-7}) &= (3.50 \times 10^{-6}) + (0.27 \times 10^{-6}) \\ &= (3.50 + 0.27) \times 10^{-6} = 3.77 \times 10^{-6} \end{aligned}$$

Multiplying & dividing: multiply or divide the significands. If multiplying, add the exponents. If dividing, subtract the exponents.

$$\frac{6.2 \times 10^8}{3.1 \times 10^{10}} = \frac{6.2}{3.1} \times 10^{8-10} = 2.0 \times 10^{-2}$$

Exponents: raise the significand to the exponent. Multiply the exponent of the power of ten by the exponent to which the number is raised.

$$(3.00 \times 10^8)^2 = (3.00)^2 \times (10^8)^2 = 9.00 \times 10^{(8 \times 2)} = 9.00 \times 10^{16}$$

Use this space for summary and/or additional notes.



### Using Scientific Notation on Your Calculator

Scientific calculators are designed to work with numbers in scientific notation. It's possible to can enter the number as a math problem (always use parentheses if you do this!) but math operations can introduce mistakes that are hard to catch.

Scientific calculators all have some kind of scientific notation button. The purpose of this button is to enter numbers directly into scientific notation and make sure the calculator stores them as a single number instead of a math equation. (This prevents you from making PEMDAS errors when working with numbers in scientific notation on your calculator.) On most Texas Instruments calculators, such as the TI-30 or TI-83, you would do the following:

What you type	What the calculator shows	What you would write
6.6 $\boxed{\text{EE}}$ -34	6.6E-34	$6.6 \times 10^{-34}$
1.52 $\boxed{\text{EE}}$ 12	1.52E12	$1.52 \times 10^{12}$
-4.81 $\boxed{\text{EE}}$ -7	-4.81E-7	$-4.81 \times 10^{-7}$

On some calculators, the scientific notation button is labeled  $\boxed{\text{EXP}}$  or  $\boxed{\times 10^x}$  instead of  $\boxed{\text{EE}}$ .

**Important note:** many high school students are afraid of the  $\boxed{\text{EE}}$  button because it is unfamiliar. If you are afraid of your  $\boxed{\text{EE}}$  button, you need to get over it and start using it anyway. However, if you insist on clinging to your phobia, you need to at least use parentheses around all numbers in scientific notation, in order to minimize the likelihood of PEMDAS errors in your calculations.

Use this space for summary and/or additional notes.

## Trigonometry

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- trigonometry functions that are used heavily in physics

**Skills:**

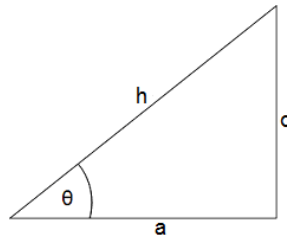
- find the  $x$ -component and  $y$ -component of a vector

**Language Objectives:**

- Understand and correctly use the terms “sine,” “cosine,” and “tangent.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

**Notes:**

If we have the following triangle:



- side “h” (the longest side, opposite the right angle) is the hypotenuse.
- side “o” is the side of the triangle that is opposite (across from) angle  $\theta$ .
- side “a” is the side of the triangle that is adjacent to (connected to) angle  $\theta$  (and is not the hypotenuse).

Use this space for summary and/or additional notes:

In a right triangle, the ratios of the lengths of the sides will be a function of the angles, and vice-versa.

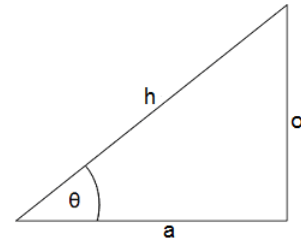
Trigonometry (from “trig” = “triangle” and “ometry” = “measurement”) is the study of these relationships.

The three primary trigonometry functions are defined as follows:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{h}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{h}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{o}{a}$$

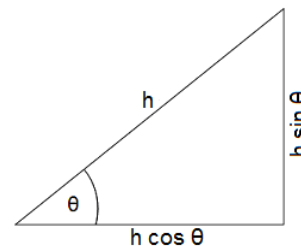


There are a lot of stupid mnemonics for remembering which sides are involved in which functions. My favorite of these is “Oh hell, another hour of algebra!”

The most common use of trigonometry functions in physics is to decompose a vector into its components in the x- and y-directions. In this situation, the vector is the hypotenuse. If we know the angle of the vector, we can use trigonometry and algebra to find the components of the vector in the x- and y-directions:

$$\cos\theta = \frac{a}{h} \text{ which means } a = h \cos\theta$$

$$\sin\theta = \frac{o}{h} \text{ which means } o = h \sin\theta$$



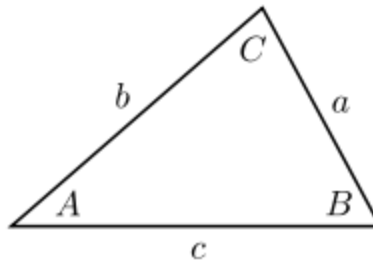
This is often necessary in physics problems involving gravity. Because gravity acts only in the y-direction, the formulas that apply in the y-direction are often different from the ones that apply in the x-direction.

Use this space for summary and/or additional notes.

### The Laws of Sines and Cosines

The Law of Sines and the Law of Cosines are often needed to calculate distances or angles in physics problems.

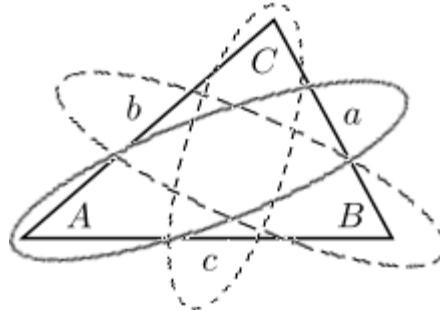
Consider the following triangle  $ABC$ , with sides  $a$ ,  $b$ , and  $c$ , and angles  $A$ ,  $B$ , and  $C$ . Angle  $A$  has its vertex at point  $A$ , and side  $a$  is opposite vertex  $A$  (and hence is also opposite angle  $A$ ).



#### The Law of Sines

The law of sines states that, for any triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



You can use the law of sines if you know one angle and the length of the opposite side. If you know those two things plus any other side or any other angle, you can work your way around the triangle and calculate every side and every angle.

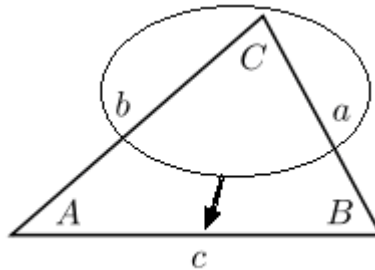
Use this space for summary and/or additional notes.

### The Law of Cosines

The law of cosines states that, for any triangle:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

You can use the law of cosines to find any angle or the length of the third side of a triangle as long as you know any two sides and the included angle:



You can also use the law of cosines to find one of the angles if you know the lengths of all three sides.

Remember that which sides and angles you choose to be  $a$ ,  $b$  and  $c$ , and  $A$ ,  $B$  and  $C$  are arbitrary. This means you can switch the labels around to fit your situation, as long as angle  $C$  is opposite side  $c$  and so on.

Note that the Pythagorean Theorem is simply the law of cosines in the special case where  $C = 90^\circ$  (because  $\cos 90^\circ = 0$ ).

The law of cosines is algebraically less convenient than the law of sines, so a good strategy would be to use the law of sines whenever possible, reserving the law of cosines for situations when it is not possible to use the law of sines.

Use this space for summary and/or additional notes.

## Vectors

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- what a vector is

**Skills:**

- adding & subtracting vectors

**Language Objectives:**

- Understand and correctly use the terms “vector,” “scalar,” and “magnitude.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

vector: a quantity that has both a magnitude (value) and a direction.

scalar: a quantity that has a value but does not have a direction. (A scalar is what you think of as a “regular” number, including its unit.)

magnitude: the scalar part of a vector (*i.e.*, the number and its units, but without the direction). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if  $\vec{F}$  is 25 N to the east, then  $\|\vec{F}\| = 25\text{ N}$

resultant: a vector that results from a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes:

unit vector: a vector that has a magnitude of 1.

Unit vectors are typeset as vectors, but with a “hat” instead of an arrow.

The purpose of a unit vector is to turn a scalar into a vector without changing its magnitude (value). For example, if  $d$  represents the scalar quantity 25 cm, and  $\hat{n}$ \* represents a unit vector pointing southward, then  $d\hat{n}$  would represent a vector of 25 cm to the south.

The letters  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are often used to represent unit vectors along the  $x$ ,  $y$ , and  $z$  axes, respectively.

Variables that represent vectors are traditionally typeset in ***bold italics***. Vector variables may also optionally have an arrow above the letter:

$$J, \vec{F}, v$$

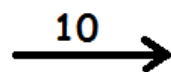
Variables that represent scalars are traditionally typeset in *plain Italics*:

$$V, t, \lambda$$

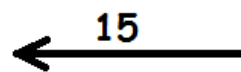
Note that a variable that represents only the magnitude of a vector quantity is generally typeset as a scalar:

For example,  $\vec{F}$  is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.) The magnitude would be 25 N, and would be represented by the variable  $F$ .

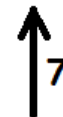
Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:



magnitude 10  
direction:  $0^\circ$   
(to the right)



magnitude 15  
direction:  $+180^\circ$   
(to the left)



magnitude 7  
direction:  $+90^\circ$  (up)

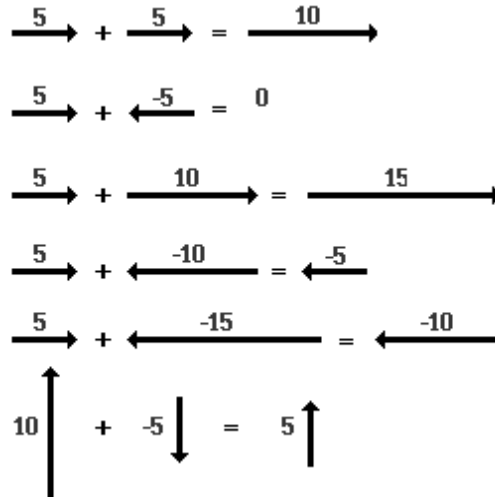
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\*  $\hat{n}$  is pronounced “n hat”

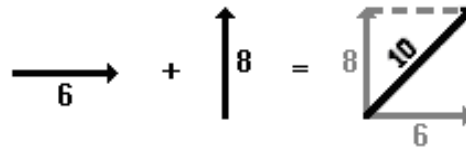
Use this space for summary and/or additional notes.

### Adding & Subtracting Vectors

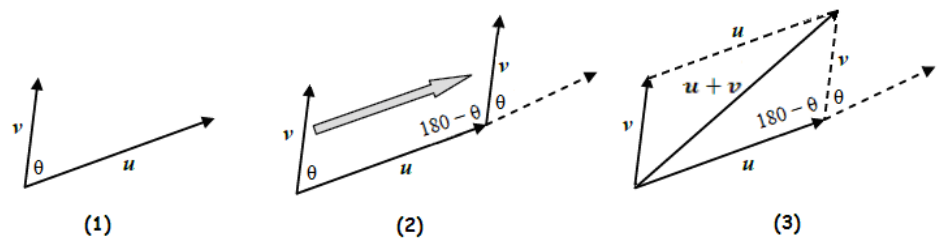
If the vectors have the same direction or opposite directions, the resultant is easy to envision:



If the vectors are not in the same direction, we move them so they start from the same place and complete the parallelogram. If they are perpendicular, we can add them using the Pythagorean theorem:



The same process applies to adding vectors that are not perpendicular:

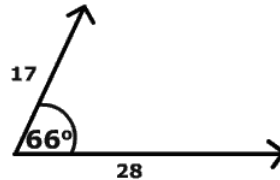


However, the trigonometry needed for the calculations is more involved.

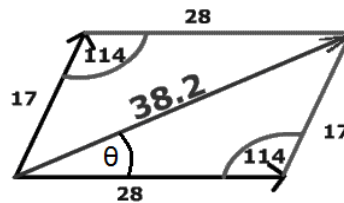
Use this space for summary and/or additional notes.



If the vectors have different (and not opposite) directions, the resultant vector is given by the diagonal of the parallelogram created from the two vectors. For example, the following diagram shows addition of a vector with a magnitude of 28 and a direction of  $0^\circ$  added to a vector with a magnitude of 17 and a direction of  $66^\circ$ :



The resultant vector is given by the parallelogram created by the two vectors:



The magnitude can be calculated using the law of cosines:

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$c^2 = 28^2 + 17^2 - 2(28)(17) \cos(114^\circ)$$

$$c^2 = 1460$$

$$c = \sqrt{1460} = 38.2$$

For the direction, use the law of sines:

$$\frac{38.2}{\sin 114^\circ} = \frac{17}{\sin \theta}$$

$$\sin \theta = \frac{17 \sin 114^\circ}{38.2} = \frac{(17)(0.914)}{38.2} = 0.407$$

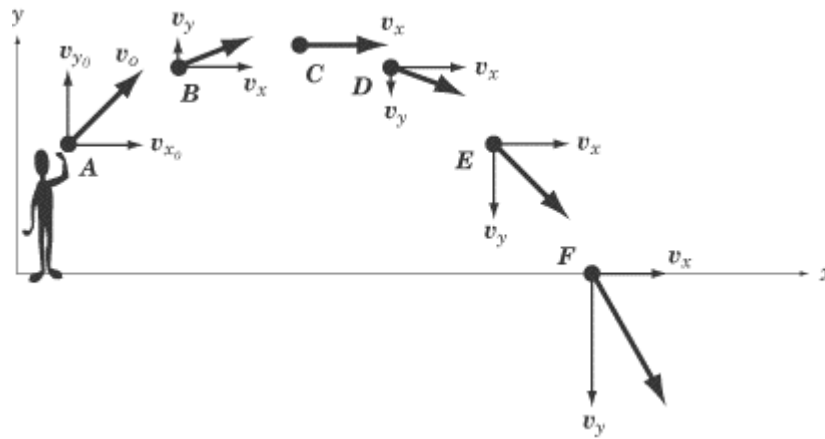
$$\theta = \sin^{-1} 0.407 = 24.0^\circ$$

Thus the resultant vector has a magnitude of 38.2 and a direction of  $+24.0^\circ$  (or  $24.0^\circ$  above the horizontal).

Use this space for summary and/or additional notes.

One type of physics problem that commonly uses vectors in this way is two-dimensional projectile motion. If the motion of the projectile is represented by a vector,  $\vec{v}$ , at angle  $\theta$ , the vector can be represented as the sum of a horizontal vector and a vertical vector. This is useful because the horizontal vector gives us the component (portion) of the vector in the x-direction, and the vertical vector gives us the component of the vector in the y-direction.

For example, in the following diagram, the velocity of the projectile is  $\vec{v}$ . The component of the velocity in the x-direction is  $\vec{v}_x$ , and the component in the y-direction is  $\vec{v}_y$ .



Notice that  $\vec{v}_x$  remains constant, but  $\vec{v}_y$  changes (because of the effects of gravity).

Use this space for summary and/or additional notes.

## Vectors vs. Scalars in Physics

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how vectors are used in an algebra-based physics course

**Language Objectives:**

- Accurately describe and apply the concepts described in this section, including the difference between “vector” and “scalar,” using appropriate academic language.

**Notes:**

In physics, most numbers represent quantities that can be measured or calculated from measurements. Most of the time, there is no concept of a “deficit” of a measured quantity. For example, quantities like mass, energy, and power can only be nonnegative, because in classical mechanics there is no such thing as “anti-mass,” “anti-energy,” or “anti-power.”

However, vector quantities have a direction as well as a magnitude, and direction can be positive or negative.

A rule of thumb that works *most* of the time in this class is:

**Scalar quantities.** These are almost always positive. (Note, however, that we will encounter some exceptions during the year. An example is electric charge, which can be positive or negative.)

**Vector quantities.** Vectors can be positive or negative. In any given problem, you will choose which direction is positive. Vectors in the positive direction will be expressed as positive numbers, and vectors in the opposite (negative) direction will be expressed as negative numbers.

Use this space for summary and/or additional notes:

In some cases, you will need to split a vector in two vectors, one vector in the  $x$ -direction, and a separate vector in the  $y$ -direction. In these cases, you will need to choose which direction is positive and which direction is negative for both the  $x$ - and  $y$ -axes. Once you have done this, every vector quantity must be assigned a positive or negative value, according to the directions you have chosen.

**Example:**

Suppose you have a problem that involves throwing a ball straight upwards with a velocity of  $15 \frac{\text{m}}{\text{s}}$ . Gravity is slowing the ball down with a downward acceleration of  $9.8 \frac{\text{m}}{\text{s}^2}$ . You want to know how far the ball has traveled in 0.5 s.

Displacement, velocity, and acceleration are all vectors. The motion is happening in the  $y$ -direction, so we need to choose whether “up” or “down” is the positive direction. Suppose we choose “up” to be the positive direction. This means:

- When the ball is first thrown, it is moving upwards. This means its velocity is in the positive direction, so we would represent the initial velocity as  $\vec{v}_o = +15 \frac{\text{m}}{\text{s}}$ .
- Gravity is accelerating the ball downwards, which is the negative direction. We would therefore represent the acceleration as  $\vec{a} = -9.8 \frac{\text{m}}{\text{s}^2}$ .
- Time is a scalar quantity, so it can only be positive.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

we would do so as follows:

$$\vec{d} = (+15)(0.5) + \left(\frac{1}{2}\right)(-9.8)(0.5)^2$$

and we would find out that  $\vec{d} = +6.275 \text{ m}$ .

The answer is positive. Earlier, we defined positive as “up”, so the answer tells us that the displacement is upwards from the starting point.

Use this space for summary and/or additional notes.

What if, instead, we had chosen “down” to be the positive direction?

- When the ball is first thrown, it is moving upwards. This means its velocity is now in the negative direction, so we would represent the initial velocity as  $\vec{v}_o = -15 \frac{\text{m}}{\text{s}}$ .
- Gravity is accelerating the ball downwards, which is the positive direction. We would therefore represent the acceleration as  $\vec{a} = +9.8 \frac{\text{m}}{\text{s}^2}$ .
- Time is a scalar quantity, so it can only be positive.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

we would do so as follows:

$$\vec{d} = (-15)(0.5) + \left(\frac{1}{2}\right)(9.8)(0.5)^2$$

and we would find out that  $\vec{d} = -6.275 \text{ m}$ .

The answer is negative. Remember that “down” was positive, which means “up” is the negative direction. This means the displacement is upwards from the starting point, as before.

Remember: in any problem you solve, the choice of which direction is positive vs. negative is arbitrary. The only requirement is that every vector quantity in the problem needs to be consistent with your choice.

Use this space for summary and/or additional notes.

## Vector Multiplication

**Unit:** Mathematics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- dot product & cross product of two vectors

**Language Objectives:**

- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

cross product: multiplication of two vectors that results in a new vector.

tensor product: multiplication of two vectors that results in a tensor. (A tensor is an array of vectors that describes the effect of each vector on each other vector within the array. We will not use tensors in a high school physics course.)

Use this space for summary and/or additional notes:

### Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{d} = \vec{v}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

### The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the “dot product”. Dot product multiplication of vectors is represented with a dot:

$$\vec{A} \bullet \vec{B}^*$$

The dot product of  $\vec{A}$  and  $\vec{B}$  is:

$$\vec{A} \bullet \vec{B} = AB \cos \theta$$

where  $A$  is the magnitude of  $\vec{A}$ ,  $B$  is the magnitude of  $\vec{B}$ , and  $\theta$  is the angle between the two vectors  $\vec{A}$  and  $\vec{B}$ .

For example, in physics, work (a scalar quantity) is the dot product of the vectors force and displacement (distance):

$$W = \vec{F} \bullet \vec{d} = Fd \cos \theta$$

---

\* pronounced “A dot B”

Use this space for summary and/or additional notes.

### The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{A} \times \vec{B}^*$$

The cross product of vectors  $\vec{A}$  and  $\vec{B}$  that have an angle of  $\theta$  between them is given by the formula:

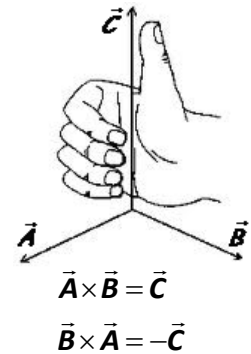
$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

where the magnitude is  $AB \sin \theta$ , and the vector  $\hat{n}$  is the direction. ( $AB \sin \theta$  is a scalar. The unit vector  $\hat{n}$  is what gives the vector its direction.)

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the “right hand rule”:

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector ( $\hat{n}$ ).

Note that this means that the resultant vectors for  $\vec{A} \times \vec{B}$  and  $\vec{B} \times \vec{A}$  will point in *opposite* directions, *i.e.*, the cross product of two vectors is not commutative!



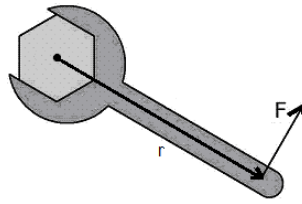

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\* pronounced “A cross B”

Use this space for summary and/or additional notes.



In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force  $\vec{F}$  acting at a radius  $\vec{r}$  is given by the equation:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta \hat{n}$$

Because the direction of the force is usually perpendicular to the displacement, it is usually true that  $\sin\theta = \sin 90^\circ = 1$ . This means the magnitude  $rF \sin\theta = rF(1) = rF$ . Using the right-hand rule, we determine that the *direction* of the resultant torque vector ( $\hat{n}$ ) is coming out of the page.

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

### Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

A:    $\sin\theta \hat{n}$

Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

Use this space for summary and/or additional notes.



## Introduction: Kinematics (Motion)

**Unit:** Kinematics (Motion)

### Topics covered in this chapter:

Motion, Speed & Velocity .....	89
Acceleration .....	94
Solving Motion Problems.....	100
Projectile Motion .....	105

In this chapter, you will study how things move and how the relevant quantities are related.

- *Motion, Speed & Velocity* and *Acceleration* deal with understanding and calculating the velocity (change in position) and acceleration (change in velocity) of an object, and with representing and interpreting graphs involving these quantities.
- *Projectile Motion* deals with an object that has two-dimensional motion—moving horizontally and also affected by gravity.

Some of the challenging tasks include identifying quantities from their units, choosing the equation that relates the quantities of interest, and keeping track of positive and negative directions when working with vector quantities.

### Standards addressed in this chapter:

#### Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

#### Massachusetts Curriculum Frameworks (2006):

- 1.4** Interpret and apply Newton’s three laws of motion.
- 1.5** Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.

Use this space for summary and/or additional notes:

**1.6** Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.

**1.7** Describe Newton's law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

**Kinematics**, such as velocity, acceleration, motion in one dimension, and motion of projectiles

1. Displacement
2. Speed, velocity and acceleration
3. Kinematics with graphs
4. One-dimensional motion with uniform acceleration
5. Two-dimensional motion with uniform acceleration

**Skills learned & applied in this chapter:**

- Choosing from a set of equations based on the quantities present.
- Working with vector quantities.
- Relating the slope of a graph and the area under a graph to equations.
- Using graphs to represent and calculate quantities.
- Keeping track of things happening in two directions at once.

Use this space for summary and/or additional notes.

## Motion, Speed & Velocity

**Unit:** Kinematics (Motion)

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.1, 1.2

### Knowledge/Understanding Goals:

- understand terms relating to position, speed & velocity
- understand the difference between speed and velocity

### Language Objectives:

- Understand and correctly use the terms “position,” “distance,” “displacement,” “speed,” and “velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

### Notes:

coördinate system: a framework for describing an object's position (location), based on its distance (in one or more directions) from a specifically-defined point (the origin). (You should remember these terms from math.)

direction: which way an object is oriented or moving within its coördinate system. Note that direction can be positive or negative.

position ( $s$ ): the location of an object relative to the origin (zero point) of its coördinate system. We will consider position to be a zero-dimensional vector, which means it can be positive or negative with respect to the chosen coördinate system.

distance ( $d$ ): [scalar] how far an object has moved.

displacement ( $\vec{d}$ ): [vector] how far an object's current position is from its starting position (“initial position”). Displacement can be positive or negative (or zero), depending on the chosen coördinate system.

rate: the change in a quantity over a specific period of time.

motion: when an object's position is changing over time.

Use this space for summary and/or additional notes:

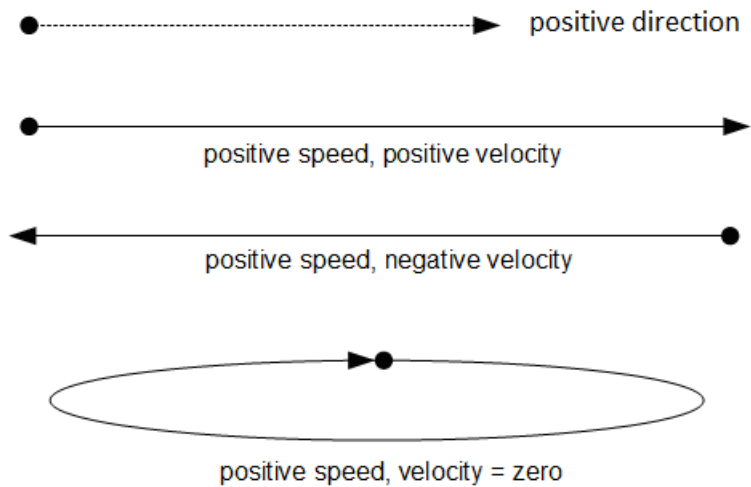
speed: [scalar] the rate at which an object is moving at an instant in time. Speed does not depend on direction, and is always nonnegative.

velocity: ( $\vec{v}$ ) [vector] an object's displacement over a given period of time.

Because velocity is a vector, it has a direction as well as a magnitude.  
Velocity can be positive, negative, or zero.

uniform motion: motion at a constant velocity (*i.e.*, with constant speed and direction)

An object that is moving has a positive speed, but its velocity may be positive, negative, or zero, depending on its position.



Use this space for summary and/or additional notes.

Variables Used to Describe Motion

Variable	Quantity	MKS Unit
$s, x$	position	m
$d, \vec{d}$	distance, displacement	m
$h$	height	m
$\vec{v}$	velocity	$\frac{m}{s}$
$\bar{v}$	average velocity	$\frac{m}{s}$

The average velocity of an object is its displacement divided by the time, or its change in position divided by the (change in) time:

$$\bar{v} = \frac{\vec{d}}{t} = \frac{s - s_0}{t} = \frac{\Delta s}{t} = \frac{\Delta s}{\Delta t}$$

We can use calculus to turn  $\bar{v}$  into  $v$  by taking the limit as  $\Delta t$  approaches zero:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

*i.e.*, velocity is the first derivative of displacement with respect to time.

We can rearrange this formula to show that displacement is average velocity times time:

$$\vec{d} = \bar{v}t$$

Position is the object's starting position plus its displacement:

$$s = s_0 + \vec{d} = s_0 + \bar{v}t$$

where  $s_0^*$  means "position at time = 0". This formula is often expressed as:

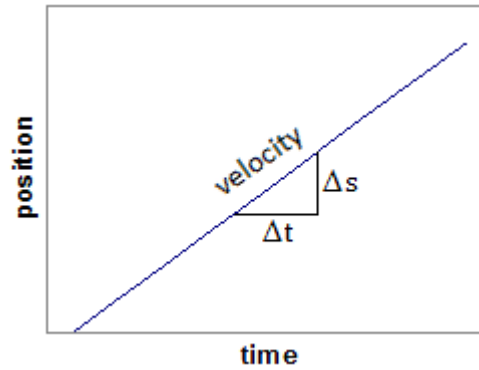
$$s - s_0 = \vec{d} = \bar{v}t$$

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\*  $s_0$  is pronounced "s-zero" or "s-naught".

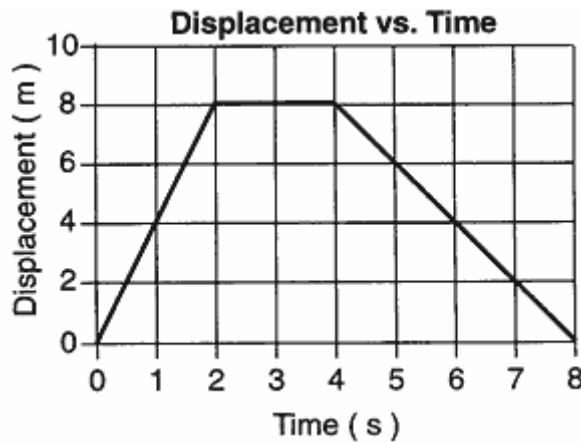
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Note that  $\frac{\Delta s}{\Delta t}$  is the slope of a graph of position (s) vs. time (t). Because  $\frac{\Delta s}{\Delta t} = v$ , this means that the slope of a graph of position vs. time is equal to the velocity.



This is true of any graph. The slope of the line is  $\frac{\Delta y}{\Delta x}$ , which means the quantity defined by  $\frac{y\text{-axis}}{x\text{-axis}}$  will always be the slope.

Recall that velocity is a vector, which means it can be positive, negative, or zero. On the graph below, the velocity is  $+4 \frac{m}{s}$  from 0 s to 2 s, zero from 2 s to 4 s, and  $-2 \frac{m}{s}$  from 4 s to 8 s.



Use this space for summary and/or additional notes.



**Sample problems:**

Q: A car travels 1200 m in 60 seconds. What is its average velocity?

A:  $\bar{v} = \frac{d}{t}$

$$\bar{v} = \frac{1200 \text{ m}}{60 \text{ s}} = 20 \frac{\text{m}}{\text{s}}$$

Q: A person walks 320 m at an average velocity of  $1.25 \frac{\text{m}}{\text{s}}$ . How long did it take?

A: "How long" means what length of time.

$$\bar{v} = \frac{d}{t}$$

$$1.25 = \frac{320}{t}$$

$$t = 256 \text{ s}$$

It took 256 seconds for the person to walk 320 m.

Use this space for summary and/or additional notes.

## Acceleration

**Unit:** Kinematics (Motion)

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.1, 1.2

### Knowledge/Understanding Goals:

- what acceleration means
- what positive vs. negative acceleration means

### Skills:

- calculate position, velocity and acceleration for problems that involve movement in one direction

### Language Objectives:

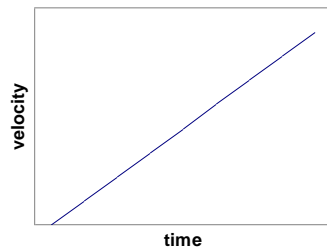
- Understand and correctly use the term “acceleration.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

### Notes:

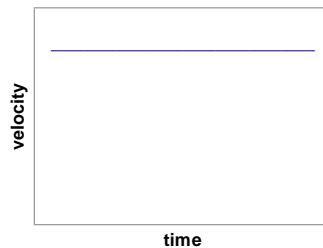
acceleration: a change in velocity over a period of time.

If an object is “speeding up,” we say it has positive acceleration.

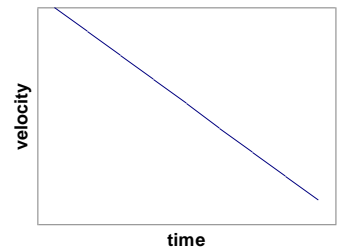
If an object is “slowing down,” we say it has negative acceleration.



positive acceleration



acceleration = zero



negative acceleration

uniform acceleration: when an object’s rate of acceleration (*i.e.*, the rate at which its velocity changes) is constant.

Use this space for summary and/or additional notes:

**Variables Used to Describe Acceleration**

Variable	Quantity	MKS Units
$\vec{a}$	acceleration	$\frac{m}{s^2}$
$\vec{g}$	acceleration due to gravity	$\frac{m}{s^2}$

By convention, physicists use the variable  $\vec{g}$  to mean acceleration due to gravity, and  $\vec{a}$  to mean acceleration caused by something other than gravity.

Because acceleration is a change in velocity over a period of time, the formula for acceleration is:

$$\vec{a} = \frac{v - v_o}{t} = \frac{\Delta v}{t} = \frac{\Delta v}{\Delta t} \quad \text{and, from calculus:} \quad a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The units must match the formula, which means the units for acceleration must be velocity (distance/time) divided by time, which equals distance divided by time squared.

Because  $v = \frac{ds}{dt}$ , this means that acceleration is the second derivative of position

with respect to time:  $a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$

*i.e.*, acceleration is the second derivative of displacement with respect to time. However, in an algebra-based physics course, we will limit ourselves to problems in which acceleration is constant.

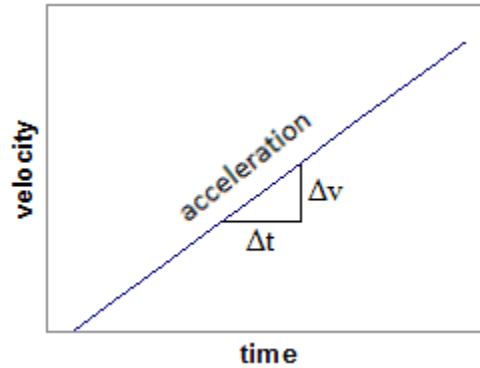
We can rearrange this formula to show that the change in velocity is acceleration times time:

$$\Delta v = v - v_o = at$$

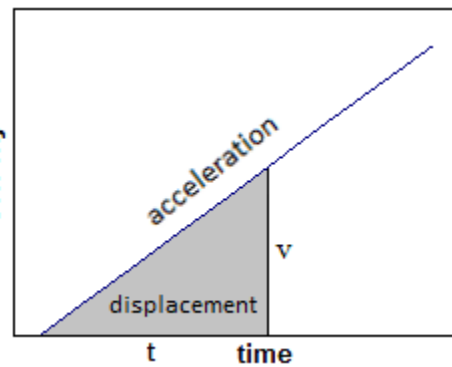
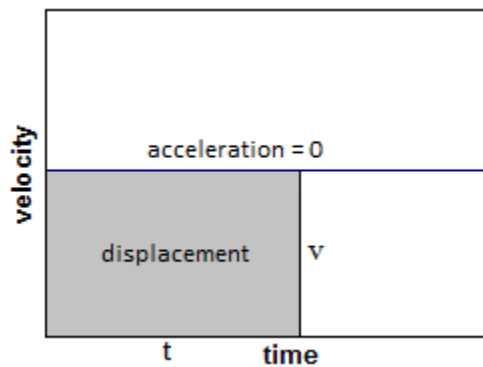
Note that when an object's velocity is changing, the final velocity,  $v$ , is not the same as the average velocity,  $\bar{v}$ . (This is a common mistake that first-year physics students make.)

Use this space for summary and/or additional notes.

$\frac{\Delta v}{\Delta t}$  is the slope of a graph of velocity ( $v$ ) vs. time ( $t$ ). Because  $\frac{\Delta v}{\Delta t} = a$ , this means that acceleration is the slope of a graph of velocity vs. time:



Note also that  $\bar{v}t$  is the area under a graph of velocity ( $v$ ) vs. time ( $t$ ). Because  $\bar{v}t = d$ , this means the area under a graph of velocity vs. time is the displacement. Note that this also works both for constant velocity (the graph on the left) and changing velocity (as shown in the graph on the right).



In calculus, the area under a curve is the integral. This means:

$$d = \int_0^t v dt$$

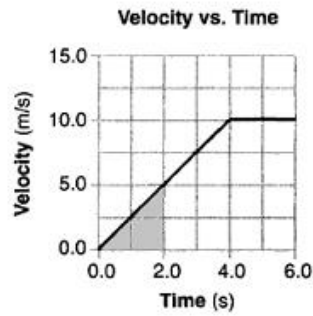
where  $v$  can be any function of  $t$ .

Use this space for summary and/or additional notes.

In the graph below, between 0 s and 4 s the object is accelerating at a rate of  $+2.5 \frac{m}{s^2}$ .

Between 4 s and 6 s the object is moving at a constant velocity (of  $+10 \frac{m}{s}$ ), so the acceleration is zero.

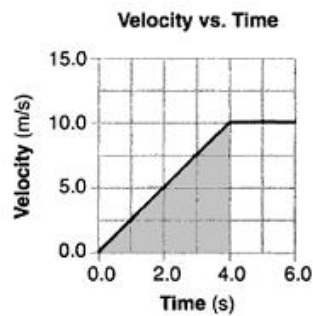
The area under a velocity/time graph equals the total distance traveled.



$$a = 2.5 \frac{m}{s^2}$$

$$d = \frac{1}{2}(2.5)(2^2) = 5 \text{ m}$$

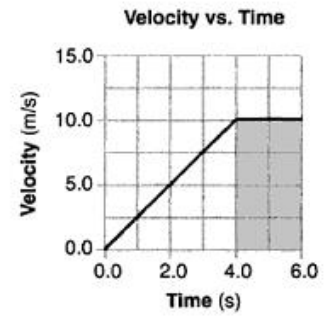
$$A = \frac{1}{2}(2)(5) = 5 \text{ m}$$



$$a = 2.5 \frac{m}{s^2}$$

$$d = \frac{1}{2}(2.5)(4^2) = 20 \text{ m}$$

$$A = \frac{1}{2}(4)(10) = 20 \text{ m}$$



$$a = 0$$

$$d = \bar{v}t = (10)(2) = 20 \text{ m}$$

$$A = (2)(10) = 20 \text{ m}$$

Use this space for summary and/or additional notes.

To show the relationship between  $v$  and  $\bar{v}$ , we can combine the formula for average velocity with the formula for acceleration in order to get a formula for the position of an object that is accelerating.

$$d = \bar{v}t$$

$$v = at$$

However, the problem is that  $v$  in the formula  $v = at$  is the velocity at the *end*, which is not the same as the *average* velocity  $\bar{v}$ .

If the velocity of an object is changing (*i.e.*, the object is accelerating), the average velocity is given by the formula:

$$\bar{v} = \frac{v_o + v}{2}$$

If the object starts at rest (not moving, which means  $v_o = 0$ ) and it accelerates at a constant rate, the average velocity is therefore

$$\bar{v} = \frac{v_o + v}{2} = \frac{0 + v}{2} = \frac{v}{2} = \frac{1}{2}v$$

Combining all of these gives, for an object starting from rest:

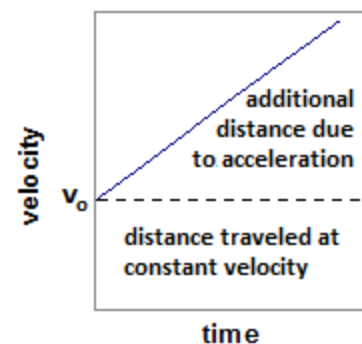
$$d = \bar{v}t = \frac{1}{2}vt = \frac{1}{2}(at)t = \frac{1}{2}at^2$$

If an object was moving before it started to accelerate, it had an initial velocity, or a velocity at time = 0. We will represent this initial velocity as  $v_o^*$ . Now, the formula becomes:

$$s - s_o = d = v_o t + \frac{1}{2}at^2$$

distance the object  
would travel at its  
initial velocity

*additional* distance  
the object will travel  
because it is  
accelerating



\* pronounced "v-zero" or "v-naught"

Use this space for summary and/or additional notes.

This equation can be combined with the equation for velocity to give the following equation, which relates initial and final velocity and distance:

$$v^2 - v_o^2 = 2ad$$

Finally, when an object is accelerating because of gravity, we say that the object is in “free fall”.

On earth, the average acceleration due to gravity is approximately  $9.807 \frac{\text{m}}{\text{s}^2}$  at sea level (which we will usually round to  $9.8 \frac{\text{m}}{\text{s}^2}$ , or sometimes just  $10 \frac{\text{m}}{\text{s}^2}$ ). Any time gravity is involved (and the problem takes place on Earth), assume that  $a = g = 9.8 \frac{\text{m}}{\text{s}^2}$ .

Use this space for summary and/or additional notes.

## Solving Motion Problems

**Unit:** Kinematics (Motion)

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.2

**Skills:**

- solve problems involving motion in two dimensions

**Language Objectives:**

- Set up and solve word problems relating to motion.

**Notes:**

We have learned five equations for solving motion problems. They are:

Equation	Comments
$\vec{d} = \Delta s = s - s_o$	Definition of displacement.
$\bar{\vec{v}} = \frac{\vec{d}}{t} = \frac{\Delta s}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Used to calculate <u>average</u> velocity. Note that you can't use $\bar{v}$ if there is acceleration (the velocity is changing).
$\Delta \vec{v} = \vec{v} - \vec{v}_o = \vec{a}t$	Relates velocity ( $v$ ), initial velocity ( $v_o$ ), acceleration ( $a$ ), and time ( $t$ ).
$s - s_o = \vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a}t^2$	Relates distance or displacement ( $d$ ), initial velocity ( $v_o$ ), acceleration ( $a$ ), and time ( $t$ ).
$\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}d$	Relates distance or displacement ( $d$ ), velocity ( $v$ ), initial velocity ( $v_o$ ), and acceleration ( $a$ ).

Note that vector quantities (shown in bold with an arrow) can be positive or negative, depending on direction.

Use this space for summary and/or additional notes:



### Selecting the Right Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation must contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
  - If an object starts at rest (not moving), then  $\vec{v}_o = 0$ .
  - If an object comes to a stop, then  $\vec{v} = 0$
  - If gravity is involved (e.g., the object is falling),  $\vec{a} = \vec{g} = 9.8 \frac{\text{m}}{\text{s}^2}$

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

Use this space for summary and/or additional notes.

### Additional Strategies

Motion problems in physics often involve gravity. These problems usually fall into one of two categories:

1. If you have an object in free fall, the problem will probably give you either the distance it fell ( $d$ ), or the time it fell ( $t$ ). Use the formula  $\vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a} t^2$  to calculate whichever one you don't know. (If the object starts from rest, that means  $\vec{v}_o = 0$ .)
2. If an object is thrown upwards, it will decelerate at a rate of  $-9.8 \frac{m}{s^2}$  (assuming "up" is the positive direction) until it stops moving ( $\vec{v} = 0$ ). Then it will fall. This means you need to split the problem into two parts:
  - a. When the object is moving upward, the initial velocity,  $\vec{v}_o$ , is usually given and  $\vec{v}$  (at the top) = zero. From these, you can use  $\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}d$  to figure out the distance it traveled, which gives you the maximum height.
  - b. Once you know the maximum height, you know the distance to the ground,  $\vec{v}_o = 0$ , and you can use  $\vec{d} = \frac{1}{2} \vec{g} t^2$  (this time with an acceleration of  $+9.8 \frac{m}{s^2}$ ) to find the time it spends falling. The total time (up + down) will be twice as much.

Use this space for summary and/or additional notes.

**Sample Problems:**

Q: If a cat jumps off a 1.8 m tall refrigerator, how long does it take to hit the ground?



A: The problem gives us  $d = 1.8$  m. The cat is starting from rest ( $v_a = 0$ ), and gravity is accelerating the cat at a rate of  $a = g = 9.8 \frac{\text{m}}{\text{s}^2}$ . We need to find  $t$ .

Looking at the equations, the one that has what we need ( $t$ ) and only quantities we know is:

$$d = v_o t + \frac{1}{2} a t^2$$

$v_o = 0$ , so this reduces to:

$$d = \frac{1}{2} a t^2$$

$$1.8 = (\frac{1}{2})(9.8) t^2$$

$$\frac{1.8}{4.9} = 0.367 = t^2$$

$$t = \sqrt{0.367} = 0.61 \text{ s}$$

Use this space for summary and/or additional notes.

Q: An apple falls from a tree branch at a height of 5 m and lands on Isaac Newton's head. (Assume Isaac Newton was 1.8 m tall.)



How fast was the apple traveling at the time of impact?

A: We know  $d = s - s_o = 5 - 1.8 = 3.2$  m. We also know that the apple is starting from rest ( $v_a = 0$ ), and gravity is accelerating the apple at a rate of  $a = g = 9.8 \frac{\text{m}}{\text{s}^2}$ . We want to find  $v$ .

The equation that relates all of our variables is:

$$v^2 - v_o^2 = 2ad$$

Substituting, we get:

$$v^2 - 0 = (2)(9.8)(3.2)$$

$$v^2 = 62.7$$

$$v = \sqrt{62.7} = 7.9 \frac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes.

## Projectile Motion

**Unit:** Kinematics (Motion)

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.2

**Skills:**

- solve problems involving motion in two dimensions

**Language Objectives:**

- Understand and correctly use the term “projectile.”
- Set up and solve word problems involving projectiles.

**Notes:**

projectile: an object that is propelled (thrown, shot, *etc.*) horizontally and also falls due to gravity.

Gravity affects projectiles the same way regardless of whether the projectile is moving horizontally. Gravity does not affect the horizontal motion of the projectile. This means the vertical and horizontal motion of the projectile can be considered separately, using a separate set of equations for each.

Assuming we can neglect friction and air resistance (which is usually the case in first-year physics problems), we make two important assumptions:

- All projectiles have a constant horizontal velocity,  $v_h$ , in the positive horizontal direction. The equation for the horizontal motion is:

$$d_h = v_h t$$

- All projectiles have a constant downward acceleration of  $g = 9.8 \frac{m}{s^2}$  (in the vertical direction), due to gravity. (You can choose whether the positive vertical direction is up or down, depending on the situation.) The equation for the vertical motion is:

$$d_v = v_{o,v} t + \frac{1}{2} g t^2$$

- The time that the projectile spends falling must be the same as the time that the projectile spends moving horizontally. This means time ( $t$ ) is the same in both equations, which means time is the variable that links the vertical problem to the horizontal problem.

Use this space for summary and/or additional notes.

The consequences of these assumptions are:

- The *time* that the object takes to fall is determined by its movement only in the vertical direction.
- The *horizontal distance* that the object travels is determined by the time (calculated above) and its velocity in the horizontal direction.

Therefore, the general strategy for most projectile problems is:

1. Solve the vertical problem first, to get the time.
2. Use the time from the vertical problem to solve the horizontal problem.

**Sample problem:**

Q: A ball is thrown horizontally at a velocity of  $5 \frac{m}{s}$  from a height of 1.5 m. How far does the ball travel (horizontally)?

A: We're looking for the horizontal distance,  $d_h$ . We know the vertical distance,  $d_v = 1.5 \text{ m}$ , and we know that  $v_{o,v} = 0$  (there is no initial vertical velocity because the ball is thrown horizontally), and we know that  $a = g = 9.8 \frac{m}{s^2}$ .

We need to separate the problem into the horizontal and vertical components.

Horizontal:

$$d_h = v_h t$$

$$d_h = 5 t$$

At this point we can't get any farther, so we need to turn to the vertical problem.

Vertical:

$$d_v = v_{o,v} t + \frac{1}{2} g t^2$$

$$d_v = \frac{1}{2} g t^2$$

$$1.5 = (\frac{1}{2})(9.8) t^2$$

$$\frac{1.5}{4.9} = 0.306 = t^2$$

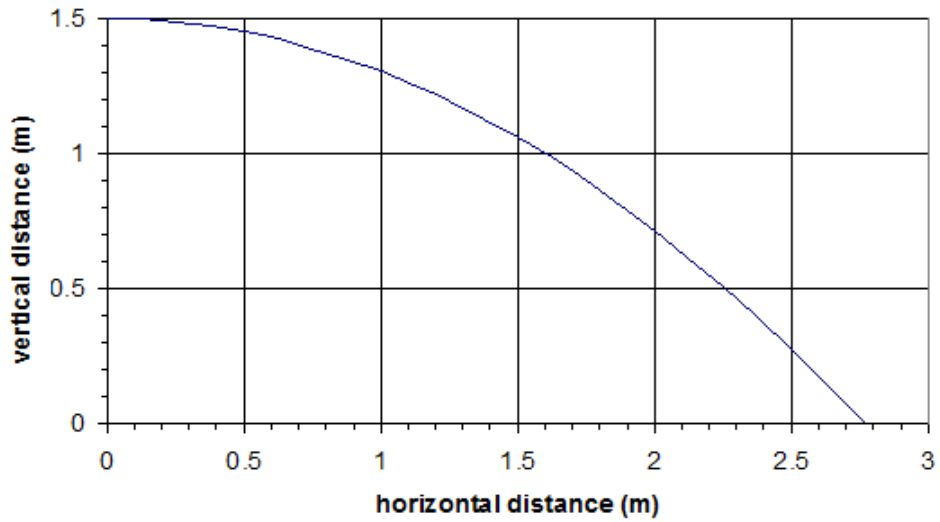
$$t = \sqrt{0.306} = 0.55 \text{ s}$$

Now that we know the time, we can substitute it back into the horizontal equation, giving:

$$d_h = (5)(0.55) = 2.77 \text{ m}$$

Use this space for summary and/or additional notes.

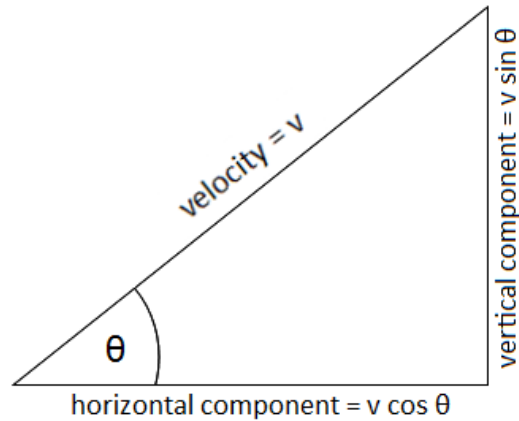
A graph of the vertical vs. horizontal motion of the ball looks like this:



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### Projectiles Launched at an Angle

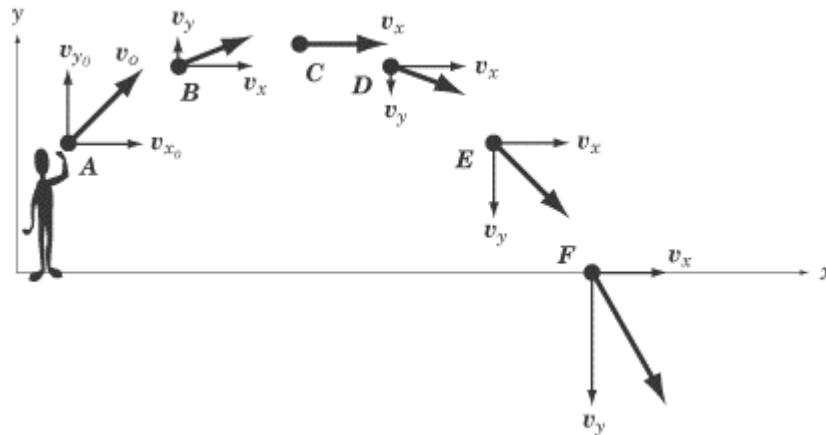
If the object is thrown/launched at an angle, you will need to use trigonometry to separate the velocity vector into its horizontal ( $x$ ) and vertical ( $y$ ) components:



Thus:

- horizontal velocity =  $v_h = v \cos \theta$
- *initial* vertical velocity =  $v_{o,v} = v \sin \theta$

Note that the vertical component of the velocity,  $v_y$ , is constantly changing because of acceleration due to gravity.



Use this space for summary and/or additional notes.



**Sample Problems:**

Q: An Angry Bird is launched upward from a slingshot at an angle of  $40^\circ$  with a velocity of  $20 \frac{m}{s}$ . The bird strikes the pigs' fortress at the same height that it was launched from. How far away is the fortress?

A: We are looking for the horizontal distance,  $d_h$ .

We know the magnitude and direction of the launch, so we can find the horizontal and vertical components of the velocity using trigonometry:

$$v_h = v \cos \theta = 20 \cos 40^\circ = (20)(0.766) = 15.3 \frac{m}{s}$$

$$v_{o,v} = v \sin \theta = 20 \sin 40^\circ = (20)(0.643) = 12.9 \frac{m}{s}$$

We call the vertical component  $v_{o,v}$  because it is both the initial velocity ( $v_o$ ) and the vertical velocity ( $v_v$ ), so we need both subscripts.

Let's make upward the positive vertical direction.

We want the horizontal distance ( $d_h$ ), which is in the horizontal equation, so we start with:

$$d_h = v_h t$$

$$d_h = 15.3 t$$

At this point we can't get any farther because we don't know the time, so we need to get it by solving the vertical equation.

The Angry Bird lands at the same height as it was launched, which means the vertical displacement ( $d_v$ ) is zero. We already calculated that the initial vertical velocity is  $12.9 \frac{m}{s}$ . If upward is the positive direction, acceleration due to gravity needs to be negative (because it's downward), so  $a = g = -9.8 \frac{m}{s^2}$ .

Use this space for summary and/or additional notes.

The vertical equation is:

$$d_v = v_o t + \frac{1}{2} a t^2$$

$$0 = 12.9 t + \left(\frac{1}{2}\right)(-9.8) t^2$$

$$0 = 12.9 t - 4.9 t^2$$

$$0 = t(12.9 - 4.9 t)$$

$$t = 0, \quad 12.9 - 4.9 t = 0$$

$$12.9 = 4.9 t$$

$$t = \frac{12.9}{4.9} = 2.6 \text{ s}$$

Finally, we return to the horizontal equation to find  $d_h$ .

$$d_h = 15.3 t$$

$$d_h = (15.3)(2.6) = 15.3 \text{ m}$$

Q: A ball is thrown upward at an angle of  $30^\circ$  from a height of 1 m with a velocity of  $18 \frac{\text{m}}{\text{s}}$ . How far does the ball travel?

A: As before, we are looking for the horizontal distance,  $d_h$ .

Again, we'll make upward the positive vertical direction.

Again we find the horizontal and vertical components of the velocity using trigonometry:

$$v_h = v \cos \theta = 18 \cos 30^\circ = (18)(0.866) = 15.6 \frac{\text{m}}{\text{s}}$$

$$v_{o,v} = v \sin \theta = 18 \sin 30^\circ = (18)(0.5) = 9 \frac{\text{m}}{\text{s}}$$

Starting with the horizontal equation:

$$d_h = v_h t$$

$$d_h = 15.6 t$$

Again, we can't get any farther, so we need to get the time from the vertical problem.

Use this space for summary and/or additional notes.

The ball moves 1 m downwards. Its initial position is  $s_o = +1 \text{ m}$ , and its final position is  $s = 0$ , so we can use the equation:

$$s - s_o = v_o t + \frac{1}{2} a t^2$$

$$0 - 1 = 9 t + \left(\frac{1}{2}\right)(-9.8) t^2$$

$$0 = 1 + 9 t - 4.9 t^2$$

This time we can't factor the equation, so we need to solve it using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-9 \pm \sqrt{9^2 - (4)(-4.9)(1)}}{(2)(-4.9)}$$

$$t = \frac{9 \pm \sqrt{81 + 19.6}}{9.8} = \frac{9 \pm \sqrt{100.6}}{9.8}$$

$$t = \frac{9 \pm 10.03}{9.8} = \frac{19.03}{9.8} = 1.94 \text{ s}$$

Now we can go back to the horizontal equation and use the horizontal velocity ( $15.6 \frac{\text{m}}{\text{s}}$ ) and the time (1.94 s) to find the distance:

$$d_h = v_h t = (15.6)(1.94) = 30.3 \text{ m}$$

Another way to solve the vertical problem is to realize that the ball goes up to its maximum height, then comes back down. The ball starts from a different height than it falls down to, so unfortunately we can't just find the time at the halfway point and double it.

At the maximum height, the vertical velocity is zero. For the ball going up, this gives:

$$v - v_o = at$$

$$0 - 9 = (-9.8) t_{up}$$

$$t_{up} = \frac{-9}{-9.8} = 0.918 \text{ s}$$

Use this space for summary and/or additional notes.

At this point, the ball has reached a height of:

$$s = s_o + v_o t + \frac{1}{2} a t^2$$

$$s = 1 + (9)(0.918) + \left(\frac{1}{2}\right)(-9.8)(0.918)^2$$

$$s = 1 + 8.26 - 4.13 = 5.13 \text{ m}$$

Now the ball falls from its maximum height of 5.13 m to the ground. The time this takes is:

$$d = \frac{1}{2} a t_{down}^2$$

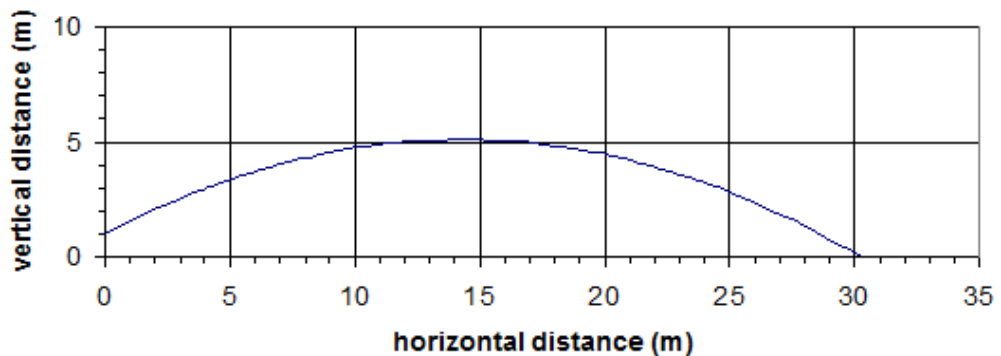
$$5.13 = \left(\frac{1}{2}\right)(9.8) t_{down}^2$$

$$t_{down}^2 = \frac{5.13}{4.9} = 1.05$$

$$t_{down} = \sqrt{1.05} = 1.02 \text{ s}$$

Thus the total elapsed time is  $t_{up} + t_{down} = 0.918 + 1.02 = 1.94 \text{ s}$ . (Q.E.D.)

The motion of this ball looks like this:



Use this space for summary and/or additional notes.

## Introduction: Forces

**Unit:** Forces

### Topics covered in this chapter:

Newton's Laws of Motion .....	116
Forces .....	118
Free-Body Diagrams.....	124
Newton's Second Law .....	128
Force Applied at an Angle .....	133
Ramp Problems.....	140
Pulleys & Tension .....	143
Friction .....	146
Aerodynamic Drag .....	151
Universal Gravitation .....	153

In this chapter you will learn about different kinds of forces and how they relate.

- *Newton's Laws and Forces* describe basic scientific principles of how objects affect each other.
- *Free-Body Diagrams* describes a way of drawing a picture that represents forces acting on an object.
- *Forces Applied at an Angle, Ramp Problems, and Pulleys & Tension* describe some common situations involving forces and how to calculate the forces involved.
- *Friction and Aerodynamic Drag* describe situations in which a force is created by the action of another force.
- *Newton's Law of Universal Gravitation* describes how to calculate the force of gravity caused by massive objects such as planets and stars.

Use this space for summary and/or additional notes.

One of the first challenges will be working with variables that have subscripts. Each type of force uses the variable  $F$ . Subscripts will be used to keep track of the different kinds of forces. This chapter also makes extensive use of vectors.

Another challenge in this chapter will be to “chain” equations together to solve problems. This involves finding the equation that has the quantity you need, and then using a second equation to find the quantity that you are missing from the first equation.

**Standards addressed in this chapter:**

**Next Generation Science Standards (NGSS):**

**HS-PS2-1.** Analyze data to support the claim that Newton’s second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

**HS-PS2-3.** Apply scientific and engineering ideas to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

**HS-PS2-4.** Use mathematical representations of Newton’s Law of Gravitation and Coulomb’s Law to describe and predict the gravitational and electrostatic forces between objects.

**Massachusetts Curriculum Frameworks (2006):**

**1.4** Interpret and apply Newton’s three laws of motion.

**1.5** Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.

**1.6** Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.

**1.7** Describe Newton’s law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.

Use this space for summary and/or additional notes.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Dynamics**, such as force, Newton's laws, statics, and friction.
- **Gravity**, such as the law of gravitation, ~~orbits, and Kepler's laws.~~

1. What are Forces?
2. Types of Forces
3. Newton's Laws
4. Problem Solving With Newton's Laws
5. Pulleys
6. Inclined Planes
7. Newton's Law of Universal Gravitation
8. Weightlessness

**Skills learned & applied in this chapter:**

- Solving chains of equations.
- Using trigonometry to extract a vector in a desired direction.
- Working with material-specific constants from a table.
- Estimating the effect of changing one variable on another variable in the same equation.

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## Newton's Laws of Motion

**Unit:** Forces

**NGSS Standards:** HS-PS2-1

**MA Curriculum Frameworks (2006):** 1.4

**Knowledge/Understanding Goals:**

- Newton's three laws of motion

**Language Objectives:**

- Understand and correctly use the terms "at rest," "in motion," and "force."
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to forces.

**Notes:**

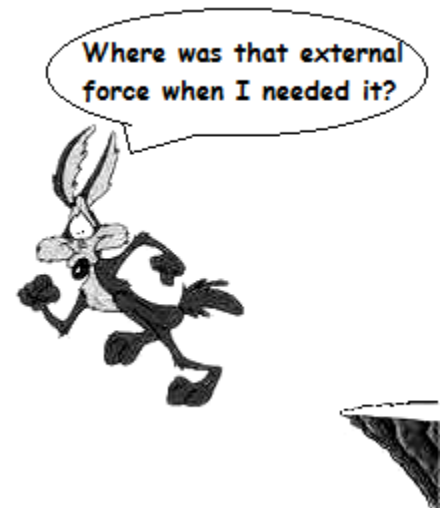
force: a push or pull on an object.

**Newton's First Law:** (the law of inertia)

Everything keeps doing what it was doing unless a force acts to change it. "An object at rest remains at rest, unless acted upon by a net force. An object in motion remains in motion, unless acted upon by a net force."

For example, a brick sitting on the floor will stay at rest on the floor forever unless an outside force moves it.

Wile E. Coyote, on the other hand, remains in motion...



Use this space for summary and/or additional notes.



**Newton's Second Law:** Forces cause acceleration (a change in velocity). "A net force,  $\vec{F}$ , acting on an object causes the object to accelerate in the direction of the net force." In equation form:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

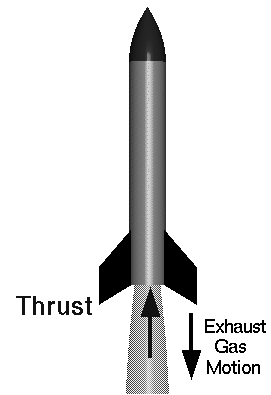
Newton's second law is probably the most important concept regarding forces.

- If there is a net force, the object accelerates (its velocity changes). If the object's velocity is changing (*i.e.*, if it accelerates), then there must be a net force acting on it.
- If there is no net force, the object's velocity will not change (Newton's First Law). If the object's velocity is not changing (*i.e.*, if it is not accelerating), there must be no net force. (If there are no net forces, either there are no forces acting on the object at all, or all of the forces on the object are equal and opposite, and their effects cancel.)

**Newton's Third Law:** Every force involves two objects. The first object exerts a force on the second, and the second object exerts the same force back on the first. "For every action, there is an equal and opposite reaction."

Burning fuel in a rocket causes exhaust gases to escape from the back of the rocket. The force from the gases exiting (the action) applies thrust to the rocket (the reaction), which propels the rocket forward.

If you punch a hole in a wall and break your hand in the process, your hand applied the force that broke the wall (the action). This caused a force from the wall, which broke your hand (the reaction). This may seem obvious, though you will find that someone who has just broken his hand by punching a hole in a wall is unlikely to be receptive to a physics lesson!



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## Forces

**Unit:** Forces

**NGSS Standards:** HS-PS2-1

**MA Curriculum Frameworks (2006):** 1.5

**Knowledge/Understanding Goals:**

- what force is
- net force
- types of forces

**Skills:**

- identify the forces acting on an object

**Language Objectives:**

- Understand and correctly use the terms “force,” “normal force,” “contact force,” “opposing force,” and “weight.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

force: (vector) a push or pull on an object.



weight: the force of gravity pulling an object downward. In physics, we represent weight as the vector  $\vec{F}_g$ . Note that from Newton’s second law,  $\vec{F}_g = m\vec{g}$ , which means on Earth,  $\vec{F}_g = m(9.8)$ .

opposing force: a force in the opposite direction of another force that reduces the effect of the original force.

net force: the overall force on an object after opposing forces cancel out.

contact force: a opposing force that exists only while another force is acting on an object. Examples include friction and the normal force.

normal force: a force exerted by a surface (such as the ground or a wall) that resists the force of gravity on an object.

Use this space for summary and/or additional notes.

Because force is a vector, two forces in opposite directions counteract each other; if the magnitudes are equal, the forces completely cancel. For example, in the following picture:



the forces acting on the bird are:

- gravity, which is pulling the bird down, and
- the force from the ground, which is pushing the bird up.

The force from the ground is called the normal force. (The term “normal” is borrowed from math and means “perpendicular”.)

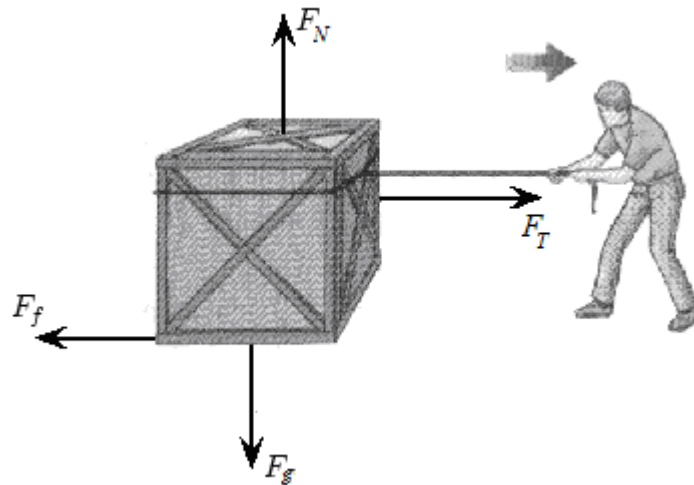
The normal force is the force exerted by a surface (such as the ground, a table, or a wall) that counteracts another force on the object. The normal force is called a contact force, because it is caused by the action of another force, and it exists *only* while the objects are in contact. The normal force is also an opposing force because it acts in the opposite direction from the applied force, and acts to lessen or diminish the applied force.

For example, if you push on a wall with a force of 10 N and the wall doesn't move, that means the force you apply to the wall causes a normal force of 10 N pushing back from the wall. This normal force continues for as long as you continue pushing.

friction: Like the normal force, friction is also both a contact force (caused by the action of another force) that opposes the force that causes it. The direction is parallel to the surfaces that are in contact, and opposite to the direction of motion.

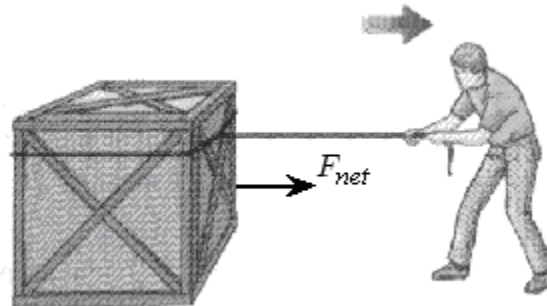
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An object can have several forces acting on it at once:



On the box in the above diagram, the forces are gravity ( $\vec{F}_g$ ), the normal force ( $\vec{F}_N$ ), the tension in the rope ( $\vec{F}_T$ ), and friction ( $\vec{F}_f$ ). Notice that in this problem, the arrow for tension is longer than the arrow for friction, because the force of tension is stronger than the force of friction.

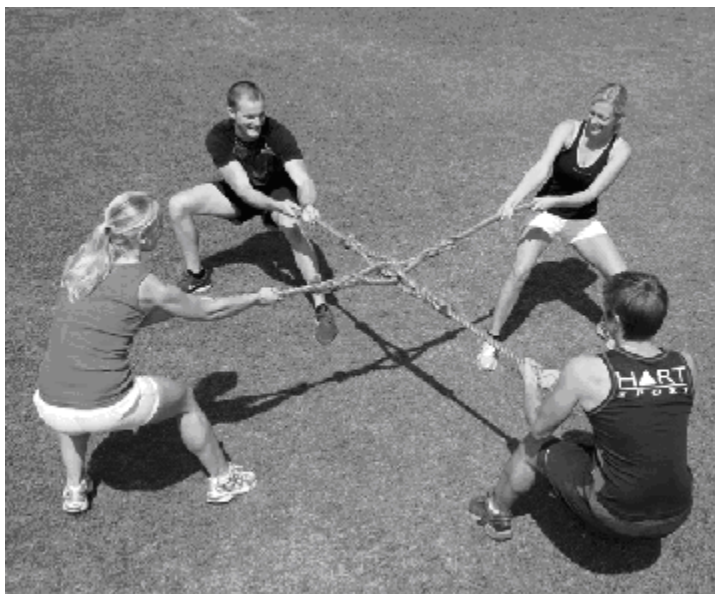
net force: the remaining force on an object after canceling opposing forces. The net force on the box (after canceling out gravity and the normal force, and subtracting friction from the tension) would be represented as:



Because there is a net force to the right, the box will accelerate to the right as a result of the force.

Use this space for summary and/or additional notes.

You can think of forces as participants in a multi-direction tug-of-war:



In the above situation, the net force is in the direction that the ropes will move.

**Forces cause acceleration.** If a net force acts on an object, it will speed up, slow down or change direction. Remember that *if the object's velocity is not changing, there is no net force, which means all of the forces on the object cancel.*

In the MKS system, the unit of force is the newton (N). One newton is defined as the amount of force that it would take to cause a 1 kg object to accelerate at a rate of  $1 \frac{\text{m}}{\text{s}^2}$ .

$$1 \text{ N} \equiv 1 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

Because the acceleration due to gravity on Earth is  $9.8 \frac{\text{m}}{\text{s}^2}$ ,  $F = mg$  indicates that a 1 kg mass has a weight on Earth of 9.8 N.

In more familiar terms, one newton is approximately 3.6 ounces, which is the weight of an average-sized apple. One pound is approximately 4.45 N.

Use this space for summary and/or additional notes.

**Common Types of Forces**

Force	Symbol	Definition	Direction
gravity (weight)	$\vec{F}_g$	pull of gravity between two objects with mass, one of which is usually the Earth	between the centers of mass of the objects (usually toward the center of the Earth)
tension	$\vec{F}_T$	pull exerted by a rope/string/cable	away from the object in the direction of the string/rope/cable
normal	$\vec{F}_N$	contact force by a surface on an object	perpendicular to and away from surface
friction	$\vec{F}_f$	contact force that opposes sliding between surfaces	parallel to surface; opposite to direction of applied force
thrust	$\vec{F}_t$	push that accelerates objects such as rockets, planes & cars	in the same direction as acceleration
spring	$\vec{F}_s$	the push or pull exerted by a spring	opposite the displacement of the object
buoyancy	$\vec{F}_B$	the upward force by a fluid on objects less dense than the fluid	opposite to gravity
drag	$\vec{F}_D$	friction caused by the molecules of a fluid as an object moves through it	opposite to the direction of motion
lift	$\vec{F}_\ell$	the upward push (reaction force) by a fluid on an object (such as an airplane wing) moving through it at an angle	opposite to gravity.

Use this space for summary and/or additional notes.

Force	Symbol	Definition	Direction
electrostatic	$\vec{F}_e$	the attraction or repulsion between two objects with an electrical charge	like charges repel; opposite charges attract
magnetic	$\vec{F}_B$	the magnetic attraction or repulsion between two objects	like poles repel; opposite poles attract

You may have noticed that both buoyancy and magnetic force use the same subscript (both are represented as  $\vec{F}_B$ ). This should not cause too much confusion, because there are very few situations in which both of these forces would be applied to the same object. If that were to happen, we would need to change one of the subscripts. (For example, we might use  $\vec{F}_b$  for the buoyant force and  $\vec{F}_B$  for the magnetic force.)

Use this space for summary and/or additional notes.

## Free-Body Diagrams

**Unit:** Forces

**NGSS Standards:** HS-PS2-1

**MA Curriculum Frameworks (2006):** 1.5

**Skills:**

- draw a free-body diagram representing the forces on an object

**Language Objectives:**

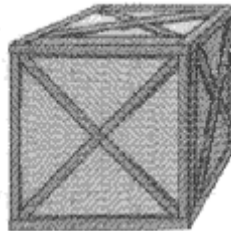
- Understand and correctly use the term “free-body diagram.”
- Identify forces by name and draw a free-body diagram.

**Notes:**

free-body diagram (force diagram): a diagram representing the forces acting on an object.

In a free-body diagram, we represent the object as a dot, and each force as an arrow. The direction of the arrow represents the direction of the force, and the relative lengths of the arrows represent the relative magnitudes of the forces.

Consider a box that is at rest on the floor:



The forces on the box are:

- gravity, which is pulling the box toward the center of the Earth
- the normal force from the floor opposing gravity and holding up the box.

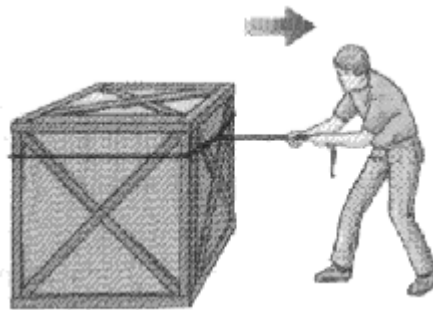
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The free-body diagram for the box looks like this:



Now consider the following situation of a box that accelerates to the right as it is pulled across the floor by a rope:

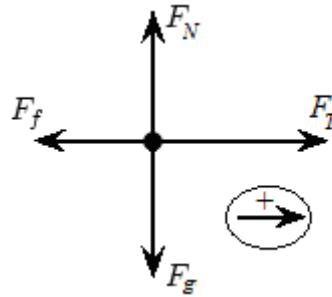


From the picture and description, we can assume that:

- The box has weight, which means gravity is pulling down on it.
- The floor is holding up the box.
- The rope is pulling on the box.
- Friction between the box and the floor is resisting the force from the rope.
- Because the box is accelerating to the right, the force applied by the rope must be stronger than the force from friction.

Use this space for summary and/or additional notes.

In the free-body diagram, we again represent the object (the box) as a point, and the forces (vectors) as arrows. Because there is a net force, we need to include a legend that shows which direction is positive.



In the above diagram, the forces are:

$\vec{F}_g$  = the force of gravity pulling down on the box

$\vec{F}_N$  = the normal force (the floor holding the box up)

$\vec{F}_T$  = the force of tension from the rope. (This might also be designated  $F_a$  because it is the force applied to the object.)

$\vec{F}_f$  = the force of friction resisting the motion of the box.

In the bottom right corner of the diagram, the arrow with the “+” sign shows that we have chosen to make the positive direction to the right.

Notice that the arrows for the normal force and gravity are equal in length, because in this problem, these two forces are equal in magnitude.

Notice that the arrow for friction is shorter than the arrow for tension, because in this problem, the tension is stronger than the force from friction. The difference between the lengths of these two vectors represents the net force, which is what causes the box to accelerate to the right.

Use this space for summary and/or additional notes.

### Steps for Drawing Free-Body Diagrams

In general, the following are the steps for drawing most free-body diagrams.

1. Is gravity involved?
  - Represent gravity as  $\vec{F}_g$  pointing straight down.
2. Is something holding the object up?
  - If it is a flat surface, it is the normal force ( $\vec{F}_N$ ), perpendicular to the surface.
  - If it is a rope, chain, *etc.*, it is the force of tension ( $\vec{F}_T$ ) acting along the rope, chain, *etc.*
3. Is there an external force pulling or pushing on the object?
4. Is there an opposing force?
  - If there are two surfaces in contact, there is almost always friction ( $\vec{F}_f$ ), unless the problem specifically states that the surfaces are frictionless.
  - At low velocities, air resistance is very small and can usually be ignored, even if the problem does not explicitly say so.
  - Usually, all sources of friction are shown as one combined force. *E.g.*, if there is sliding friction along the ground and also air resistance, the  $\vec{F}_f$  vector includes both.
5. Are forces acting in opposing directions?
  - If the problem requires calculations involving opposing forces, you need to indicate which direction is positive.

Use this space for summary and/or additional notes.

## Newton's Second Law

**Unit:** Forces

**NGSS Standards:** HS-PS2-1

**MA Curriculum Frameworks (2006):** 1.4

**Skills:**

- Solve problems relating to Newton's Second Law ( $F = ma$ )
- Solve problems that combine kinematics (motion) and forces

**Language Objectives:**

- Identify and correctly use the quantities involved in a Newton's Second Law problem.
- Identify and correctly use the quantities involved in a problem that combines kinematics and forces.

**Notes:**

Newton's Second Law: Forces cause acceleration (a change in velocity). "A net force,  $\vec{F}$ , acting on an object causes the object to accelerate in the direction of the net force."

If there is a net force, the object accelerates (its velocity changes). If there is no net force, the object's velocity remains the same.

If an object accelerates (its velocity changes), there was a net force on it. If an object's velocity remains the same, there was no net force on it.

Remember that forces are vectors. No net force can mean no forces at all, or it can mean that there were equal forces in opposite directions and their effects cancel.

Use this space for summary and/or additional notes.

In equation form:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

This form is preferred, because acceleration is what results from a force applied to a mass. (*i.e.*, force and mass are the independent variables and acceleration is the dependent variable. Forces cause acceleration, not the other way around.) However, the equation is more commonly written:

$$\vec{F}_{\text{net}} = m\vec{a}$$

### Sample Problems

Most of the physics problems involving forces require the application of Newton's Second Law,  $\vec{F}_{\text{net}} = m\vec{a}$ .

Q: A net force of +50 N is applied to a cart that has a mass of 35 kg. How fast does the cart accelerate?

A: Applying Newton's Second Law:

$$\begin{aligned}\vec{F} &= m\vec{a} \\ +50 &= 35\vec{a} \\ \vec{a} &= +1.43 \frac{\text{m}}{\text{s}^2}\end{aligned}$$

Q: What is the weight of (*i.e.*, the force of gravity acting on) a 10 kg block?

$$\begin{aligned}\text{A: } \vec{F} &= m\vec{g} \\ \vec{F} &= (10.)(+9.8) = +98 \text{ N}\end{aligned}$$

(Remember that we use the variable  $\vec{g}$  instead of  $\vec{a}$  when the acceleration is caused by gravity.)

Use this space for summary and/or additional notes.

### Free Body Diagrams and Newton's Second Law

Free-body diagrams are often used in combination with Newton's second law ( $\vec{F}_{net} = m\vec{a}$ ); the free-body diagram enables you to calculate the net force, from which you can calculate mass or acceleration.

**Sample Problem:**

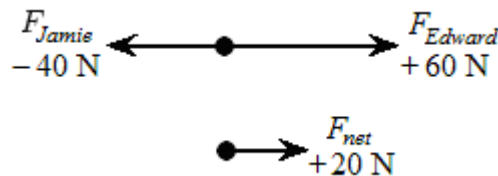
Q: Two children are fighting over a toy.



Jamie pulls to the left with a force of 40 N, and Edward pulls to the right with a force of 60 N. If the toy has a mass of 0.6 kg, what is the resulting acceleration of the toy?

A: Let us decide that the positive direction is to the right. (This is convenient because the force to the right is larger, which means the net force will come out to a positive number.)

The force diagram looks like this:



$$\vec{F}_{net} = m\vec{a}$$

$$+ 20 = (0.6) \vec{a}$$

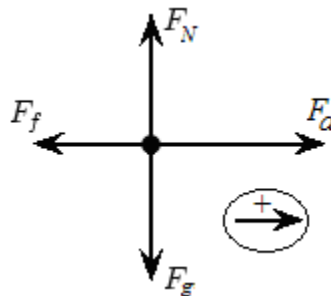
$$\vec{a} = \frac{20}{0.6} = +33.3 \frac{m}{s^2} \text{ (to the right)}$$

Use this space for summary and/or additional notes.

Q: A 5.0 kg block is resting on a horizontal, flat surface. How much force is needed to overcome a force of 2.0 N of friction and accelerate the block from rest to a velocity of  $6.0 \frac{m}{s}$  over a 1.5-second interval?

A: This is a combination of a Newton's second law problem, and a motion problem. We will need a free-body diagram to be able to visualize what's going on.

The free-body diagram for the block looks like this:



1. The net force is given by:

$$F_{net} = F_a - F_f = F_a - 2$$

$$F_a = F_{net} + 2$$

This means we need to find the net force, and then add 2 N to get the applied force.

2. To find the net force, we need the equation:

$$F_{net} = ma$$

3. We know that  $m = 5.0 \text{ kg}$ , but we don't know  $a$ . We need to find  $a$  in order to calculate  $F_{net}$ . For this, we will turn to the motion problem.

4. The problem tells us that  $v_o = 0$ ,  $v = 6.0 \frac{m}{s}$ , and  $t = 1.5 \text{ s}$ . Looking at the motion equations, we see that we have all of the variables except for  $a$  in the equation:

$$v - v_o = at$$

Use this space for summary and/or additional notes.

5. Our strategy is to solve this equation for  $a$ , then substitute into  $F_{net} = ma$  to find  $F_{net}$ , then use the relationship we found from the free-body diagram to find  $F_a$ .

The acceleration is:

$$v - v_o = at$$

$$6.0 - 0 = a(1.5)$$

$$6.0 = 1.5a$$

$$a = 4.0 \frac{\text{m}}{\text{s}^2}$$

Substituting back into  $F_{net} = ma$  gives:

$$F_{net} = ma$$

$$F_{net} = (5.0)(4.0)$$

$$F_{net} = 20. \text{N}$$

Finally:

$$F_a = F_{net} + 2$$

$$F_a = 20 + 2$$

$$F_a = 22 \text{ N}$$

Problems like this are straightforward to solve, but they are challenging because you need to keep chasing the quantities that you don't know until you have enough information to calculate them. However, you need to keep track of each step, because once you have found the last equation you need, you have to follow the steps in reverse order to get back to the answer.

Use this space for summary and/or additional notes.



## Force Applied at an Angle

**Unit:** Forces

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.5

**Skills:**

- calculate forces applied at different angles using trigonometry

**Language Objectives:**

- Set up and solve word problems involving forces applied at an angle.

**Notes:**

An important property of vectors is that a vector has no effect on a second vector that is perpendicular to it. As we saw with projectiles, this means that the velocity of an object in the horizontal direction has no effect on the velocity of the same object in the vertical direction. This allowed us to solve for the horizontal and vertical velocities as separate problems.

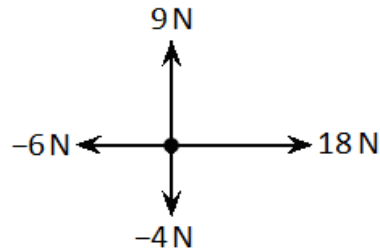
The same is true for forces. If forces are perpendicular to each other, they act independently, and the two can be separated into separate, independent mathematical problems:

$$\vec{F}_{\text{horizontal}} = m\vec{a}_{\text{horizontal}}$$

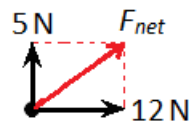
$$\vec{F}_{\text{vertical}} = m\vec{a}_{\text{vertical}}$$

Use this space for summary and/or additional notes.

For example, if we have the following forces acting on an object:



The net horizontal force ( $F_h$ ) would be  $18\text{ N} + (-6\text{ N}) = +12\text{ N}$ , and the net vertical force ( $F_v$ ) would be  $9\text{ N} + (-4\text{ N}) = +5\text{ N}$ . The total net force would be the resultant of the net horizontal and net vertical forces:



Using the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = F_{net}^2$$

$$169 = F_{net}^2$$

$$\sqrt{169} = F_{net} = 13\text{ N}$$

We can get the angle from trigonometry:

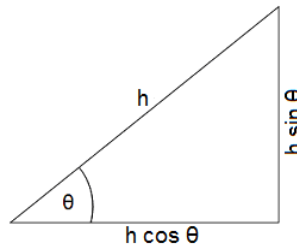
$$\tan\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{12} = 0.417$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.417) = 22.6^\circ$$

Use this space for summary and/or additional notes.

If we have one or more forces that is neither vertical nor horizontal, we can use trigonometry to split the force into a vertical component and a horizontal component. Once all of the forces are either vertical or horizontal, we can add them as above.

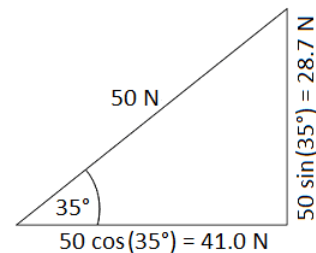
Recall the following relationships from trigonometry:



Suppose we have a force of 50 N at a direction of  $35^\circ$  above the horizontal. In the above diagram, this would mean that  $h = 50$  N and  $\theta = 35^\circ$ :

The horizontal force is  $h \cos(\theta) = 50 \cos(35^\circ) = 41.0$  N

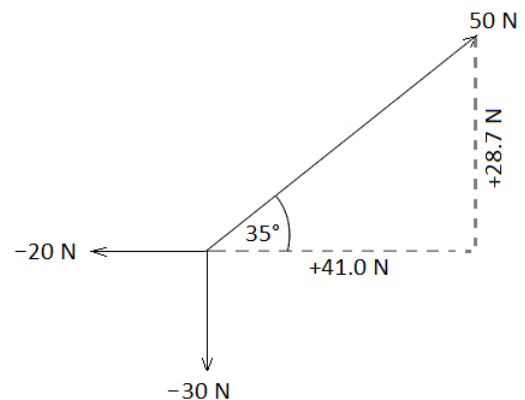
The vertical force is  $h \sin(\theta) = 50 \sin(35^\circ) = 28.7$  N



Now, suppose an object was subjected to the same 50 N force at an angle of  $35^\circ$  above the horizontal, but also a 20 N force to the left and a 30 N force downward.

The net horizontal force would therefore be  $41 + (-20) = 21$  N to the right.

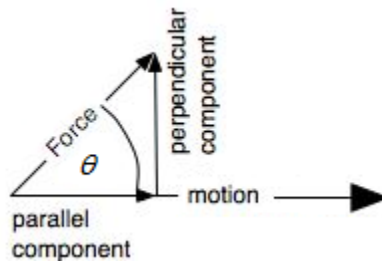
The net vertical force would therefore be  $28.7 + (-30) = -1.3$  N upwards, which is the same as 1.3 N downwards.



Once you have calculated the net vertical and horizontal forces, you can resolve them into a single net force, as in the above example.

Use this space for summary and/or additional notes.

In some physics problems, a force is applied at an angle but the object can move in only one direction. A common problem is a force applied at an angle to an object resting on a flat surface, which causes the object to move horizontally:



In this situation, only the horizontal (parallel) component of the applied force actually causes the object to move. If magnitude of the total force is  $F$ , then the horizontal component of the force is given by:

$$F_h = F \cos \theta$$

If the object accelerates horizontally, that means only the horizontal component is causing the acceleration, which means the net force must be  $F \cos \theta$  and we can ignore the vertical component.

For example, suppose the worker in the diagram at the right pushes on the hand truck with a force of 200 N, at an angle of  $60^\circ$ .

The force in the direction of motion (horizontally) would be:

$$F \cos \theta = 200 \cos (60^\circ) = (200)(0.5) = 100 \text{ N}$$

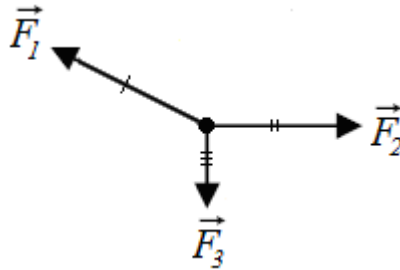
In other words, if the worker applies 200 N of force at an angle of  $60^\circ$ , the resulting horizontal force will be 100 N.



Use this space for summary and/or additional notes.

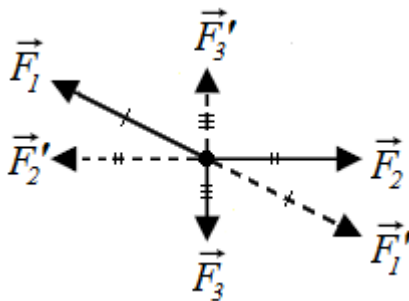
### Static Problems Involving Forces at an Angle

Many problems involving forces at an angle are based on an object with no net force (either a stationary object or an object moving at constant velocity) that has three or more forces acting at different angles. In the following diagram, the forces are  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$ .



$\vec{F}_1$  needs to cancel the combination of  $\vec{F}_2$  and  $\vec{F}_3$ . This means:

1. If we split  $\vec{F}_1$  into its horizontal and vertical components, those components would be exactly opposite  $\vec{F}_2$  and  $\vec{F}_3$ , so we will call them  $\vec{F}_2'$  and  $\vec{F}_3'$ .
2. If we calculate the resultant of vectors  $\vec{F}_2$  and  $\vec{F}_3$ , this would be exactly opposite  $\vec{F}_1$ , so we will call this vector  $\vec{F}_1'$ .



As you can see, each vector is canceled by the resultant of the other two vectors, which shows why there is no net force.

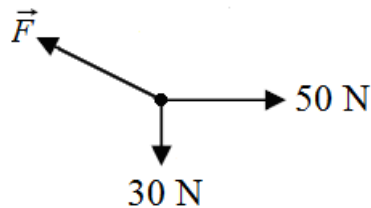
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**Strategy**

1. Resolve all known forces into their horizontal and vertical components.
2. Add the horizontal and vertical components separately.
3. Use the Pythagorean Theorem to find the magnitude of forces that are neither horizontal nor vertical.
4. Because you know the vertical and horizontal components of the resultant force, use arcsine ( $\sin^{-1}$ ), arccosine ( $\cos^{-1}$ ) or arctangent ( $\tan^{-1}$ ) to find the angle.

**Sample Problems:**

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitude and direction of  $\vec{F}$  ?

A:  $\vec{F}$  is equal and opposite to the resultant of the other two vectors. The magnitude of the resultant is:

$$\|\vec{F}'\| = \sqrt{30^2 + 50^2} = \sqrt{3400} = 58.3 \text{ N}$$

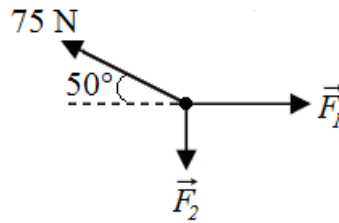
The direction is:

$$\tan\theta = \frac{30}{50} = 0.6$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.6) = 31.0^\circ \text{ up from the left (horizontal)}$$

Use this space for summary and/or additional notes.

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitudes of  $\vec{F}_1$  and  $\vec{F}_2$ ?

A:  $\vec{F}_1$  and  $\vec{F}_2$  are equal and opposite to the vertical and horizontal components of the 75 N force, which we can find using trigonometry:

$$\|\vec{F}_1\| = \text{horizontal} = 75 \cos(50^\circ) = (75)(0.643) = 48.2 \text{ N}$$

$$\|\vec{F}_2\| = \text{vertical} = 75 \sin(50^\circ) = (75)(0.766) = 57.5 \text{ N}$$

Use this space for summary and/or additional notes.

## Ramp Problems

**Unit:** Forces

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- calculate the forces on a ramp

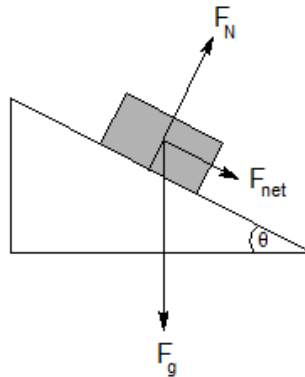
**Language Objectives:**

- Set up and solve word problems involving forces on a ramp.

**Notes:**

The direction of the normal force does not always directly oppose gravity. For example, if a block is resting on a (frictionless) ramp, the weight of the block is  $\vec{F}_g$ , in the direction of gravity. However, the normal force is perpendicular to the ramp, not to gravity.

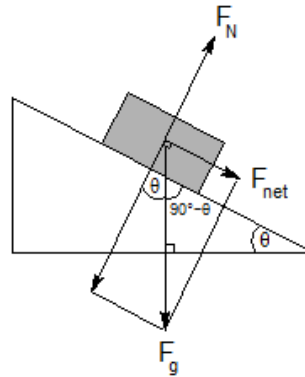
If we were to add the vectors representing the two forces, we would see that the resultant—the net force—acts down the ramp:



Use this space for summary and/or additional notes.



From geometry, we can determine that the angle of the ramp,  $\theta$ , is the same as the angle between gravity and the normal force.

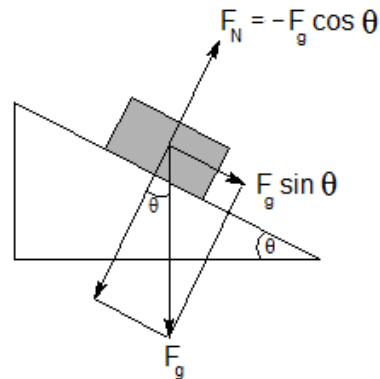


From trigonometry, we can calculate that the component of gravity parallel to the ramp (which equals the net force down the ramp) is the side opposite angle  $\theta$ . This means:

$$F_{net} = F_g \sin \theta$$

The component of gravity perpendicular to the ramp is  $F_g \cos \theta$ , which means the normal force is:

$$F_N = -F_g \cos \theta$$



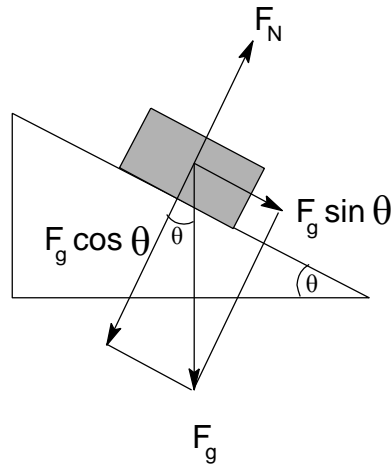
(The negative sign is because we have chosen down to be the positive direction.)

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: A block with a mass of 2.5 kg sits on a frictionless ramp with an angle of inclination of  $35^\circ$ . How fast does the block accelerate down the ramp?

A: The weight of the block is  $F_g = ma = (2.5)(9.8) = 24.5 \text{ N}$ . However, the component of the force of gravity in the direction that the block slides down the ramp is  $F_g \sin \theta$ :



$$F_g \sin \theta = 24.5 \sin 35^\circ = (24.5)(0.574) = 14.1 \text{ N}$$

Now that we know the net force (in the direction of motion), we can apply Newton's Second Law:

$$F = ma$$

$$14.1 = 2.5 a$$

$$a = 5.64 \frac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes.

## Pulleys & Tension

**Unit:** Forces

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- set up & solve problems involving ropes under tension

**Language Objectives:**

- Understand and correctly use the terms “pulley” and “tension.”
- Set up and solve word problems involving pulleys and/or tension.

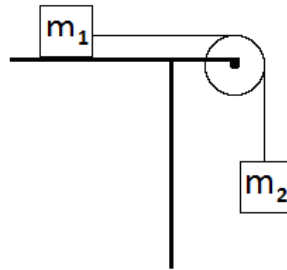
**Notes:**

tension: the pulling force on a rope, cable, etc.

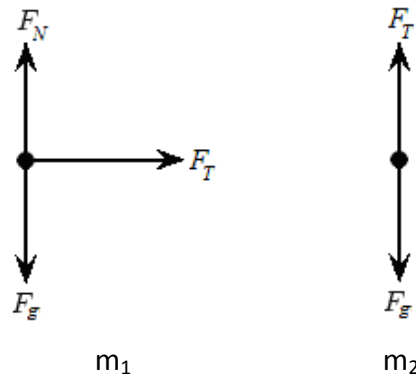
pulley: a wheel used to change the direction of tension on a rope

Use this space for summary and/or additional notes.

A typical problem involving pulleys and tension might be to find the acceleration of blocks  $m_1$  and  $m_2$  in the following situation. (Assume the surface and the pulley are frictionless.)



Free-body diagrams for the two masses would look like the following:



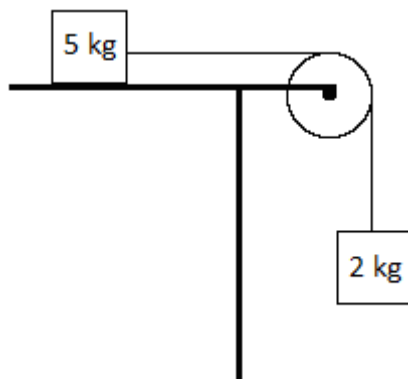
We know the following:

- $F_T$  is the net force on  $m_1$ . Therefore, for  $m_1$ ,  $F_{net} = m_1 a$ .
- For  $m_2$ , gravity and tension are pulling in opposite directions. The net force is therefore  $F_{net} = F_g - F_T = m_2 a$
- Because the blocks are connected, both  $F_T$  and  $a$  must be the same for both blocks.

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: Suppose we had the following situation:



Calculate the acceleration of the pair of blocks.

A: For the block on the table:

$$F_T = ma = (5)(a)$$

For the block hanging from the pulley:

$$F_{net} = F_g - F_T = ma = (2)(a)$$

$$(2)(9.8) - F_T = 2a$$

$$19.6 - F_T = 2a$$

Now we substitute  $F_T = 5a$  into the second equation:

$$19.6 - 5a = 2a$$

$$19.6 = 7a$$

$$a = \frac{19.6}{7} = 2.8 \frac{m}{s^2}$$

Use this space for summary and/or additional notes.

## Friction

**Unit:** Forces

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.6

**Knowledge/Understanding Goals:**

- difference between static & kinetic friction
- direction of the vector representing friction

**Skills:**

- calculate the frictional force on an object
- calculate net force in problems involving friction

**Language Objectives:**

- Understand and correctly use the terms “friction,” “static friction,” “kinetic friction,” and “coefficient of friction.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving friction.

**Notes:**

friction: a contact force caused by the roughness of the materials in contact, deformations in the materials, and molecular attraction between materials. Frictional forces are always parallel to the plane of contact between two surfaces, and opposite to the direction of motion or applied force.

static friction: friction acting on an object at rest that resists its ability to start moving.

kinetic friction: friction resisting the motion of an object.

In almost all situations, static friction is a stronger force than kinetic friction.

Use this space for summary and/or additional notes.

coefficient of friction: a constant that relates the frictional force on an object to the normal force. The coefficient of friction is represented by the Greek letter  $\mu$  (mu). It is a dimensionless number, which means that it has no units. (This is because  $\mu$  is a ratio of two forces, so the units cancel.)

coefficient of static friction:  $\mu_s$  represents the coefficient of friction for an object that is stationary (not moving)

coefficient of kinetic friction:  $\mu_k$  represents the coefficient of friction for an object that is moving.

The coefficient of friction takes into account the surface areas and surface characteristics of the objects in contact.

The force of friction on an object is given by the equations:

$$F_f \leq \mu_s F_N \quad \text{for an object that is stationary, and}$$

$$F_f = \mu_k F_N \quad \text{for an object that is moving,}$$

Where  $F_f$  is the magnitude of the force of friction,  $\mu_s$  and  $\mu_k$  are the coefficients of static and kinetic friction, respectively, and  $F_N$  is the magnitude of the normal force.

Note that the force of static friction is an inequality. For a stationary object, the force that resists sliding is, of course, equal to the force applied. However, once the applied force exceeds  $\mu_s F_N$ , the object starts moving and the equation for kinetic friction applies.

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### Friction as a Vector Quantity

Like other forces, the force of friction is, of course, actually a vector. Its direction is opposite to the direction of motion. However, the direction of the friction vector is opposite to the direction of the force attempting to cause sliding (in the case of static friction) or opposite to the direction of motion (in the case of kinetic friction). This is perpendicular to the normal force, but the direction of the normal force cannot tell us the direction of the force of friction.

We could write:

$\vec{F}_f \leq -\mu_s F_N \hat{F}_{applied}$  for an object that is stationary (because the force of friction is in the direction opposite to the applied force)

$\vec{F}_f = -\mu_k F_N \hat{v}$  for an object that is moving (because the force of friction is in the direction opposite to the velocity)

However, in practice, it is more cumbersome to try to explain the unit vectors than it is to simply declare that the force of static friction is opposite to the force that is attempting to cause sliding, and the force of kinetic friction is opposite to the direction of motion.

This means that whether the force of friction should be positive or negative needs to be determined directly from the coordinate system chosen for the problem.

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### Solving Simple Friction Problems

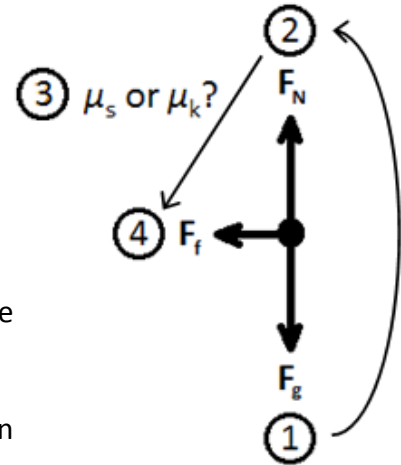
Because friction is a contact force, all friction problems involve friction in addition to some other (usually externally applied) force.

To calculate the force from friction, you need to:

1. Calculate the force of gravity. On Earth,  $F_g = m(9.8)$
2. Calculate the normal force. If the object is resting on a horizontal surface (which is usually the case), the normal force is usually equal in magnitude to the force of gravity. This means that for an object sliding across a horizontal surface:

$$F_N = F_g$$

3. Figure out whether the friction is static (there is an applied force, but the object is not moving), or kinetic (the object is moving). Look up the appropriate coefficient of friction ( $\mu_s$  for static friction, or  $\mu_k$  for kinetic friction).



4. Calculate the force of friction from the equation:

$$F_f \leq \mu_s F_N \quad \text{or} \quad F_f = \mu_k F_N$$

Make the force of friction positive or negative, as appropriate.

5. If the problem is asking for net force, remember to go back and calculate it now that you have calculated the force of friction.

If friction is causing the object to slow down and eventually stop and there is no separate applied force, then:

$$F_{net} = F_f$$

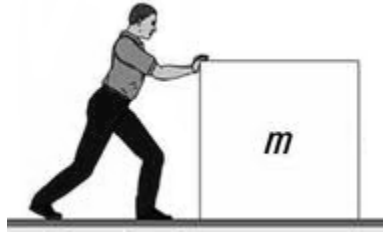
However, if there is an applied force and friction is opposing it, then the net force would be:

$$F_{net} = F_{applied} - F_f$$

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: A person pushes a box at a constant velocity across a floor:



The box has a mass of 40 kg, and the coefficient of kinetic friction between the box and the floor is 0.35. What is the magnitude of the force that the person exerts on the box?

A: The box is moving at a constant velocity, which means there is no acceleration, and therefore no net force on the box. This means the force exerted by the person is exactly equal to the force of friction.

The force of friction between the box and the floor is given by the equation:

$$F_f = \mu_k F_N$$

The normal force is equal in magnitude to the weight of the box ( $F_g$ ), which is given by the equation:

$$F_N = F_g = ma = (40)(9.8) = 392 \text{ N}$$

Therefore, the force of friction is:

$$F_f = \mu_k F_N$$

$$F_f = (0.35)(392) = 137 \text{ N}$$

Use this space for summary and/or additional notes.

## Aerodynamic Drag

**Unit:** Forces

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

### Knowledge/Understanding Goals:

- an intuitive sense of how aerodynamic drag works
- how aerodynamic drag is calculated

### Language Objectives:

- Understand and correctly use the term “drag” when it refers to an object that is slowed down by a fluid.
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to aerodynamic drag.

### Notes:

Most of the physics I problems that involve aerodynamic drag fall into two categories:

1. The drag force is small enough that we ignore it.
2. The drag force is equal to some other force that we can measure or calculate.

For simple situations involving aerodynamic drag, the drag force is given by the following equation:

$$\vec{F}_D = -\frac{1}{2}\rho\vec{v}^2C_D A$$

where:

- $\vec{F}_D$  = drag force
- $\rho$  = density of the fluid
- $\vec{v}$  = velocity of the object (relative to the fluid)
- $C_D$  = drag coefficient of the object (based on its shape)
- $A$  = cross-sectional area of the object






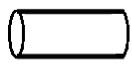



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This equation applies when the object has a blunt form factor, and the object's velocity relative to the properties of the fluid (such as viscosity) causes turbulence in the object's wake (*i.e.*, behind the object).

The drag coefficient,  $C_D$ , is a dimensionless number (meaning that it has no units) that encompasses all of the types of friction associated with aerodynamic drag. It serves the same purpose in drag problems that the coefficient of friction,  $\mu$ , serves in problems involving friction between solid surfaces.

Approximate drag coefficients for simple shapes are given below, assuming that the fluid motion relative to the object is in the direction of the arrow.

Measured Drag Coefficients

Shape	Drag Coefficient
Sphere → 	0.47
Half-sphere → 	0.42
Cone → 	0.50
Cube → 	1.05
Angled Cube → 	0.80
Long Cylinder → 	0.82
Short Cylinder → 	1.15
Streamlined Body → 	0.04
Streamlined Half-body → 	0.09

Use this space for summary and/or additional notes.

## Universal Gravitation

**Unit:** Forces

**NGSS Standards:** HS-PS2-4

**MA Curriculum Frameworks (2006):** 1.7

**Skills:**

- solve problems involving Newton's Law of Universal Gravitation

**Language Objectives:**

- Understand and correctly use the terms "gravity" and "gravitation."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Newton's Law of Universal Gravitation.

**Notes:**

Gravity is a force of attraction between two objects because of their mass. The cause of this attraction is not currently known. More mass causes a stronger force, but the force gets weaker as the object gets farther away.

This means that the gravitational pull of a single object is directly proportional to its mass, and inversely proportional to its distance, i.e.,

$$F_g \propto \frac{m}{d}$$

(The symbol  $\propto$  means "is proportional to".)

If we have two objects, "1" and "2":

$$F_{g,1} \propto \frac{m_1}{d_1} \quad \text{and} \quad F_{g,2} \propto \frac{m_2}{d_2}$$

Because each object is pulling on the other one, the total force is therefore:

$$F_g \propto \frac{m_1}{d_1} \cdot \frac{m_2}{d_2} = \frac{m_1 m_2}{d_1 d_2} = \frac{m_1 m_2}{d^2}$$

Use this space for summary and/or additional notes.

Finally, if we are using MKS units, the masses are in kilograms, the distance is in meters. If we want the force in newtons to be correct, we have to multiply by the appropriate conversion factor, which turns out to be  $6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ . (The units are chosen because they cancel the  $\text{m}^2$  and  $\text{kg}^2$  from the formula and give newtons, which is the desired unit. Thus the formula becomes:

$$F_g = (6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}) \frac{m_1 m_2}{d^2}$$

The number  $6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$  is called the universal gravitation constant, and is represented by the symbol "G". Thus we have Isaac Newton's Law of Universal Gravitation in equation form:

$$F_g = \frac{G m_1 m_2}{d^2}$$

However, you may recall that on Earth, where the acceleration due to gravity is  $g = 9.8 \frac{\text{m}}{\text{s}^2}$ , we also have the formula:

$$F_g = mg = m(9.8)$$

In the expression  $g = \frac{G m_1}{d^2}$ ,  $G$  is  $6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ ,  $m_1$  is the mass of the Earth ( $5.98 \times 10^{24}$  kg) and  $d$  is the radius of the Earth ( $6.37 \times 10^6$  m). If you plug these numbers into the formula, you get  $g = 9.8 \frac{\text{m}}{\text{s}^2}$  as expected, *i.e.*, near the surface of the Earth:

$$F_g = \frac{G m_1}{d^2} m_2 = g \cdot m_2 = 9.8 \cdot m_2$$

Use this space for summary and/or additional notes.

**Sample Problems:**

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is  $5.98 \times 10^{24}$  kg, and its radius is  $6.37 \times 10^6$  m.

$$F_g = \frac{Gm_1m_2}{d^2}$$

A: 
$$F_g = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(75)}{(6.37 \times 10^6)^2}$$

$$F_g = 737 \text{ N}$$

Q: Find the acceleration due to gravity on the moon.

A: 
$$g_{\text{moon}} = \frac{Gm_{\text{moon}}}{d_{\text{moon}}^2}$$

$$g_{\text{moon}} = \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})}{(1.74 \times 10^6)^2} = 1.62 \frac{\text{m}}{\text{s}^2}$$

Q: If the distance between an object and the center of mass of a planet is tripled, what happens to the force of gravity between the planet and the object?

A: Starting with  $F_g = \frac{Gm_1m_2}{d^2}$ , if we replace  $d$  with  $3d$ , we would get:

$$F'_g = \frac{Gm_1m_2}{(3d)^2} = \frac{Gm_1m_2}{9d^2} = \frac{1}{9} \cdot \frac{Gm_1m_2}{d^2}$$

Thus  $F'_g$  is  $\frac{1}{9}$  of the original  $F_g$ .

Use this space for summary and/or additional notes.

## Introduction: Angular Motion & Torque

**Unit:** Angular Motion & Torque

### Topics covered in this chapter:

Angular Motion .....	158
Torque.....	159
Centripetal vs. Centrifugal Force .....	164

This chapter involves motion around a fixed point.

- *Angular Motion* describes concepts and equations relating to motion in a circle.
- *Torque* describes forces that cause rotational motion and the equations relating to them.
- *Centripetal vs. Centrifugal Force* describes the forces on an object that is moving in a circular path.

This chapter will present some new challenges with keeping directions correct. The torque section will introduce the idea of having multiple instances of the same quantity in an equation and adding them up.

### Standards addressed in this chapter:

#### Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

#### Massachusetts Curriculum Frameworks (2006):

**1.8** Describe conceptually the forces involved in circular motion.

Use this space for summary and/or additional notes.



**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Circular Motion**, such as uniform circular motion and centripetal force
- **Gravity**, such as ~~the law of gravitation~~, orbits, and Kepler's laws
  1. Rotational Kinematics
  2. Uniform Circular Motion
  3. Rotational Dynamics
  4. Kinetic Energy
  5. Angular Momentum
  6. Kepler's Laws

**Skills learned & applied in this chapter:**

- Working with more than one instance of the same quantity in a problem.

Use this space for summary and/or additional notes.

## Angular Motion

**Unit:** Angular Motion & Torque

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.8

**Knowledge/Understanding Goals:**

- angular velocity

**Language Objectives:**

- Understand and correctly use the terms “angular motion” and “angular velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

angular velocity ( $\omega$ ): the speed at which an object moves through a given angle as it travels in a circular path.

The angular velocity of an object is the distance of arc traveled in a unit of time.

If the arc length equals the angle in radians, then  $\omega = \frac{\Delta\theta}{\Delta t}$ , and  $\omega$  is measured in  $\frac{\text{radians}}{\text{second}}$ .

Because the radian is based on the unit circle, the linear velocity of an object traveling in a circle is:

$$v = r\omega$$

similarly, if the angular acceleration is  $\alpha = \frac{\Delta\omega}{\Delta t}$ , then the linear acceleration would be:

$$a = r\alpha$$

Use this space for summary and/or additional notes.

## Torque

**Unit:** Angular Motion & Torque

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- center of mass and torque

**Skills:**

- calculate torque

**Language Objectives:**

- Understand and correctly use the terms “torque,” “axis of rotation,” “fulcrum,” “lever arm,” and “center of mass.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems involving torque.

**Notes:**

torque ( $\vec{\tau}$ ): a vector quantity that measures the effectiveness of a force in causing rotation.

axis of rotation: the point around which an object rotates.

fulcrum: the point around which a lever pivots.

lever arm: the distance from the axis of rotation that a force is applied, causing a torque.

center of mass: if all of the mass were condensed to one point in a way that kept all forces and torques the same, the point would be the center of mass.

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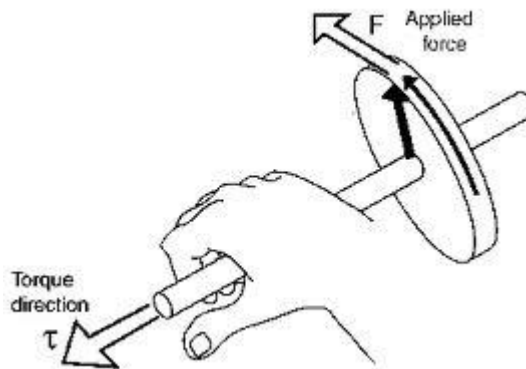
Torque is calculated as the cross product of lever arm  $\times$  force:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

We use the variable  $r$  for the lever arm (which is a distance) because torque causes rotation, and  $r$  is the distance from the center of the circle (radius) at which the torque is applied.

The magnitude of  $\vec{\tau}$  is  $rF \sin \theta$  where  $\theta$  is the angle between the lever arm and the applied force. If  $\theta = 90^\circ$ , then the magnitude of  $\tau = rF$

Because torque is the cross product of two vectors, it is a vector whose direction is perpendicular to both the lever arm and the force:



This is an application of the “right hand rule.” If your fingers of your right hand curl from the first vector ( $\vec{r}$ ) to the second ( $\vec{F}$ ), then your thumb points in the direction of the resultant vector ( $\vec{\tau}$ ).

In this course, we will concern ourselves only with the magnitude of the torque.

Torque is measured in Newton-meters:

$$1 \text{ N} \cdot \text{m} = 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

Note that work and energy are also measured in Newton-meters. However, work and energy are scalar quantities. The derived S.I. unit Joule (J) is a scalar unit that measures scalar Newton-meters. Because torque is a *vector* quantity, one Newton-meter of torque can not be described as one Joule. This is true even if we are only concerning ourselves with the magnitude.

Use this space for summary and/or additional notes.

### Sample Problem

Q: If a force of 20 N is applied to a wrench with a 25 cm handle, what is the torque applied to the bolt?

A:  $\vec{\tau} = \vec{r} \times \vec{F}$

$$\tau = rF \sin \theta$$

$$\tau = rF(1) = rF$$

$$\tau = (20 \text{ N})(0.25 \text{ m})$$

$$\tau = 4 \text{ N} \cdot \text{m}$$

### Seesaw Problems

A seesaw problem is one in which objects on opposite sides of a lever (such as a seesaw) balance one another.

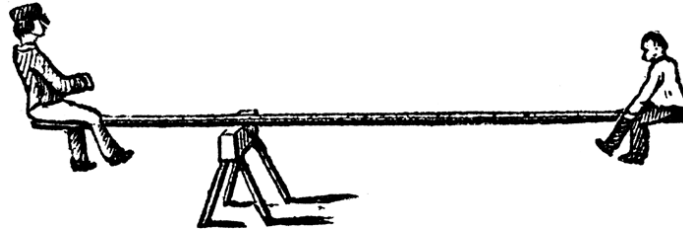
To solve seesaw problems, if the seesaw is not moving, then the torques must balance and the net torque must be zero.

The total torque on each side is the sum of the separate torques caused by the separate masses. Each of these masses can be considered as a point mass (infinitely small object) placed at the object's center of mass.

Use this space for summary and/or additional notes.

**Sample Seesaw Problem**

Q: In the following diagram, the mass of the person on the left is 90. kg and the mass of the person on the right is 50. kg. The board is 6.0 m long and has a mass of 20. kg.



Where should the board be positioned in order to balance the seesaw?

A: The torque from the person on the left acts at a distance  $x$ :

$$\tau_{left} = rF$$

$$\tau_{left} = x(mg) = x(90 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})$$

$$\tau_{left} = 882x$$

The torque from the weight of the half of the board that is to the left of the fulcrum acts at a distance of  $x/2$ :

Center of mass is at  $x/2$ .

Fraction of mass to the left of the fulcrum is  $x/6.0$ .

$$\tau_{left} = rF$$

$$\tau_{left} = r(mg) = (\frac{x}{2})(\frac{x}{6.0})(9.8)$$

$$\tau_{left} = 0.817x^2$$

Total torque on the left:

$$\tau_{left} = 0.817x^2 + 882x$$

Use this space for summary and/or additional notes.

The torque from the person on the right acts at a distance of  $6 - x$ :

$$\tau_{right} = rF$$

$$\tau_{right} = r(mg) = (6 - x)(50)(9.8)$$

$$\tau_{right} = 490(6 - x)$$

$$\tau_{right} = 2940 - 490x$$

The torque from the right side of the board acts at a distance of  $\frac{6 - x}{2}$ :

$$\tau_{right} = rF$$

$$\tau_{right} = r(mg) = \frac{(6 - x)}{2} \cdot \frac{(6 - x)}{6.0} (9.8)$$

$$\tau_{right} = 0.817(36 - 12x + x^2)$$

$$\tau_{right} = 29.4 - 9.8x + 0.817x^2$$

Total torque on the right:

$$\tau_{right} = 0.817x^2 - 490x - 9.8x + 29.4 + 2940$$

$$\tau_{right} = 0.817x^2 - 499.8x + 2969.4$$

Because the torques on both sides are equal, set the equations equal and solve for  $x$ :

$$0.817x^2 + 882x = 0.817x^2 - 499.8x + 2969.4$$

$$882x = -499.8x + 2969.4$$

$$1381.8x = 2969.4$$

$$x = 2.15 \text{ m}$$

Therefore, the board must be placed with 2.15 m to the left of the fulcrum, and  $6.0 - 2.15 = 3.85$  m to the right.

Use this space for summary and/or additional notes.

## Centripetal vs. Centrifugal Force

**Unit:** Angular Motion & Torque

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 1.8

**Knowledge/Understanding Goals:**

- the difference between centripetal and centrifugal force

**Skills:**

- calculate the centripetal force of an object moving in a circle

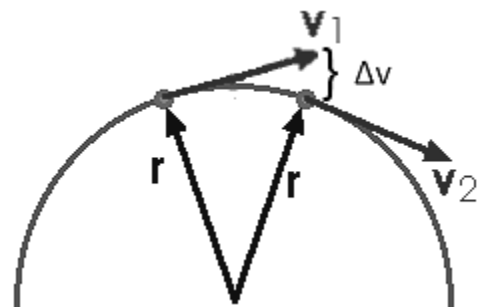
**Language Objectives:**

- Understand and correctly use the terms “rotation,” “centripetal force,” and “centrifugal force.”
- Explain the difference between centripetal force and centrifugal force.

**Notes:**

Suppose an object is moving at a constant speed around a circle. Although its speed is constant, its direction keeps changing as it goes around. Because velocity is a vector (speed and direction), this means its velocity is constantly changing. (To be precise, the magnitude is staying the same, but the direction is changing.)

Because a change in velocity over time is acceleration, this means the object is constantly accelerating. The continuous change in velocity is toward the center of the circle, which means *there is continuous acceleration toward the center of the circle.*

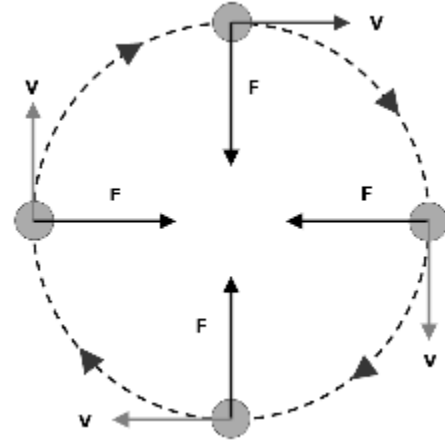


Use this space for summary and/or additional notes:



Finally, because acceleration is caused by a net force (Newton's second law of motion), if there is continuous acceleration toward the center of the circle, then there must be a continuous force toward the center of the circle.

This force is called "centripetal force".



centripetal force: the inward force that keeps an object moving in a circle. If the centripetal force were removed, the object would fly away from the circle in a straight line that starts from a point tangent to the circle.

The formula for centripetal acceleration ( $\alpha$ ) is:

$$\alpha = \frac{v^2}{r}$$

Given that  $F = ma$ , the equation for centripetal force is therefore:

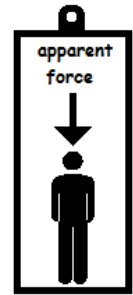
$$F_c = m\alpha = \frac{mv^2}{r}$$

Use this space for summary and/or additional notes.

centrifugal "force": the apparent outward force felt by an object that is moving in a circle.

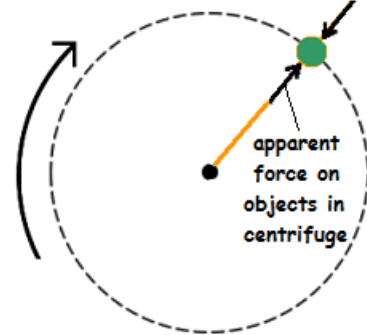
Centrifugal "force" is technically not a force as we would define it in physics. Centrifugal "force" is actually the inertia of objects resisting motion as they are continuously pulled toward the center of a circle by centripetal acceleration.

As an analogy, imagine that you are standing in an elevator. While the elevator is accelerating upward, the force between you and the floor of the elevator increases. An increase in the normal force from the floor because of the upward acceleration of the elevator feels the same as an increase in the downward force of gravity.



force from  
bottom of  
centrifuge tube

Similarly, a sample being spun in a centrifuge is subjected to the force from the bottom of the centrifuge tube as it is accelerated toward the center. The faster the rotation, the stronger the force. Again, an increase in the normal force from the bottom of the centrifuge tube would "feel" the same as a downward force toward the bottom of the centrifuge tube.



Use this space for summary and/or additional notes.

## Introduction: Energy & Momentum

**Unit:** Energy & Momentum

### Topics covered in this chapter:

Linear Momentum .....	170
Impulse.....	176
Work.....	178
Energy .....	183
Escape Velocity .....	190
Newton's Cradle.....	193
Power .....	196

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Linear Momentum* describes a way to represent the movement of an object and what happens when objects collide, and the equations that relate to it. *Impulse* describes changes in momentum.
- *Work* and *Energy* describe the ability to cause something to move and the related equations. *Power* describes the rate at which energy is applied.
- *Escape Velocity* and *Newton's Cradle* describe interesting applications of energy and momentum.

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

### Standards addressed in this chapter:

#### Next Generation Science Standards (NGSS):

- HS-PS2-2.** Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

Use this space for summary and/or additional notes:

**HS-PS3-1.** Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

**HS-PS3-2.** Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

**HS-PS3-3.** Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

**Massachusetts Curriculum Frameworks (2006):**

**2.1** Interpret and provide examples that illustrate the law of conservation of energy.

**2.2** Interpret and provide examples of how energy can be converted from gravitational potential energy to kinetic energy and vice versa.

**2.3** Describe both qualitatively and quantitatively how work can be expressed as a change in mechanical energy.

**2.4** Describe both qualitatively and quantitatively the concept of power as work done per unit time.

**2.5** Provide and interpret examples showing that linear momentum is the product of mass and velocity, and is always conserved (law of conservation of momentum). Calculate the momentum of an object.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Energy and Momentum**, such as potential and kinetic energy, work, power, impulse, and conservation laws.

1. What is Linear Momentum?
2. Impulse
3. Conservation of Momentum
4. Collisions
5. Center of Mass
6. Work
7. Energy
8. Forms of Energy

Use this space for summary and/or additional notes.

9. Power

**Skills learned & applied in this chapter:**

- Working with more than one instance of the same quantity in a problem.
- Conservation laws (before/after problems).

Use this space for summary and/or additional notes.

## Linear Momentum

**Unit:** Energy & Momentum

**NGSS Standards:** HS-PS2-2

**MA Curriculum Frameworks (2006):** 2.5

**Knowledge/Understanding Goals:**

- definition of momentum

**Skills:**

- calculate the momentum of an object
- solve problems involving the conservation of momentum

**Language Objectives:**

- Understand and correctly use the terms “collision,” “elastic collision,” “inelastic collision,” “inertia” and “momentum.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving collisions and momentum.

**Notes:**

collision: when two or more objects come together and hit each other.

elastic collision: a collision in which the objects bounce off each other after they collide, without any loss of kinetic energy.

inelastic collision: a collision in which the objects remain together after colliding. In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before vs. after the collision.

Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large scale impacts are ever perfectly elastic.

inertia: an object’s ability to resist the action of a force.

Use this space for summary and/or additional notes:

momentum ( $\vec{p}$ ): the quantity of movement of an amount of matter. You can also think of momentum as the amount of force that a moving object could transfer each second in a collision. Momentum is a vector quantity given by the formula:

$$\vec{p} = m\vec{v}$$

and is measured in units of  $\text{N}\cdot\text{s}$ , or  $\frac{\text{kg}\cdot\text{m}}{\text{s}}$ .

Note that an object at rest has a momentum of zero.

Inertia and momentum are related, but are not the same thing; an object has inertia even at rest. An object's momentum changes if either its mass or its velocity changes, but an the inertia of an object can change only if its mass changes.

In a closed system, momentum is conserved. This means that unless there is an outside force, the combined momentum of objects after they collide is equal to the combined momentum of the objects before the collision.

### Solving Momentum Problems

Almost all momentum problems involve the conservation of momentum law:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

The symbol  $\sum$  is the Greek capital letter "sigma". In mathematics, the symbol  $\sum$  means "summation".  $\sum \vec{p}$  means the sum of the momentums. The subscript "i" means initial (before the collision), and the subscript "f" means final (after the collision). In plain English, find each individual value of  $\vec{p}$  (positive or negative, depending on the direction) and then add them all up to find the total. The conservation of momentum law means that the total before a collision must be equal to the total after.

For example, if you had a momentum problem with two objects, the law of conservation of momentum becomes:

$$\vec{p}_{i,1} + \vec{p}_{i,2} = \vec{p}_{f,1} + \vec{p}_{f,2}$$

Notice that we have two subscripts after each " $\vec{p}$ ", because we have two separate things to keep track of. The "i" and "f" mean "initial" and "final," and the "1" and "2" mean object #1 and object #2.

Use this space for summary and/or additional notes.

Because  $\vec{p} = m\vec{v}$ , we can replace each  $\vec{p}$  with  $m\vec{v}$ .

For our momentum problem with two objects, this becomes:

$$m_1\vec{v}_{i,1} + m_2\vec{v}_{i,2} = m_1\vec{v}_{f,1} + m_2\vec{v}_{f,2}$$

Note that there are six separate quantities in this problem:  $m_1$ ,  $m_2$ ,  $\vec{v}_{i,1}$ ,  $\vec{v}_{i,2}$ ,  $\vec{v}_{f,1}$ , and  $\vec{v}_{f,2}$ . A typical momentum problem will give you (or enable you to calculate) five of these, and will ask you for the sixth.

Note also that most momentum problems do not mention the word “momentum.” The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that the problem involves conservation of momentum.

Most momentum problems involve collisions. Usually, there are two objects initially, and the objects either bounce off each other (elastic collision) or stick together (inelastic collision).

For an elastic collision between two objects, the problem is exactly as described above: there are six quantities to consider: the two masses, the two initial velocities, and the two final velocities. The equation relating them is:

$$m_1\vec{v}_{i,1} + m_2\vec{v}_{i,2} = m_1\vec{v}_{f,1} + m_2\vec{v}_{f,2}$$

To solve the problem, you need to obtain the quantities given in the word problem and solve for the missing one.

For an inelastic collision between two objects, the objects stick together after the collision, which means there is only one “object” afterwards. The total mass of the object is  $m_T = m_1 + m_2$ , and there is only one “object” with a final velocity.

there are five quantities: the two masses, the two initial velocities, and the final velocity of the combined object. The equation relating them is:

$$m_1\vec{v}_{i,1} + m_2\vec{v}_{i,2} = m_T\vec{v}_f$$

To solve this problem, you need to obtain the quantities given in the word problem, add the two masses to find  $m_T$ , and solve for the missing quantity.

Use this space for summary and/or additional notes.



**Sample Problems**

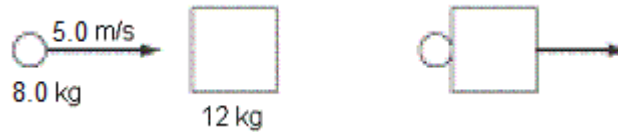
Q: What is the momentum of a 15 kg object moving at a velocity of  $+3.0 \frac{m}{s}$ ?

A:  $\vec{p} = m\vec{v}$

$$\vec{p} = (15\text{kg})(+3.0 \frac{m}{s}) = 45 \frac{\text{kg}\cdot\text{m}}{\text{s}} = +45 \text{ N}\cdot\text{s}$$

The answer is given as  $+45 \text{ N}\cdot\text{s}$  because momentum is a vector, and we indicate the direction as positive or negative.

Q: An object with a mass of 8.0 kg moving with a velocity of  $+5.0 \frac{m}{s}$  collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?



A: The momentum of the moving object before the collision is:

$$\vec{p} = m\vec{v} = (8.0)(+5.0) = +40 \text{ N}\cdot\text{s}$$

The stationary object has a momentum of zero, so the total momentum of the two objects combined is  $+40 \text{ N}\cdot\text{s}$ .

After the collision, the total mass is  $8.0 \text{ kg} + 12 \text{ kg} = 20 \text{ kg}$ . The momentum after the collision must still be  $+40 \text{ N}\cdot\text{s}$ , which means the velocity is:

$$\vec{p} = m\vec{v} \quad 40 = 20\vec{v} \quad \vec{v} = +2 \frac{m}{s}$$

Using the equation, we would solve this as follows:

$$m_1\vec{v}_{i,1} + m_2\vec{v}_{i,2} = m_T\vec{v}_f$$

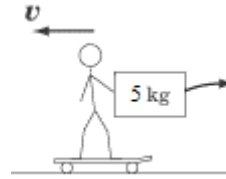
$$(8)(5) + (12)(0) = (8 + 12)\vec{v}_f$$

$$40 = 20\vec{v}_f$$

$$\vec{v}_f = \frac{40}{20} = +2 \frac{m}{s}$$

Use this space for summary and/or additional notes.

Q: Mr. Stretchy has a mass of 60. kg and is holding a 5.0 kg box as he rides on a skateboard toward the west at a speed of  $3.0 \frac{m}{s}$ . (Assume the 60. kg is the mass of Mr. Stretchy and the skateboard combined.) He throws the box behind him, giving it a velocity of  $2.0 \frac{m}{s}$  to the east.



What is Mr. Stretchy's velocity after throwing the box?

A: This problem is like an inelastic collision in reverse; Mr. Stretchy and the box are together before the "collision" and apart afterwards. The equation would therefore look like this:

$$m_T \vec{v}_i = m_s \vec{v}_{f,s} + m_b \vec{v}_{f,b}$$

Where the subscript "s" is for Mr. Stretchy, and the subscript "b" is for the box. Note that after Mr. Stretchy throws the box, he is moving one direction and the box is moving the other, which means we need to be careful about our signs. Let's choose the direction Mr. Stretchy is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be  $\vec{v}_{f,b} = -2.0 \frac{m}{s}$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned} m_T \vec{v}_i &= m_s \vec{v}_{f,s} + m_b \vec{v}_{f,b} \\ (60 + 5)(+3) &= 60 \vec{v}_{f,s} + (5)(-2) \\ + 195 &= 60 \vec{v}_{f,s} + (-10) \\ + 205 &= 60 \vec{v}_{f,s} \\ \vec{v}_{f,s} &= \frac{+ 205}{60} = +3.4 \frac{m}{s} \end{aligned}$$

Use this space for summary and/or additional notes.

Q: A soccer ball that has a mass of 0.43 kg is rolling east with a velocity of  $5.0 \frac{\text{m}}{\text{s}}$ . It collides with a volleyball that has a mass of 0.27 that is rolling west with a velocity of  $6.5 \frac{\text{m}}{\text{s}}$ . After the collision, the soccer ball is rolling to the west with a velocity of  $3.87 \frac{\text{m}}{\text{s}}$ . Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

$$m_s \vec{v}_{i,s} + m_v \vec{v}_{i,v} = m_s \vec{v}_{f,s} + m_v \vec{v}_{f,v}$$

Where the subscript "s" is for the soccer ball and the subscript "v" is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don't know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

quantity	direction	value
initial velocity of soccer ball	east	$+5.0 \frac{\text{m}}{\text{s}}$
initial velocity of volleyball	west	$-6.5 \frac{\text{m}}{\text{s}}$
final velocity of soccer ball	west	$-3.87 \frac{\text{m}}{\text{s}}$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned} m_s \vec{v}_{i,s} + m_v \vec{v}_{i,v} &= m_s \vec{v}_{f,s} + m_v \vec{v}_{f,v} \\ (0.43)(5.0) + (0.27)(-6.5) &= (0.43)(-3.87) + (0.27) \vec{v}_{f,v} \\ 2.15 + (-1.755) &= -1.664 + 0.27 \vec{v}_{f,v} \\ 0.395 &= -1.664 + 0.27 \vec{v}_{f,v} \\ 2.059 &= 0.27 \vec{v}_{f,v} \\ \vec{v}_{f,v} &= \frac{+2.059}{0.27} = +7.63 \frac{\text{m}}{\text{s}} \end{aligned}$$

Positive means east, so the volleyball is moving with a velocity of  $7.63 \frac{\text{m}}{\text{s}}$  to the east.

Use this space for summary and/or additional notes.

## Impulse

**Unit:** Energy & Momentum

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 2.5

**Knowledge/Understanding Goals:**

- what impulse is

**Skills:**

- calculate the impulse given to an object
- calculate the change in momentum as the result of an impulse

**Language Objectives:**

- Understand and correctly use the term “impulse.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving impulse.

**Notes:**

impulse ( $\vec{J}$ ): a force applied to an object over an interval of time. Impulse is a vector quantity.

An impulse causes a change in momentum, and the amount of the impulse is equal to the change in momentum. The impulse also equals force times time:

$$\vec{J} = \vec{F}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Impulse is measured in the same unit as momentum (newton-seconds):

$$1 \text{ N} \cdot \text{s} = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

**Sample Problem**

Q: A baseball has a mass of 0.145 kg and is pitched with a velocity of  $-38 \frac{\text{m}}{\text{s}}$  toward home plate. After the ball is hit, its velocity is  $+52 \frac{\text{m}}{\text{s}}$  toward the outfield fence. If the impact between the ball and bat takes place over an interval of 3.0 ms, find the impulse given to the ball by the bat, and the force applied to the ball by the bat.

A: The impulse is:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{J} = (0.145)(52 - (-38)) = +13.05 \text{ N}\cdot\text{s}$$

The collision takes place over a time interval of 3.0 ms = 0.0030 s.

$$\vec{J} = \vec{F}\Delta t$$

$$+13.05 = \vec{F}(0.0030)$$

$$\vec{F} = \frac{13.05}{0.0030} = +4350 \text{ N}$$

Use this space for summary and/or additional notes.

## Work

**Unit:** Energy & Momentum

**NGSS Standards:** HS-PS3-1

**MA Curriculum Frameworks (2006):** 2.3

**Knowledge/Understanding Goals:**

- understand the definition of work

**Skills:**

- calculate work done by a force applied to an object

**Language Objectives:**

- Understand and correctly use the term “work.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving work.

**Notes:**

work: the effort applied over a distance against a force.

For example, if you lift a heavy object off the ground, you are doing work against the force of gravity.

Mathematically, work is the dot product of the force vector and the displacement vector:

$$W = \vec{F} \cdot \vec{d}$$

The dot product is one of three ways of multiplying vectors. The dot product is a scalar (a number without a direction), and is equal to the product of the magnitudes of the force and distance, and the cosine of the angle between them. This means:

$$W = Fd\cos\theta$$

Where  $F$  is the magnitude of the force vector  $\vec{F}$ ,  $d$  is the magnitude of the displacement vector  $\vec{d}$ , and  $\theta$  is the angle between the two vectors.

Use this space for summary and/or additional notes:

Note that if the force and displacement are in the same direction the angle  $\theta = 0^\circ$ , which means  $\cos\theta = \cos(0^\circ) = 1$ . In this case, the equation reduces to  $W = Fd$ .

Work is measured in newton-meters (N·m).

$$1 \text{ N}\cdot\text{m} \equiv 1 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

As we will see later, work is equivalent to energy, which means a newton-meter is equivalent to a Joule.

**Note that:**

- If the displacement is zero, no work is done by the force. *E.g.*, if you hold a heavy box without moving it, you are exerting a force (counteracting the force of gravity) but you are not doing work.
- If the net force is zero, no work is done by the displacement (change in location) of the object. *E.g.*, if a cart is sliding across a frictionless air track at a constant velocity, the net force on the cart is zero, which means no work is being done.
- If the displacement is perpendicular to the direction of the applied force ( $\theta = 90^\circ$ , which means  $\cos\theta = 0$ ), no work is done by the force. *E.g.*, you can slide a very heavy object along a roller conveyor (pictured below), because the force of gravity is acting vertically and the object's displacement is horizontal, which means gravity is doing no work, and therefore you do not have to do any work against gravity.



Use this space for summary and/or additional notes.

In industry, gravity roller conveyors are usually set up at a slight downward angle:



In this situation, gravity is doing work to pull the boxes downward. (Vertical force; vertical displacement). Friction is doing work to slow the boxes down. (Horizontal force; horizontal displacement.) The angle is designed so that the work done by gravity equals the work done by friction, which means the workers do not have to do any work (in the physics sense ☺) to keep the boxes moving.

Use this space for summary and/or additional notes.



**Sample Problems**

Q: If you lifted a 60. kg box 1.5 m off the ground at a constant velocity over a period of 3.0 s, how much work did you do?

A: The box is being lifted, which means the work is done against the force of gravity.

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd \cos \theta$$

$$W = F_g d \cos 0^\circ = F_g d$$

$$W = F_g d = (mg)d$$

$$W = [(60 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})](1.5 \text{ m})$$

$$W = (588 \text{ N})(1.5 \text{ m}) = 882 \text{ N} \cdot \text{m}$$

Note that the amount of time it took to lift the box has nothing to do with the amount of work done.

It may be tempting to try to use the time to calculate velocity and acceleration in order to calculate the force. However, because the box is lifted at a constant velocity, the only force needed to lift the box is enough to overcome the weight of the box ( $F_g$ ).

In general, if work is done to move an object vertically, the work is done against gravity, and you need to use  $a = g = 9.8 \frac{\text{m}}{\text{s}^2}$  for the acceleration when you calculate  $F = ma$ .

Similarly, if work is done to move an object horizontally, the work is *not* against gravity and you need to find out the acceleration of the object before you can calculate  $F = ma$ .

Use this space for summary and/or additional notes.

Q: In the picture below, the adult is pulling on the handle of the wagon with a force of 150. N at an angle of 60.0°.



If the adult pulls the wagon for a distance of 500. m, how much work does he do?

A:  $W = \vec{F} \cdot \vec{d}$

$$W = Fd \cos \theta$$

$$W = (150. \text{N})(500. \text{m}) \cos 60.0^\circ$$

$$W = (150. \text{N})(500. \text{m})(0.500) = 37\,500 \text{ N} \cdot \text{m} = 37\,500 \text{ N} \cdot \text{m}$$

Use this space for summary and/or additional notes.

## Energy

**Unit:** Energy & Momentum

**NGSS Standards:** HS-PS3-1

**MA Curriculum Frameworks (2006):** 2.1, 2.2, 2.3

### Knowledge/Understanding Goals:

- define energy
- conservation of energy
- work-energy theorem

### Skills:

- calculate gravitational potential energy
- calculate kinetic energy

### Language Objectives:

- Understand and correctly use the terms “energy,” “kinetic energy,” and “potential energy.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving energy and the conservation of energy.

### Notes:

energy: the ability to do work

Energy is a scalar quantity that exists in several forms, including:

kinetic energy ( $E_k$ ): the energy an object has because of its motion.

heat ( $Q$ ): the energy an object has because of the kinetic energy of its molecules.

electrical work ( $W$ ): the work done by applying an electric current over a period of time. Measured in joules (J) or kilowatt-hours (kWh).

potential energy ( $U$ ) the unrealized energy that an object has because of its position, temperature, chemical reactions that could occur, *etc.*

Use this space for summary and/or additional notes:

In mechanics, kinetic energy usually refers to the energy of an object because of its mass and velocity. Potential energy usually refers to an object's position, and the ability of the force of gravity to cause it to move.

Energy is measured in Joules (J):

$$1 \text{ J} \equiv 1 \text{ N} \cdot \text{m} \equiv 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

### Kinetic Energy

The kinetic energy of an object is related to its mass and velocity:

$$E_k = \frac{1}{2}mv^2$$

Notice that kinetic energy relates the same quantities as momentum—mass and velocity. Kinetic energy is related to momentum via the formula:

$$E_k = \frac{p^2}{2m}$$

Note also that, because kinetic energy and momentum must *both* be conserved in an elastic collision, both final velocities are actually determined by the masses and the initial velocities.

### Gravitational Potential Energy

The gravitational potential energy of an object is determined by the force of gravity and its distance above the ground (which is the distance over which the force of gravity is able to expend energy to do work on the object).

$$U = F_g h = mgh$$

(Remember that  $g$  is the variable that we use for acceleration when it equals the acceleration due to gravity, which is  $9.8 \frac{\text{m}}{\text{s}^2}$  on Earth.)

Use this space for summary and/or additional notes.

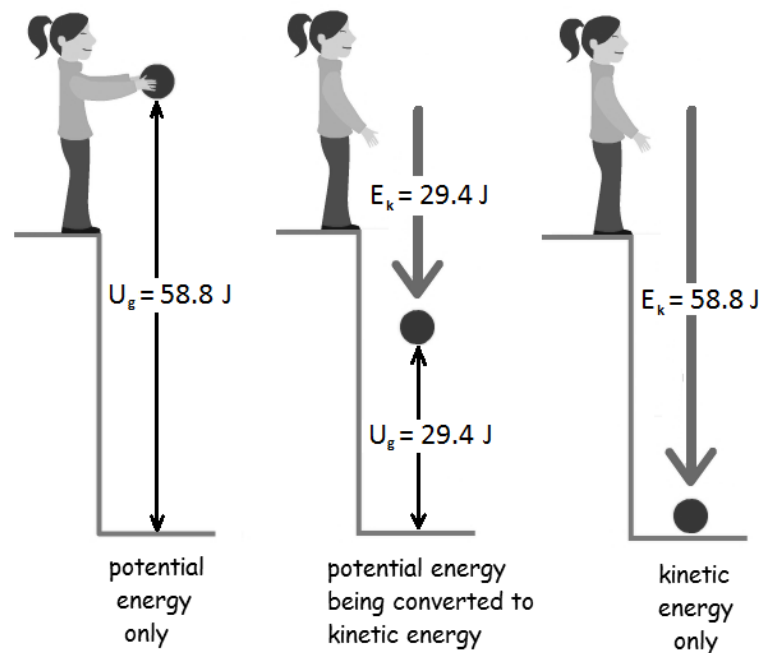
### Conservation of Energy

In a *closed system*, the total energy is constant. Energy can be converted from one form to another. When this happens, the increase in any one form of energy is the result of a corresponding decrease in another form of energy.

In a system that has potential energy, kinetic energy and heat, the total energy is given by:

$$E_{total} = U + E_k + Q$$

In the following diagram, suppose that the girl drops a ball with a mass of 2 kg from a height of 3 m.



Before the girl lets go of the ball, it has 58.8 J of potential energy. As the ball falls to the ground, potential energy is gradually converted to kinetic energy. The potential energy continuously decreases and the kinetic energy continuously increases, but the total energy is always 58.8 J. After the ball hits the ground, 58.8 N·m of work was done by gravity, and the 58.8 J of kinetic energy is converted to other forms, such as thermal energy (the temperatures of the ball and the ground increase infinitesimally), sound, etc.

Use this space for summary and/or additional notes.

### Work-Energy Theorem

Work and energy are equivalent quantities. The amount of work done by an object that has kinetic energy is given by the change in its kinetic energy:

$$W = \Delta E_k$$

Because kinetic energy lost equals potential energy gained (and *vice versa*), we can say that:

$$W = \Delta E_k = -\Delta U$$

Note that the units for work and energy—newton-meters and joules—are equivalent.

Work-energy theorem problems will give you information related to the kinetic energy of an object (such as its mass and a change in velocity) and ask you how much work was done.

A simple rule of thumb (meaning that it is *not always strictly* true) is:

- Potential energy is energy in the *future* (energy that is available for use).
- Kinetic energy is energy in the *present* (the energy of an object that is currently in motion).
- Work is the result of energy in the *past* (the result of kinetic energy having acted on an object).

Use this space for summary and/or additional notes.

### Solving Conservation of Energy Problems

Conservation of energy problems involve recognizing that energy is changing from one form to another. Once you have figured out what is being converted, calculate the amount energy that is converted, and use the equation for the new form to calculate the desired quantity.

In mechanics, conservation of energy problems usually involve work, gravitational potential energy, and kinetic energy:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta \quad (= Fd \text{ if force \& displacement are in the same direction})$$

$$U_g = mgh$$

$$E_k = \frac{1}{2}mv^2$$

There are two common types of conservation of energy problems:

One type gives you the energy at the start, and you have to calculate the desired quantity (such as velocity or height) from the energy after the conversion.

$$\text{Energy of one type} = \text{Energy of another type}$$

$$\text{Equation for first type} = \text{Equation for second type}$$

The second type requires you to calculate the same type of energy at two different times or locations. You have to use the difference to calculate the desired quantity (such as velocity or height).

$$\text{Energy after} - \text{Energy before} = \text{Energy Change}$$

$$\text{Energy change of one type} = \text{Energy change of another type or work done}$$

Use this space for summary and/or additional notes.

**Sample Problems**

Q: An 875 kg car accelerates from  $22 \frac{m}{s}$  to  $44 \frac{m}{s}$ . What were the initial and final kinetic energies of the car? How much work did the engine do to accelerate it?

A:  $E_{k,1} = \frac{1}{2}mv_1^2 = \frac{1}{2}(875\text{kg})(22\frac{m}{s})^2 = 211750 \text{ J}$

$$E_{k,2} = \frac{1}{2}mv_2^2 = \frac{1}{2}(875\text{kg})(44\frac{m}{s})^2 = 847000 \text{ J}$$

$$W = \Delta E_k = 847000 \text{ J} - 211750 \text{ J} = 635250 \text{ J} = 635250 \text{ N}\cdot\text{m}$$

The engine did approximately 635 250 N·m of work.

Q: An 80. kg physics student falls off the roof of a 15 m high school building. How much kinetic energy does he have when he hits the ground?

A: There are two approaches to answer this question.

1. Recognize that the student's potential energy at the top of the building is entirely converted to kinetic energy when he hits the ground. Calculate his potential energy at the top of the building, and that will be his kinetic energy when he hits the ground.

$$U = mgh = (80\text{kg})(9.8\frac{m}{s^2})(15\text{m}) = 11760 \text{ J}$$

The student has 11 760 J of kinetic energy when he hits the ground.

Use this space for summary and/or additional notes.



2. Use motion equations to find the student's velocity when he hits the ground, based on the height of the building and acceleration due to gravity. Then use the formula  $E_k = \frac{1}{2}mv^2$ .

$$d = \frac{1}{2}at^2$$

$$15\text{m} = \frac{1}{2}(9.8\frac{\text{m}}{\text{s}^2})t^2$$

$$t^2 = 3.061$$

$$t = \sqrt{3.061} = 1.750 \text{ s}$$

$$v = at$$

$$v = (9.8\frac{\text{m}}{\text{s}^2})(1.750\text{s}) = 17.15 \frac{\text{m}}{\text{s}}$$

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}(80\text{kg})(17.15\frac{\text{m}}{\text{s}})^2$$

$$E_k = 11760 \text{ J}$$

As before, the student has approximately 11 760 J of kinetic energy when he hits the ground.

Use this space for summary and/or additional notes.

## Escape Velocity

**Unit:** Energy & Momentum

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 2.1, 2.2, 2.3

### Knowledge/Understanding Goals:

- how fast a rocket or space ship needs to travel to escape Earth's (or any other planet's) gravity
- how escape velocity relates to gravitational potential energy, kinetic energy, and Newton's Law of Universal Gravitation

### Language Objectives:

- Understand and correctly use the term "escape velocity."
- Set up and solve word problems involving escape velocity.

### Notes:

## Escape Velocity

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth's gravity.

To explain the calculation, it is necessary to understand that our formula for gravitational potential energy is actually a simplification. Really, because the force of gravity attracts objects to the center of the Earth, the equation should be:

$$U_g = mg\Delta h = mg(h - h_o)$$

where  $h_o$  is the distance to the center of the Earth (radius of the Earth), and  $h$  is the total distance between the object and the center of the Earth. Normally, we calculate gravitational potential energy based on a difference in height between some height above the ground ( $h$ ) and the ground ( $h = 0$ ).

However, when we consider the problem of escaping from the gravity of the entire planet, we need to consider the potential energy relative to the center of the Earth, where the total force of gravity would be zero.

Use this space for summary and/or additional notes:

To do this, the rocket's kinetic energy ( $\frac{1}{2}mv^2$ ) must be greater than or equal to the Earth's gravitational potential energy ( $mgh$ ). This means  $\frac{1}{2}mv^2 = F_g h$ . We can apply Newton's Law of Universal Gravitation:

$$\frac{1}{2}mv^2 = F_g h = \left( \frac{Gm_1 m_2}{d^2} \right) h$$

However, because we need to escape the gravitational pull of the entire planet, we need to measure  $h$  from the center of the Earth, not the surface. This means that at the surface of the earth,  $h$  in the above equation is the same as  $d$  in Newton's Law of Universal Gravitation. Similarly, the mass of the spaceship is one of the masses (let's choose  $m_2$ ) in Newton's Law of Universal Gravitation. This gives:

$$F_g h = F_g d = \frac{Gm_1 m_2}{d^2} d = \frac{Gm_1 m_2}{d}$$

$$\frac{1}{2}m_2 v_e^2 = \frac{Gm_1 m_2}{d}$$

$$\frac{1}{2}v_e^2 = \frac{Gm_1}{d}$$

$$v_e = \sqrt{\frac{2Gm_{planet}}{d}}$$

At the surface of the Earth, where  $m_{planet} = 5.98 \times 10^{24}$  kg and  $d = 6.37 \times 10^6$  m,  $v_e = 1.12 \times 10^4 \frac{m}{s} = 11200 \frac{m}{s}$ . This equals approximately 25 100 miles per hour.

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth's escape velocity of  $11200 \frac{m}{s}$  to escape Earth's gravity. What velocity did the space ship need to achieve in order to escape the moon's gravity and return to Earth? (I.e., what is the escape velocity on the surface of the moon?)

A: 
$$v_e = \sqrt{\frac{2Gm_{moon}}{d_{moon}}}$$

$$v_e = \sqrt{\frac{(2)(6.67 \times 10^{-11})(7.35 \times 10^{22})}{1.74 \times 10^6}}$$

$$v_e = 2370 \frac{m}{s}$$

Use this space for summary and/or additional notes.

## Newton's Cradle

**Unit:** Energy & Momentum

**NGSS Standards:** HS-PS3-1

**MA Curriculum Frameworks (2006):** 2.5

**Knowledge/Understanding Goals:**

- how Newton's Cradle demonstrates the conservation of energy and the conservation of momentum

**Language Objectives:**

- Accurately describe how Newton's cradle illustrates the principles of conservation of energy and momentum, using appropriate academic language.

**Notes:**

### Newton's Cradle

Newton's Cradle is the name given to a set of identical balls that are able to swing suspended from wires:



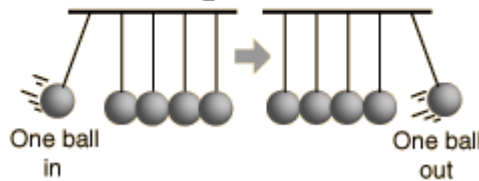
When one ball is swung and allowed to collide with the rest of the balls, the momentum transfers through the balls and one ball is knocked out from the opposite end. When two balls are swung, two balls are knocked out from the opposite end. And so on.

This apparatus demonstrates a mostly elastic collision, in which all of the momentum and most of the kinetic energy are conserved.

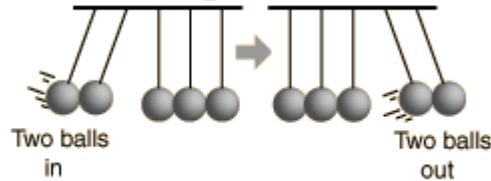
Use this space for summary and/or additional notes:

Before the collision, the moving ball(s) have momentum ( $mv$ ) and kinetic energy ( $\frac{1}{2}mv^2$ ). There are no external forces, which means momentum must be conserved. The collision is mostly elastic, which means kinetic energy is mostly also conserved. The only way for the same momentum and kinetic energy to be present after the collision is for the same number of balls to swing away from the opposite end with the same velocity.

Momentum in:  $mv = \text{momentum out}$   
Kinetic energy in:  $\frac{1}{2}mv^2 = \text{kinetic energy out}$

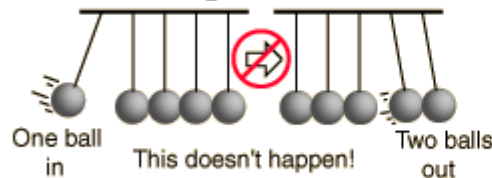


Momentum in:  $2mv = \text{momentum out}$   
Kinetic energy in:  $\frac{1}{2}2mv^2 = \text{kinetic energy out}$



If only momentum had to be conserved, two balls might move, but at  $\frac{1}{2}$  of the original velocity. But this doesn't happen.

Momentum in:  $mv = \text{momentum out}$   
Kinetic energy in:  $\frac{1}{2}mv^2 \neq \text{kinetic energy out!}$



Conserving momentum in this case requires that the two balls come out with half the speed.

$$\text{Momentum out} = 2m \frac{v}{2}$$

But this gives

$$\text{Kinetic energy out} = \frac{1}{2} 2m \frac{v^2}{4}$$

Which amounts to a loss of half of the kinetic energy!

Use this space for summary and/or additional notes.

Note also that if there were no friction, the balls would continue to swing forever. However, because of friction (between the balls and air molecules, within the strings as they stretch, *etc.*) and conversion of some of the kinetic energy to other forms (such as heat), the balls in a real Newton's Cradle will, of course, slow down and eventually stop.

Use this space for summary and/or additional notes.

## Power

**Unit:** Energy & Momentum

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 2.4

**Skills:**

- calculate power

**Language Objectives:**

- Understand and correctly use the term “power.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving power.

**Notes:**

power: a measure of the rate at which energy is applied or work is done. Power is calculated by dividing work (or energy) by time.

$$P = \frac{W}{t} = \frac{\Delta E_k}{t} = \frac{\Delta U}{t}$$

Power is a scalar quantity and is measured in Watts (W).

$$1 \text{ W} = 1 \frac{\text{J}}{\text{s}} = 1 \frac{\text{Nm}}{\text{s}} = 1 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Note that utility companies measure energy in kilowatt-hours. This is because

$$P = \frac{W}{t}, \text{ which means energy} = W = Pt.$$

Because 1 kW = 1000 W and 1 h = 3600 s, this means

$$1 \text{ kWh} = (1000 \text{ W})(3600 \text{ s}) = 3\,600\,000 \text{ J}$$

Use this space for summary and/or additional notes:



### Solving Power Problems

Most power problems require you to calculate the amount of work done or change in energy, which you should recall is:

$W = Fd$  if the force and displacement are in the same direction

$W = Fd\cos\theta$  if the force and displacement are in different directions and the angle between them is  $\theta$ .

$\Delta E_k = \frac{1}{2}m(v^2 - v_o^2)$  if the change in energy was caused by a change in velocity

$\Delta U_g = mg \Delta h$  if the change in energy was caused by a change in height

Once you have the work or energy, you plug it in for either  $W$ ,  $\Delta E_k$  or  $\Delta U$ , use the appropriate parts of the formula:

$$P = \frac{W}{t} = \frac{\Delta E_k}{t} = \frac{\Delta U}{t}$$

and solve for the missing variable.

### Sample Problems

Q: What is the power output of an engine that pulls with a force of 500. N over a distance of 100. m in 25 s?

A:  $W = Fd = (500)(100) = 50\,000 \text{ J}$

$$P = \frac{W}{t} = \frac{50\,000}{25} = 2\,000 \text{ W}$$

Use this space for summary and/or additional notes.

Q: A 60. W light bulb is powered by a generator that is powered by a falling 1.0 kg mass on a rope. Assuming the generator is 100% efficient (*i.e.*, no energy is lost when the generator converts its motion to electricity), how far must the mass fall in order to power the bulb at full brightness for 1.0 minute?

A:

$$P = \frac{\Delta U_g}{t} = \frac{mg \Delta h}{t}$$
$$60 = \frac{(1)(9.8) \Delta h}{60}$$
$$3600 = 9.8 \Delta h$$
$$\Delta h = \frac{3600}{9.8} = 367 \text{ m}$$

Note that 367 m is approximately the height of the top floor of the Empire State Building. Think about that the next time you decide that it doesn't matter if you leave the lights on when you're not in a room!

Use this space for summary and/or additional notes.

## Introduction: Special Relativity

**Unit:** Special Relativity

### Topics covered in this chapter:

Relative Motion.....	201
Relative Velocities.....	205
The Speed of Light .....	209
Length Contraction & Time Dilation .....	212
Energy-Momentum Relation .....	219

This chapter describes changes to the properties of objects when they are moving at speeds near the speed of light.

- *Relative Motion* and *Relative Velocities* describes relationships between objects that are moving with different velocities.
- *The Speed of Light* describes some familiar assumptions we have about our universe that do not apply at speeds near the speed of light.
- *Length Contraction & Time Dilation* and the *Energy-Momentum Relation* describe calculations involving changes in the length, time, mass, and momentum of objects as their speeds approach the speed of light.

New challenges in this chapter involve determining and understanding the changing relationships between two objects, both of which are moving in different directions and at different speeds.

### Standards addressed in this chapter:

#### Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

#### Massachusetts Curriculum Frameworks (2006):

No MA curriculum frameworks are addressed in this chapter.

Use this space for summary and/or additional notes:

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Relativity**, such as time dilation, length contraction, and mass-energy equivalence.

1. Special Relativity

**Skills learned & applied in this chapter:**

- keeping track of the changing relationships between two objects

Use this space for summary and/or additional notes.

## Relative Motion

**Unit:** Special Relativity

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Objectives:**

- the theory and concept of relativity

**Skill Objectives:**

- describe how a situation appears in different frames of reference

**Language Objectives:**

- Understand and correctly use the terms “relativity” and “reference frame” (“frame of reference”).
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

**Vocabulary:**

relativity: the concept that motion can be described only with respect to an observer, who may be moving or not moving relative to the object under consideration.

reference frame: the position and velocity of an observer watching an object that is moving relative to himself/herself.

Use this space for summary and/or additional notes:

**Notes:**

Consider the following picture, taken from a moving streetcar in New Orleans:



"New Orleans Streetcar." Photo by Don Chamblee.

If the streetcar is moving at a constant velocity and the track is smooth, the passengers may not notice that they are moving until they look out of the window.

In the reference frame of a person standing on the ground, the trolley and the passengers on it are moving at approximately 30 miles per hour.

In the reference frame of the trolley, the passengers sitting in the seats are stationary (not moving), and the ground is moving past the trolley at approximately 30 miles per hour.

Of course, you might want to say that the person on the ground has the "correct" reference frame. However, despite what you might prefer, neither answer is more correct than the other. Either reference frame is valid, which means either description of what is moving and what is stationary is equally valid.

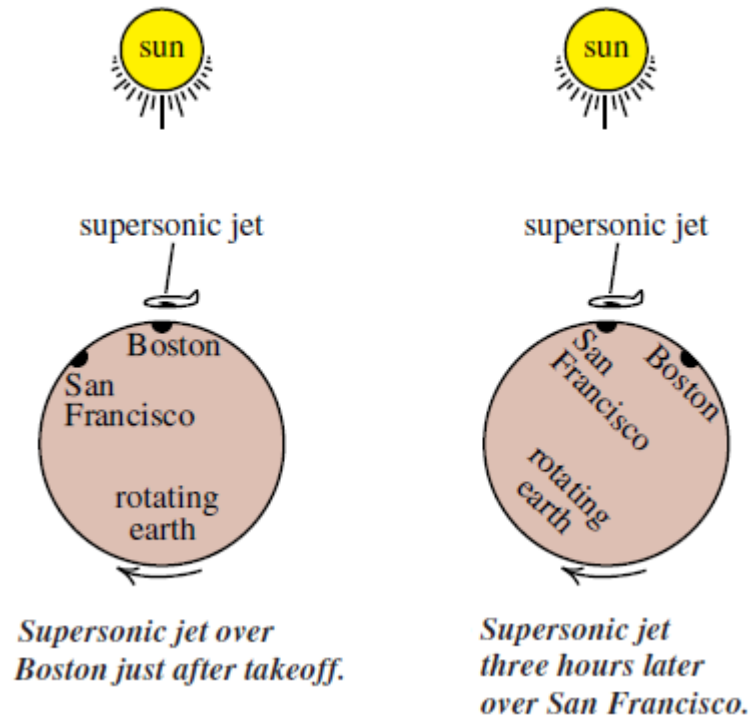
### **Principle of Relativity**

There is no experiment you can do that would allow you to determine conclusively whether or not you are moving uniformly.

Use this space for summary and/or additional notes.

Recall that “uniform motion” means moving with constant velocity, which means with a constant speed and direction. If velocity is constant, there is no acceleration, which means there is no net force.

Consider a fast airplane (such as a supersonic jet) flying from Boston to San Francisco. Imagine that the plane takes exactly three hours to fly to San Francisco, which is the same as the time difference between the two locations. Seen from outside the Earth, the situation might look like this:



You could argue that either:

1. The jet was moving from the airspace over Boston to the airspace over San Francisco.
2. The jet was stationary and the Earth rotated underneath it. (The jet needed to burn fuel to overcome the drag from the Earth's atmosphere as the Earth rotated, pulling its atmosphere and the jet with it.)

Use this space for summary and/or additional notes.

Of course, there are other reference frames you might consider as well.

3. Both the supersonic jet and the Earth are moving, because the Earth is revolving around the Sun with a speed of about  $30\,000 \frac{\text{m}}{\text{s}}$ .
4. The jet, the Earth and the Sun are all moving, because the sun is revolving around the Milky Way galaxy with a speed of about  $220\,000 \frac{\text{m}}{\text{s}}$ .
5. The jet, the Earth, the Sun, and the entire Milky Way galaxy are all moving through space toward the Great Attractor (a massive region of visible and dark matter about 150 million light-years away from us) with a speed of approximately  $1\,000\,000 \frac{\text{m}}{\text{s}}$ .
6. It is not clear whether there might be multiple Great Attractors, and what their motion might be relative to each other, or relative to some yet-to-be-discovered entity.

Regardless of which objects are moving with which velocities, if you are on the airplane and you drop a ball, you would observe that it falls straight down. In relativistic terms, we would say "In the reference frame of the moving airplane, the ball has no initial velocity, so it falls straight down."

Use this space for summary and/or additional notes.



## Relative Velocities

**Unit:** Special Relativity

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- understand how relative velocity depends on both the motion of an object and the motion of the observer

**Skills:**

- calculate relative velocities

**Language Objectives:**

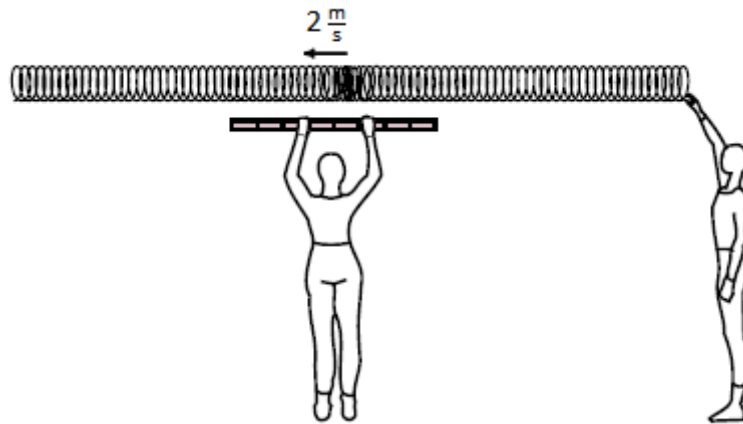
- Understand and correctly use the term “relative velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving relative velocities.

**Notes:**

Because the observation of motion depends on the reference frames of the observer and the object, calculations of velocity need to take these into account.

Use this space for summary and/or additional notes:

Suppose we set up a Slinky and a student sends a compression wave that moves with a velocity of  $2 \frac{\text{m}}{\text{s}}$  along its length:

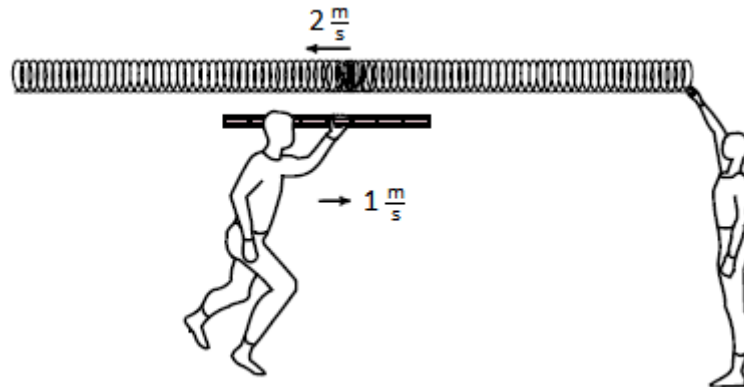


A second student holds a meter stick and times how long it takes the wave to travel from one end of the meter stick to the other. The wave would take 0.5 s to travel the length of the meter stick, and the student would calculate a velocity

$$\text{of } \frac{1\text{m}}{0.5\text{ s}} = 2 \frac{\text{m}}{\text{s}}.$$

Use this space for summary and/or additional notes.

Suppose instead that the student with the meter stick is running with a velocity of  $1 \frac{m}{s}$  toward the point of origin of the wave:



In this situation, you could use the velocities of the moving student and the wave and solve for the amount of time it would take for the wave and the end of the ruler to reach the same point. The calculation for this would be complicated, and the answer works out to be  $0.33 \text{ s}$ , which gives a velocity of  $3 \frac{m}{s}$ .

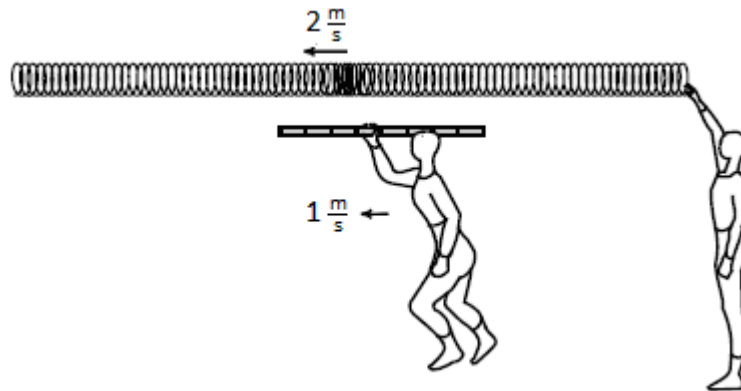
The easier way to calculate this number is to realize that the velocity of the wave relative to the moving student is simply the sum of the velocity vectors. The velocity of the wave relative to the moving student is therefore  $2 \frac{m}{s} + 1 \frac{m}{s} = 3 \frac{m}{s}$ .

This  $3 \frac{m}{s}$  is called the relative velocity, specifically the velocity of the wave relative to the moving student.

relative velocity: the apparent velocity of an object relative to an observer, which takes into account the velocities of both the object and the observer. When the object and the observer are both moving, the relative velocity is sometimes called the *approach velocity*.

Use this space for summary and/or additional notes.

Suppose instead that the student is running away from the point of origin (*i.e.*, in the same direction as the wave is traveling) with a velocity of  $1 \frac{m}{s}$ :



Now the relative velocity of the wave is  $2 \frac{m}{s} - 1 \frac{m}{s} = 1 \frac{m}{s}$  relative to the moving student.

If the student and the wave were moving with the same velocity (magnitude and direction), the relative velocity would be zero and the wave would appear stationary to the moving student.

Use this space for summary and/or additional notes.

## The Speed of Light

**Unit:** Special Relativity

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- the speed of light is constant in all reference frames

**Language Objectives:**

- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

If the principle of relativity is true, it must be true for all measurements and all reference frames, including those involving light.

In 1864, physicist James Clerk Maxwell united four calculus equations involving magnetic and electric fields into one unified theory of light. The four equations were:

1. Gauss's Law (which describes the relationship between an electric field and the electric charges that cause it).
2. Gauss's Law for Magnetism (which states that there are no discrete North and South magnetic charges).
3. Faraday's Law (which describes how a magnetic field creates an electric field).
4. Ampère's Law (which describes how an electric current can create a magnetic field), including Maxwell's own correction (which describes how a changing electric field can also create a magnetic field).

According to Maxwell's theory, light travels as an electromagnetic wave, *i.e.*, a wave of both electrical and magnetic energy. The moving electric field produces a magnetic field, and the moving magnetic field produces an electric field. Thus the electric and magnetic fields of the electromagnetic wave reinforce each other as they travel through space.

Use this space for summary and/or additional notes:

From Maxwell's equations, starting from the measured values for two physical constants: the electric permittivity of free space ( $\epsilon_0$ ) and the magnetic permeability of a vacuum ( $\mu_0$ ), Maxwell determined that the speed of light in a

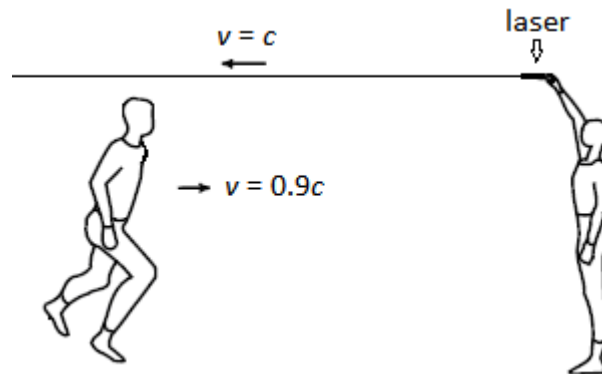
vacuum must be  $\frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \frac{\text{m}}{\text{s}}$ .

Both  $\mu_0$  and  $\epsilon_0$  are physical constants, which do not depend on the reference frame. Maxwell theorized that the speed of light in a vacuum must therefore also be a physical constant, and it therefore cannot depend on the reference frame that is used to measure it.

This means:

1. Light travels at a constant velocity, regardless of whether the light is produced by something that is moving or stationary.
2. The velocity of light is the same in all reference frames. This means a photon of light moves at the same velocity, regardless of whether that velocity is measured by an observer who is stationary or by an observer who is moving.

If the wave in the above relative velocity examples was a beam of laser light instead of a Slinky, and the observer was running at a relativistic speed (meaning a speed close to the speed of light), the velocity of the light, both students would measure exactly the same velocity for the light!



Because the speed of light (in a vacuum) is a constant, we use the variable  $c$  (which stands for "constant") to represent it in equations.

Use this space for summary and/or additional notes.

This idea seemed just as strange to 19<sup>th</sup> century physicists as it does today, and most physicists did not believe Maxwell for more than 45 years. However, several experiments have confirmed Maxwell's conclusion, and no experiment has ever successfully refuted it.

Note, however, that while the speed of light does not depend on the reference frame, it does depend on the medium it is traveling through. Light travels with a velocity of  $2.99792458 \times 10^8 \frac{\text{m}}{\text{s}}$  through a vacuum (empty space). However, when light travels through matter, (*e.g.*, air, glass, plastic, *etc.*), the velocity can be substantially slower.

Use this space for summary and/or additional notes.

## Length Contraction & Time Dilation

**Unit:** Special Relativity

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how and why distance and time change at speeds near the speed of light

**Skills:**

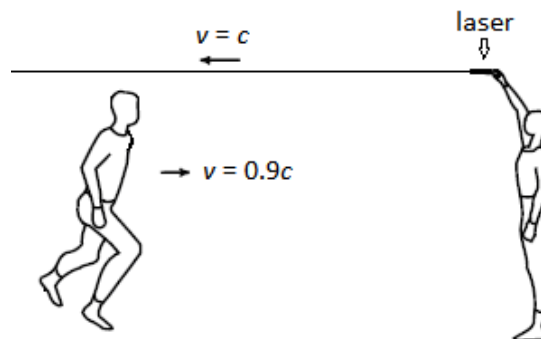
- calculate the apparent and proper lengths and times for objects moving at speeds close to the speed of light

**Language Objectives:**

- Understand and correctly use the terms “length contraction” and “time dilation.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to relativistic changes in length and time.

**Notes:**

Based on Maxwell’s conclusions, if an observer were somehow running with a relativistic speed of light toward an oncoming beam of light, the student should measure the same velocity of light as a stationary observer:



Use this space for summary and/or additional notes:



However, the amount of time it takes for a photon of light to pass the moving observer must be significantly less than the amount of time it would take a photon to pass a stationary observer.

Because velocity depends on distance and time, if the velocity of light cannot change, then as the observer approaches the speed of light, this means the distance and/or time must change!

For most people, the idea that distance and time depend on the reference frame is just as strange and uncomfortable as the idea that the speed of light cannot depend on the reference frame.

Use this space for summary and/or additional notes.

### Length Contraction

If an object is moving at relativistic speeds and the velocity of light must be constant, then distances must become shorter as velocity increases. This means that as the velocity of an object approaches the speed of light, distances in its reference frame approach zero. The Dutch physicist Hendrick Lorentz determined that the apparent change in length should vary according to the formula:

$$L = L_o \sqrt{1 - v^2/c^2}$$

where:

$L$  = length of moving object

$L_o$  = "proper length" of object (length of object at rest)

$v$  = velocity of object

$c$  = velocity of light

The ratio of  $L_o$  to  $L$  is named after Lorentz and is called the Lorentz factor ( $\gamma$ ):

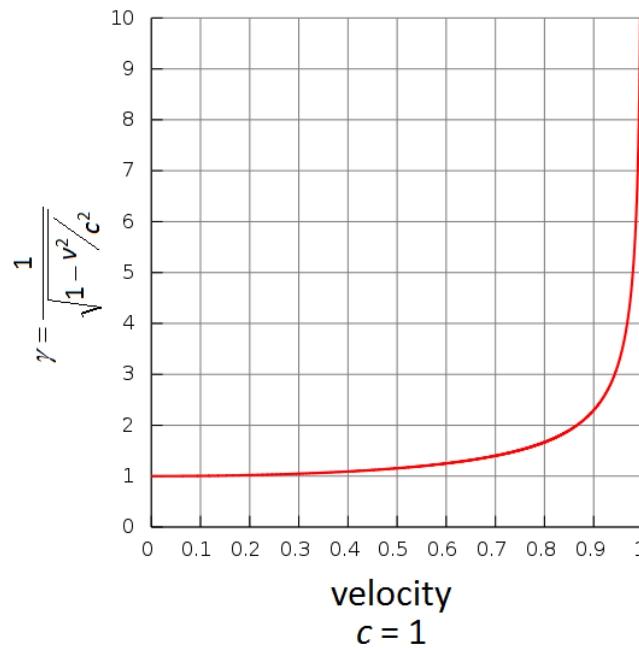
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

The contracted length is therefore given by the equations:

$$L = \frac{L_o}{\gamma} \quad \text{or} \quad \frac{L_o}{L} = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Use this space for summary and/or additional notes.

The Lorentz factor,  $\gamma$ , is 1 at rest and approaches infinity as the velocity approaches the speed of light:



In the discussion that follows, we will see that::

1. Length decreases as velocity increases. At the speed of light,  $\gamma$  approaches infinity, which means length ( $L$ ) shrinks to zero:

$$L = \frac{L_o}{\gamma}$$

2. Time slows down as velocity increases. At the speed of light,  $\gamma$  approaches infinity, which means the time that passes for the object moving at the speed of light ( $\Delta t$ ) shrinks to zero:

$$\Delta t' = \gamma \Delta t$$

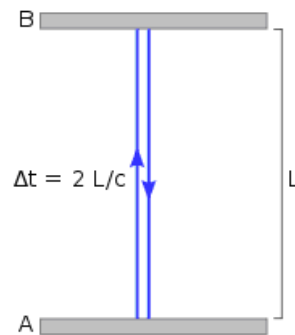
3. Mass increases as velocity increases. At the speed of light,  $\gamma$  approaches infinity, which means that the mass ( $m$ ) also approaches infinity:

$$m = \gamma m_o$$

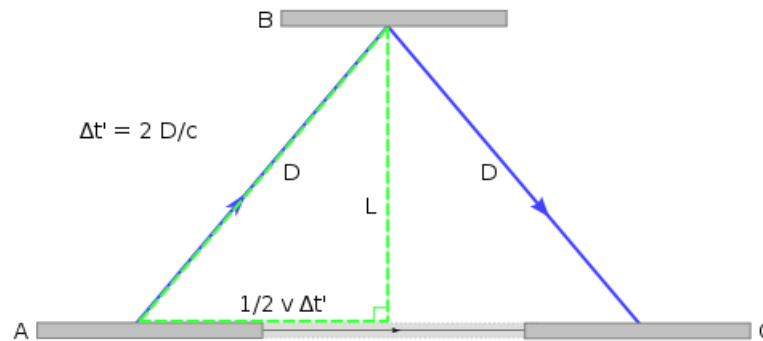
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### Time Dilation

In order to imagine how time is affected at relativistic speeds, imagine a clock that keeps time by sending a pulse of light from point (A), bouncing it off a mirror at point (B) and then measuring the time it takes to reach a detector back at point (A). The distance between the two surfaces is  $L$ , and the time for a pulse of light to travel to the mirror and back is therefore  $\Delta t = \frac{2L}{c}$ .



However, suppose the clock is moving at a relativistic speed. In the moving reference frame the situation looks exactly like the situation above. However, from an inertial (stationary) reference frame, the situation would look like the following:



Notice that in the stationary reference frame, the pulses of light must travel farther because of the motion of the mirror and detector. Because the speed of light is constant, the longer distance takes a longer time.

Use this space for summary and/or additional notes.

In other words, time is longer in the inertial (stationary) reference frame than it is in the moving reference frame!

This conclusion has significant consequences. For example, events that happen in two different locations could be simultaneous in one reference frame, but occur at different times in another reference frame!

Using arguments similar to those for length contraction, the equation for time dilation turns out to be:

$$\Delta t' = \gamma \Delta t \quad \text{or} \quad \frac{\Delta t'}{\Delta t} = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

where:

$\Delta t'$  = time difference between two events in stationary reference frame

$\Delta t$  = time difference between two events in moving reference frame

$v$  = velocity of moving reference frame

$c$  = velocity of light

Use this space for summary and/or additional notes.

**Sample problem:**

Q: In order to avoid detection by the Borg, the starship Enterprise must make itself appear to be less than 25 m long. If the rest length of the Enterprise is 420 m, how fast must it be travelling? What fraction of the speed of light is this?

A:  $L = 25 \text{ m}$

$$L_o = 420 \text{ m}$$

$$c = 2.998 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$\gamma = \frac{L_o}{L} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\frac{420}{25} = \frac{1}{\sqrt{1 - v^2 / (2.998 \times 10^8)^2}}$$

$$\frac{25}{420} = 0.0595 = \sqrt{1 - v^2 / (2.998 \times 10^8)^2}$$

$$(0.0595)^2 = 0.00354 = 1 - \frac{v^2}{8.988 \times 10^{16}}$$

$$\frac{v^2}{8.988 \times 10^{16}} = 1 - 0.00354 = 0.99646$$

$$v^2 = (0.99646)(8.988 \times 10^{16})$$

$$v^2 = 8.956 \times 10^{16}$$

$$v = \sqrt{8.956 \times 10^{16}} = 2.993 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$\frac{2.993 \times 10^8}{2.998 \times 10^8} = 0.998 c$$

Use this space for summary and/or additional notes.

## Energy-Momentum Relation

**Unit:** Special Relativity

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how and why mass changes at relativistic speeds
- the relationship between energy, momentum, and mass

**Language Objectives:**

- Accurately describe and apply the concepts described in this section, including Albert Einstein's famous equation  $E = mc^2$ , using appropriate academic language.

**Notes:**

The momentum of an object also changes according to the Lorentz factor as it approaches the speed of light:

$$p = \gamma p_o \quad \text{or} \quad \frac{p}{p_o} = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

where:

- $p$  = momentum of object in moving reference frame
- $p_o$  = momentum of object in stationary reference frame
- $v$  = velocity of moving reference frame
- $c$  = velocity of light

Because momentum is conserved, an object's momentum in its own reference frame must remain constant. Therefore, the object's mass must change at relativistic speeds!

Use this space for summary and/or additional notes:

The equation for relativistic mass is:

$$m = \gamma m_o \quad \text{or} \quad \frac{m}{m_o} = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

where:

$m$  = mass of object in moving reference frame

$m_o$  = mass of object at rest

Therefore we can write the momentum equation as:

$$p = \gamma m_o v$$

Note that as the velocity of the object approaches the speed of light, the denominator of the Lorentz factor,  $\sqrt{1 - v^2/c^2}$  approaches zero, which means that the Lorentz factor approaches infinity.

Therefore the momentum of an object must also approach infinity as the velocity of the object approaches the speed of light.

This relationship creates a potential problem. An object with infinite momentum must have infinite kinetic energy, but Einstein's equation  $E = mc^2$  is finite. While it is true that the relativistic mass becomes infinite as velocity approaches the speed of light, there is still a discrepancy. Recall from mechanics that:

$$E_k = \frac{p^2}{2m}$$

According to this formula, the energy predicted using relativistic momentum should increase faster than the energy predicted by using  $E = mc^2$  with relativistic mass. Obviously the amount of energy cannot depend on how the calculation is performed; the problem must therefore be that Einstein's equation needs a correction for relativistic speeds.

Use this space for summary and/or additional notes.



The solution is to modify Einstein's equation by adding a momentum term. The resulting energy-momentum relation is:

$$E^2 = (pc)^2 + (mc^2)^2$$

This equation gives results that are consistent with length contraction, time dilation and relativistic mass.

For an object at rest, its momentum is zero, and the equation reduces to the familiar form:

$$E^2 = 0 + (mc^2)^2$$

$$E = mc^2$$

Use this space for summary and/or additional notes.



## Introduction: Thermal Physics (Heat)

**Unit:** Thermal Physics (Heat)

### Topics covered in this chapter:

Heat & Temperature .....	226
Heat Transfer .....	230
Energy Conversion .....	234
Efficiency .....	237
Specific Heat Capacity & Calorimetry .....	241
Phase Changes & Heating Curves .....	249
Thermal Expansion.....	255

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Heat & Temperature* describes the concept of heat as a form of energy and how heat energy is different from temperature.
- *Heat Transfer, Energy Conversion* and *Efficiency* describe how to calculate the rate of the transfer of heat energy from one object to another.
- *Specific Heat Capacity & Calorimetry* describes different substances' and objects' abilities to store heat energy. *Phase Changes & Heating Curves* addresses the additional calculations that apply when a substance goes through a phase change (such as melting or boiling).
- *Thermal Expansion* describes the calculation of the change in size of an object caused by heating or cooling.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Use this space for summary and/or additional notes:

**Standards addressed in this chapter:**

**Next Generation Science Standards (NGSS):**

**HS-PS2-6.** Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

**HS-PS3-1.** Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

**HS-PS3-2.** Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

**HS-PS3-4.** Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

**Massachusetts Curriculum Frameworks (2006):**

**3.1** Explain how heat energy is transferred by convection, conduction, and radiation.

**3.2** Explain how heat energy will move from a higher temperature to a lower temperature until equilibrium is reached.

**3.3** Describe the relationship between average molecular kinetic energy and temperature. Recognize that energy is absorbed when a substance changes from a solid to a liquid to a gas, and that energy is released when a substance changes from a gas to a liquid to a solid. Explain the relationships among evaporation, condensation, cooling, and warming.

**3.4** Explain the relationships among temperature changes in a substance, the amount of heat transferred, the amount (mass) of the substance, and the specific heat of the substance.

Use this space for summary and/or additional notes.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Thermal Properties**, such as temperature, heat transfer, specific and latent heats, and thermal expansion.
- **Laws of Thermodynamics**, such as first and second laws, internal energy, entropy, and heat engine efficiency.
  1. Heat and Temperature
  2. The Kinetic Theory of Gases & the Ideal Gas Law
  3. The Laws of Thermodynamics
  4. Heat Engines

**Skills learned & applied in this chapter:**

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Use this space for summary and/or additional notes.

## Heat & Temperature

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** HS-PS3-2

**MA Curriculum Frameworks (2006):** 3.2, 3.3

### Knowledge/Understanding Goals:

- the difference between heat and temperature
- thermal equilibrium

### Language Objectives:

- Understand and correctly use the terms “heat,” “temperature,” “system,” “surroundings,” “enthalpy,” “entropy,” and “thermometer.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

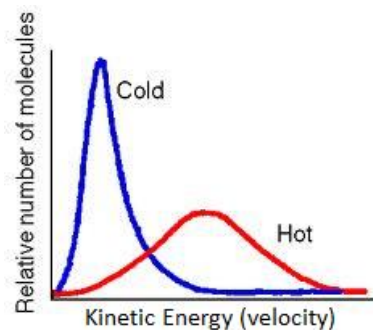
### Notes:

heat: energy that can be transferred by moving atoms or molecules via transfer of momentum.

temperature: a measure of the average kinetic energy of the particles (atoms or molecules) of a system.

thermometer: a device that measures temperature, most often via thermal expansion and contraction of a liquid or solid.

Note that heat is the energy itself, whereas temperature is a measure of the quality of the heat—the average of the kinetic energies of the individual molecules:



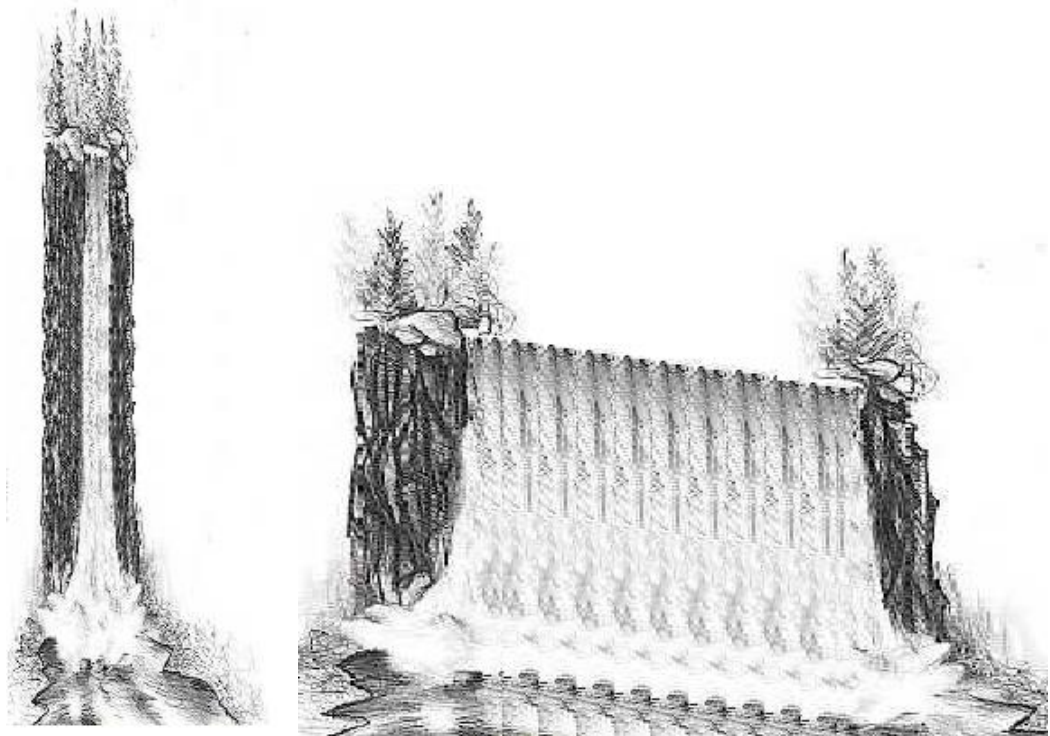
Use this space for summary and/or additional notes:

When objects are placed in contact, heat is transferred via the transfer of momentum that occurs when the individual molecules collide. Molecules that have more energy transfer more energy than they receive. Molecules that have less energy receive more energy than they transfer. This means two things:

1. Heat always transfers from objects with a higher temperature (more kinetic energy) to objects with a lower temperature (less kinetic energy).
2. If you wait long enough, all of the molecules will have the same temperature (*i.e.*, the same average kinetic energy).

In other words, the temperature of one object relative to another determines which direction the heat will flow, much like the way the elevation of one location relative to another determines which direction water will flow.

However, the total heat (energy) contained in an object depends on the mass as well as the temperature, in the same way that the total energy of the water going over a waterfall depends on the amount of water as well as the height:



Use this space for summary and/or additional notes.

### Heat Flow

system: the region being considered in a problem.

surroundings: everything that is outside of the system.

*E.g.*, if a metal block is heated, we would most likely define the system to be the block, and the surroundings to be everything outside of the block.

We generally use the variable  $Q$  to represent heat in physics equations.

Heat flow is always represented in relation to the system.

Heat Flow	Sign of $Q$	System	Surroundings
from surroundings to system	+ (positive)	gains heat (gets warmer)	lose heat (get colder)
from system to surroundings	- (negative)	loses heat (gets colder)	gain heat (get hotter)

A positive value of  $Q$  means heat is flowing into the system. Because the heat is transferred from the molecules outside the system to the molecules in the system, the temperature of the system increases, and the temperature of the surroundings decreases.

A negative value of  $Q$  means heat is flowing out of the system. Because the heat is transferred from the molecules in the system to the molecules outside the system, the temperature of the system decreases, and the temperature of the surroundings increases.

This can be confusing. Suppose you set a glass of ice water on a table. When you pick up the glass, your hand gets colder because heat is flowing from your hand (which is part of the surroundings) into the system (the glass of ice water). This means the system (the glass of ice water) is gaining heat, and the surroundings (your hand, the table, *etc.*) are losing heat. The value of  $Q$  would be positive in this example.

In simple terms, you need to remember that your hand is part of the surroundings, not part of the system.

Use this space for summary and/or additional notes.



thermal equilibrium: when all of the particles in a system have the same average kinetic energy (temperature). When a system is at thermal equilibrium, no net heat is transferred. (*i.e.*, collisions between particles may still transfer energy, but the average temperature of the particles in the system—what we measure with a thermometer—is not changing.)

## Laws of Thermodynamics

thermodynamics: the study of heat-related (thermal) energy changes (dynamics)

enthalpy: the “usable” heat content of an object or system

entropy: the “unusable” thermal energy in a system. Energy in the form of entropy is unavailable because it has “escaped” or “spread out”. (Entropy is sometimes called “disorder” or “randomness”, but this is not correct in the thermodynamic sense. The entropy of your room is a *thermodynamic* property, not a commentary on how much dirty laundry is on the floor!) A quantitative study of entropy requires an analysis of possible energy states and their probabilities, which is beyond the scope of this course.

The laws of thermodynamics state that:

0. If a system is at thermal equilibrium, every component of the system has the same temperature.
1. Heat always flows from a region of higher enthalpy to a region of lower enthalpy. This means you can't get more heat out of something than you put in. (“You can't win.”)
2. In almost every change, some energy is irretrievably lost to the surroundings. Entropy is a measure of this “lost” energy. The entropy of the universe is always increasing, which means on any practical scale, you always get out less energy than you put in. (“You can't break even.”)
3. In any closed system, the total energy (enthalpy + entropy + work) remains constant. If energy was “lost,” it turned into an increase in entropy. (“You can't get out of the game.”)

Use this space for summary and/or additional notes.

## Heat Transfer

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** HS-PS2-6, HS-PS3-2

**MA Curriculum Frameworks (2006):** 3.1

**Knowledge/Understanding:**

- heat transfer via conduction, radiation & convection

**Skills:**

- calculate heat transfer using Fourier's Law of Heat Conduction

**Language Objectives:**

- Understand and correctly use the terms "conduction," "convection," "radiation," "conductor," and "insulator."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Fourier's Law of Heat Conduction.

**Notes:**

Heat transfer is the flow of heat energy from one object to another. Heat transfer usually occurs through three distinct mechanisms: conduction, radiation, and convection.

conduction: transfer of heat through collisions of particles that are in direct contact. Conduction occurs when there is a net transfer of momentum from the molecules of an object with a higher temperature transfer to the molecules of an object with a lower temperature.

thermal conductivity ( $k$ ): a measure of the amount of heat that a given length of a substance can conduct in a specific amount of time. Thermal conductivity is measured in units of  $\frac{W}{m \cdot ^\circ C}$ .

conductor: an object that allows heat to pass through itself easily; an object with high thermal conductivity.

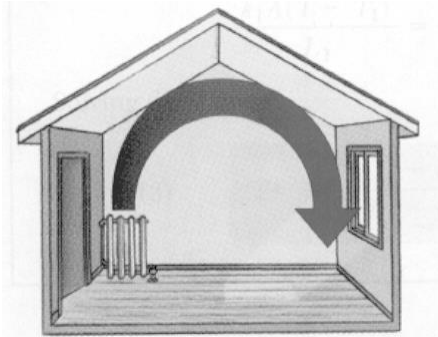
Use this space for summary and/or additional notes:

insulator: an object that does not allow heat to pass through itself easily; a poor conductor of heat; an object with low thermal conductivity.

radiation: transfer of heat through space via electromagnetic waves (light, microwaves, *etc.*)

convection: transfer of heat through exchange of particles that have a higher temperature for particles that have a lower temperature. Convection usually occurs when air moves around a room.

Natural convection occurs in a heated room because cool air is more dense than warm air. The cool air sinks to the ground, displacing the less dense warm air. When the rising warm air hits the ceiling, it moves across the room, pushing the cool air along the ceiling, down to the floor, across the floor, and back to the radiator.



Forced convection can be achieved by moving heated or cooled air using a fan. Examples of this include ceiling fans and convection ovens.

Use this space for summary and/or additional notes.

### Calculating Heat Transfer by Conduction

Heat transfer by conduction can be calculated using Fourier's Law of Heat Conduction:

$$\frac{Q}{t} = -kA \frac{\Delta T}{L}$$

where:

$Q$  = heat transferred (J)

$t$  = time (s)

$k$  = coefficient of thermal conductivity ( $\frac{W}{m \cdot ^\circ C}$ )

$A$  = cross-sectional area ( $m^2$ )

$\Delta T$  = temperature difference (K or  $^\circ C$ )

$L$  = length (m)

For insulation (the kind you have in the walls and attic of your home), the effectiveness is measured by the "R value", where

$$R = \frac{L}{k}$$

Use this space for summary and/or additional notes.

**Sample Problems:**

Q: A piece of brass is 5.0 mm (0.0050 m) thick and has a cross-sectional area of  $0.010 \text{ m}^2$ . If the temperature on one side of the metal is  $65^\circ\text{C}$  and the temperature on the other side is  $25^\circ\text{C}$ , how much heat will be conducted through the metal in 30. s? The coefficient of thermal conductivity for brass is  $120 \frac{\text{W}}{\text{m}\cdot^\circ\text{C}}$ .

$$\begin{aligned} \text{A: } \frac{Q}{t} &= -kA \frac{\Delta T}{L} \\ \frac{Q}{30} &= -(120)(0.010) \left( \frac{65 - 25}{0.0050} \right) \\ \frac{Q}{30} &= -9600 \\ Q &= -288000 \text{ J} = -288 \text{ kJ} \end{aligned}$$

(Note that because the quantities of heat that we usually measure are large, values are often given in kilojoules or megajoules instead of joules.)

Q: Suppose your house has 15 cm-thick insulation, with an R value of 16, the temperature inside your house is  $21^\circ\text{C}$  and the temperature outside is  $0.0^\circ\text{C}$ . How much heat is lost through one square meter of insulation over an 8-hour (28800 s) period?

$$\text{A: An R value of 16 means } \frac{L}{k} = 16, \text{ which means } \frac{k}{L} = \frac{1}{16}.$$

$$\begin{aligned} \frac{Q}{t} &= -kA \frac{\Delta T}{L} = -\frac{k}{L} A \Delta T \\ \frac{Q}{28800} &= -\frac{1}{16} (1)(21) \\ \frac{Q}{28800} &= -1.3125 \\ Q &= -37800 \text{ J} = -37.8 \text{ kJ} \end{aligned}$$

Use this space for summary and/or additional notes.

## Energy Conversion

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** HS-PS3-1

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- conversion of energy between forms

**Language Objectives:**

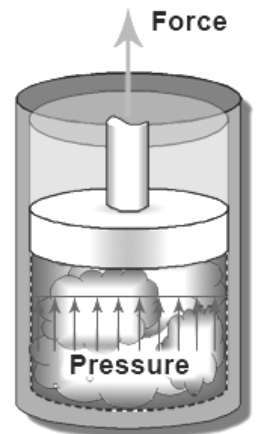
- Accurately describe the law of conservation of energy, using appropriate academic language.

**Notes:**

The law of conservation of energy states that total energy is always conserved, but that energy can be converted from one form to another.

We have already seen this in mechanics with the conversion between gravitational potential energy and kinetic energy.

Heat is energy. Like other forms of energy, it can do work. For example, in a steam engine, heat is used to boil water in a sealed container. As more water boils, there is more gas in the boiler, which makes the pressure increase. If the gas can only expand by pushing against something (like a piston), the force from the pressure can do work by moving the piston and whatever it's connected to. (We will revisit the concept of pressure as a force when we study fluid mechanics. For now, it's enough to understand that heat energy can be converted to kinetic energy.)

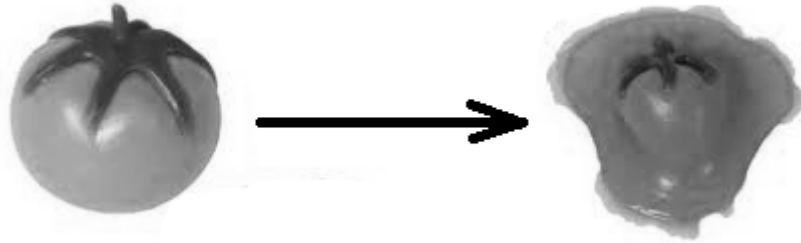


In mechanics, recall that collisions can be elastic or inelastic. In an elastic collision, kinetic energy is conserved; in an inelastic collision, some of the kinetic energy is converted to other forms, mostly heat.

Use this space for summary and/or additional notes:

We can use the law of conservation of energy to estimate the amount of energy converted to heat in a completely inelastic collision.

Consider a 0.150 kg “splat ball” hitting the wall at a velocity of  $20.0 \frac{\text{m}}{\text{s}}$ .



After the collision, the velocity of the ball and the wall are both zero. This means the kinetic energy of the ball after the collision is zero. Because energy must be conserved, this means all of the kinetic energy from the ball must have been converted to heat.

$$E_k = \frac{1}{2}mv^2$$

$$E_k = (\frac{1}{2})(0.150)(20.0)^2 = 30.0 \text{ J}$$

Use this space for summary and/or additional notes.

Now consider the same splat ball with a mass of 0.150 kg and a velocity of  $20.0 \frac{\text{m}}{\text{s}}$  hitting a 1.00 kg piece of wood that is initially at rest. This is still an inelastic collision, but now the wood is free to move, which means it has kinetic energy after the collision.

To solve this problem, we need to use conservation of momentum to find the velocity of the splat ball + wood after the collision, and then use the velocity before and after to calculate the change in kinetic energy.

Before the collision:

$$\vec{p} = m_{sb} \vec{v}_{sb} + m_w \vec{v}_w$$
$$\vec{p} = (0.150)(+20.0) + 0 = +3.00 \text{ N}\cdot\text{s}$$

$$E_k = \frac{1}{2} m_{sb} v_{sb}^2 + \frac{1}{2} m_w v_w^2$$
$$E_k = (\frac{1}{2})(0.150)(20.0)^2 + 0 = 30.0 \text{ J}$$

After the collision:

$$\vec{p} = (m_{sb} + m_w) \vec{v}$$
$$+ 3.00 = (0.150 + 1.00) \vec{v} = 1.15 \vec{v}$$
$$\vec{v} = +2.61 \frac{\text{m}}{\text{s}}$$

$$E_k = \frac{1}{2} m v^2$$
$$E_k = (\frac{1}{2})(1.15)(2.61)^2 = 3.91 \text{ J}$$

This means there is  $30.0 - 3.91 = 26.1 \text{ J}$  of kinetic energy that is “missing” after the collision. This “missing” energy is mostly converted to heat. If you could measure the temperature of the “splat ball” and the wood extremely accurately before and after the collision, you would find that both would be warmer as a result of the “missing” 26.1 J of energy.

Use this space for summary and/or additional notes.



## Efficiency

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- what efficiency means
- a sense of the efficiencies of different processes and machines

**Language Objectives:**

- Accurately describe the concept of energy efficiency, using appropriate academic language.

**Notes:**

efficiency ( $\eta$ ): the ratio of the energy consumed by a device or process to the energy output by the device or process.

Devices and processes that convert energy from one form to another are often described in terms of their energy efficiency. A device that is 100% efficient would convert 100% of the energy to a new form.

$$\eta = \frac{\text{usable energy out}}{\text{total energy in}}$$

For example, an electric motor converts electricity to motion (kinetic energy). An electric motor that is 100% efficient would convert 50 J of electric potential energy to 50 J of kinetic energy. An electric motor that is 80% efficient would convert 50 J of electric potential energy to 40 J of kinetic energy. The other 10 J of energy would be converted to other forms of energy, such as heat created by friction within the motor.

We often say that this difference is energy that is “lost”, but remember that the law of conservation of energy tells us that energy never disappears. When we say energy is “lost,” we mean it is “converted to another form that we can’t recover” (entropy), which is usually mostly heat.

Use this space for summary and/or additional notes:

The following table gives energy conversion efficiencies for common devices and processes. In all of these cases, the “lost” energy is converted to heat that is given off to the surroundings.

**Energy Conversion Efficiency**

<b>Device/Process</b>	<b>Typical Efficiency</b>
gas generator	up to 40%
coal/gas fired power plant	45%
combined cycle power plant	60%
hydroelectric power plant	up to 90%
wind turbine	up to 59%
solar cell	6–40%; usually 15%
hydrogen fuel cell	up to 85%
internal combustion engine	25%
electric motor, small (10–200 W)	50–90%
electric motor, large (> 200 W)	70–99%
photosynthesis in plants	up to 6%
human muscle	14–27%
refrigerator	20%
refrigerator, energy-saving	40–50%
light bulb, incandescent	0.7–5%
light bulb, fluorescent	8–16%
light bulb, LED	4–15%
electric heater	100%
firearm	30%

You may notice that an electric heater is 100% efficient, because all of its energy is converted to heat. However, this does not mean that electric heat is necessarily the best choice for your home, because the power plant that generated the electricity was probably only 45% efficient.

Use this space for summary and/or additional notes.

## Heating Efficiency

Heating efficiency is calculated the same way:

$$\eta = \frac{Q_{out}}{Q_{in}} = \frac{\text{usable heat out}}{\text{total energy in}}$$

“Usable heat out” means heat that is not lost to the environment. For example, if the boiler or furnace in your house is 70% efficient, that means 70% of the energy from the gas or oil that it burned was used to heat the steam, hot water or hot air that was used to heat your house. The other 30% of the energy heated the air in the boiler or furnace, and that heat was lost to the surroundings when the air went up the chimney.

Older boilers and furnaces (pre-1990s) are typically 70% efficient. New boilers are around 80% efficient, and high-efficiency boilers that use heat exchangers to collect the heat from the exhaust air before it goes up the chimney are 90–97% efficient.

Use this space for summary and/or additional notes.

### Heat Pump Efficiency

A heat pump is a device that “pumps” heat from one place to another. A refrigerator and an air conditioner are examples of heat pumps. A refrigerator uses a fluid (“refrigerant”) to transfer (or “pump”) heat from the inside of the refrigerator to the outside of the refrigerator (and into your kitchen). This is why you can’t cool off the kitchen by leaving the refrigerator door open—if you had a 100% efficient refrigerator, the heat that you pumped out would mix with the cool air and you would be back where you started. Moreover, because refrigerators are only 20–40% efficient, the refrigerator generates a significant amount of heat, which means leaving the refrigerator door open actually heats the kitchen up!

Carnot’s Theorem, named after French physicist Nicolas Carnot, states that the maximum possible efficiency of a heat pump is related to the ratio of the temperature of the heat transfer fluid (liquid or gas) when it enters the heat pump to the temperature when it exits:

$$\eta \leq 1 - \frac{T_{in}}{T_{out}}$$

If all of the heat were successfully pumped, the temperature out would be the same as the temperature in, and the efficiency would be 100%.

For example, if a gasoline engine takes in air at 20.°C (293 K) and the exhaust temperature is 800.°C (1073 K), the Carnot efficiency of the engine would be:

$$\eta \leq 1 - \frac{T_{in}}{T_{out}} = 1 - \frac{293}{1073} = 1 - 0.27 = 0.73$$

This means that the maximum possible efficiency of this engine (if the heated exhaust accounted for all of the energy loss) would be 73%.

Use this space for summary and/or additional notes.

## Specific Heat Capacity & Calorimetry

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** HS-PS2-6, HS-PS3-1

**MA Curriculum Frameworks (2006):** 3.4

**Knowledge/Understanding:**

- specific heat capacity
- calorimetry

**Skills:**

- solve calorimetry (specific heat) problems

**Language Objectives:**

- Understand and correctly use the terms “specific heat capacity” and “calorimetry.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to specific heat capacity and calorimetry.

**Notes:**

Different objects have different abilities to hold heat. For example, if you enjoy pizza, you may have noticed that the sauce holds much more heat (and burns your mouth much more readily) than the cheese or the crust.

The amount of heat that a given mass of a substance can hold is based on its specific heat capacity.

specific heat capacity ( $C$ ): a measure of the amount of heat required per gram of a substance to produce a specific temperature change in the substance.

$C_p$ : specific heat capacity, measured at constant pressure. For gases, this means the measurement was taken allowing the gas to expand as it was heated.

$C_v$ : specific heat capacity, measured at constant volume. For gases, this means the measurement was made in a sealed container, allowing the pressure to rise as the gas was heated.

Use this space for summary and/or additional notes:

For solids and liquids,  $C_p \approx C_v$  because the pressure and volume change very little as they are heated. For gases,  $C_p > C_v$  (always). For ideal gases,  $C_p - C_v = R$ , where  $R$  is the gas constant. (We will see the gas constant again when we study fluid mechanics.)

When there is a choice,  $C_p$  is more commonly used than  $C_v$  because it is easier to measure. When dealing with solids and liquids, most physicists just use  $C$  for specific heat capacity and don't worry about the distinction.

### Calculating Heat from a Temperature Change

The amount of heat gained or lost when an object changes temperature is given by the equation:

$$Q = mC\Delta T$$

where:

$Q$  = heat (J or kJ)

$m$  = mass (g or kg)

$C$  = specific heat capacity ( $\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ )

$\Delta T$  = temperature change (K or  $^{\circ}\text{C}$ )

Because problems involving heat often involve large amounts of energy, specific heat capacity is often given in kilojoules per kilogram per degree Celsius.

Note that  $1 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \equiv 1 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}} \equiv 1 \frac{\text{J}}{\text{g}\cdot^{\circ}\text{C}}$  and  $1 \frac{\text{cal}}{\text{g}\cdot^{\circ}\text{C}} \equiv 1 \frac{\text{kcal}}{\text{kg}\cdot^{\circ}\text{C}} = 4.184 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ .

You need to be careful with the units. If the mass is given in kilograms (kg), your specific heat capacity will have units of  $\frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$  and the heat energy will come out in kilojoules (kJ). If mass is given in grams, you will use units of  $\frac{\text{J}}{\text{g}\cdot^{\circ}\text{C}}$  and the heat energy will come out in joules (J).

Use this space for summary and/or additional notes.

## Specific Heat Capacities of Some Substances

Substance	Specific Heat Capacity ( $\frac{\text{kJ}}{\text{kgK}}$ )
water at 20°C	4.181
ethylene glycol (anti-freeze)	2.460
ice at -10°C	2.080
steam at 100°C	2.11
steam at 130°C	1.99
vegetable oil	2.00
air	1.012
aluminum	0.897
glass	0.84
iron	0.450
copper	0.385
brass	0.380
silver	0.233
lead	0.160
gold	0.129

Use this space for summary and/or additional notes.

## Calorimetry

calorimetry: the measurement of heat flow

In a calorimetry experiment, heat flow is calculated by measuring the mass and temperature change of an object and applying the specific heat capacity equation.

calorimeter: an insulated container for performing calorimetry experiments.

coffee cup calorimeter: a calorimeter that is only an insulated container—it does not include a thermal mass (such as a mass of water). It is usually made of styrofoam, and is often nothing more than a styrofoam coffee cup.

bomb calorimeter: a calorimeter for measuring the heat produced by a chemical reaction. A bomb calorimeter is a double-wall metal container with water between the layers of metal. The heat from the chemical reaction makes the temperature of the water increase. Because the mass and specific heat of the calorimeter (water and metal) are known, the heat produced by the reaction can be calculated from the increase in temperature of the water.

Use this space for summary and/or additional notes.



### Solving Coffee Cup Calorimetry Problems

Most coffee cup calorimetry problems involve placing a hot object in contact with a colder one. Many of them involve placing a hot piece of metal into cold water.

To solve the problems, assume that both objects end up at the same temperature. The heat lost by the hot object ( $Q_h$ ) equals the heat gained by the cold object ( $Q_c$ ). (However, remember that  $Q_h$  will be negative because the hot object is losing heat.)

$$Q_c = m_c C_c \Delta T_c$$

$$Q_h = m_h C_h \Delta T_h$$

$$Q_c = -Q_h$$

$$m_c C_c \Delta T_c = -m_h C_h \Delta T_h$$

Notice that there are six quantities that you need: the two masses ( $m_h$  and  $m_c$ ), the two specific heat capacities ( $C_h$  and  $C_c$ ), and the two temperature changes ( $\Delta T_h$  and  $\Delta T_c$ ). (You might be given initial and final temperatures for either or both, in which case you'll need to subtract.) The problem will give you all but one of these and you will need to find the missing one.

Don't fret about the negative sign. The value of  $\Delta T_h$  will be negative (because it is cooling off), and the two minus signs will cancel.

Use this space for summary and/or additional notes.

### Steps for Solving Coffee Cup Calorimetry Problems

1. Identify the variables for both the hot and cold substance. This can be tricky because you have the two masses, the two specific heat capacities, and the two temperature changes. For each quantity, you have to identify both the variable and which substance it applies to.
2. Look up the specific heat capacities of the substances involved.
3. Plug each set of numbers into the equation  $Q = mC\Delta T$ . (i.e., you'll have two separate  $Q = mC\Delta T$  equations.)
  - a. Remember that for the substance that is cooling off, heat is going out of the system, which means the equation will be  $Q = -mC\Delta T$ .
  - b. Because  $\Delta T$  will be negative for the substance that was cooling off, the two negative signs will cancel.
4. Use the fact that  $Q$  is the same for both equations to solve for the unknown quantity. This will involve doing one of the following:
  - a. Calculate the value of  $Q$  from one equation and use it in the other equation.
  - b. If you need to find the final temperature, set the two  $mC\Delta T$  expressions (or  $mC(T_f - T_i)$  expressions) equal to each other.

Use this space for summary and/or additional notes.

**Sample Problems:**

Q: An 0.050 kg block of aluminum is heated and placed in a calorimeter containing 0.100 kg of water at 20°C. If the final temperature of the water was 30°C, to what temperature was the aluminum heated?

A: The heat gained by the water equals the heat lost by the aluminum.

The heat gained by the water is:

$$Q = mC\Delta T$$

$$Q = (0.100 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(+10^\circ\text{C})$$

$$Q = 4.18 \text{ kJ}$$

The heat lost by the metal must therefore be 4.18 kJ.

$$Q = -mC\Delta T$$

$$4.18 \text{ kJ} = -(0.050 \text{ kg})(0.900 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) \Delta T$$

$$4.18 = -(0.045)(\Delta T)$$

$$\Delta T = \frac{4.18}{-0.045} = -92.9^\circ\text{C}$$

The temperature of the aluminum was  $-93^\circ\text{C}$  (*i.e.*, it went down by  $93^\circ\text{C}$ )

$$\Delta T = T_f - T_i$$

$$-92.9 = 30 - T_i$$

$$T_i = 123.1^\circ\text{C}$$

This means the initial temperature must have been  $123.1^\circ\text{C}$ .

Use this space for summary and/or additional notes.

Q: An 0.025 kg block of copper at 95°C is dropped into a calorimeter containing 0.075 kg of water at 25°C. What is the final temperature?

A: Once again, the heat lost by the copper equals the heat gained by the water.

$$\begin{aligned} -Q_c &= Q_w \\ -m_c C_c \Delta T_c &= m_w C_w \Delta T_w \\ -(0.025)(0.385)(T_f - 95) &= (0.075)(4.18)(T_f - 25) \\ -(0.009625)(T_f - 95) &= (0.3138)(T_f - 25) \\ -(0.009625 T_f - 0.9144) &= 0.3138 T_f - 7.845 \\ -0.009625 T_f + 0.9144 &= 0.3138 T_f - 7.845 \\ +0.009625 T_f &= +0.009625 T_f \\ 0.9144 &= 0.3234 T_f - 7.845 \\ +7.845 &= +7.845 \\ 8.759 &= 0.3234 T_f \\ \frac{8.759}{0.3234} &= 27^\circ\text{C} = T_f \end{aligned}$$

Note that because the specific heat of the water is so much higher than that of copper, and because the mass of the water was larger than the mass of the copper, the final temperature ended up much closer to the initial water temperature.

Use this space for summary and/or additional notes.

## Phase Changes & Heating Curves

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** HS-PS3-1

**MA Curriculum Frameworks (2006):** 3.3

### Knowledge/Understanding Goals:

- phases and phase changes
- how heat is transferred in a phase change
- why evaporation causes cooling

### Skills:

- calculate the heat absorbed or produced during phase changes
- plot and make calculations from heating curves

### Language Objectives:

- Understand and correctly use the terms “phase change,” “heat of fusion,” “heat of vaporization,” and “heating curve.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve heating curve problems.

### Notes:

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

solid: molecules are rigidly connected. A solid has a definite shape and volume.

liquid: molecules are loosely connected—bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of particles with rapidly changing charges.

Use this space for summary and/or additional notes:

phase change: when an object or substance changes from one phase to another through gaining or losing heat.

Breaking bonds requires energy. Forming bonds releases energy. This is true for the bonds that hold a solid or liquid together as well as for chemical bonds (regardless of what your biology teacher may have told you!)

*I.e.*, you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it). Energy is released when a gas condenses or a liquid freezes. (*E.g.*, ice in your ice tray needs to give off heat in order to freeze. Your freezer needs to remove that heat in order to make this happen.)

The reason evaporation causes cooling is because the system (the water) needs to absorb heat from its surroundings (*e.g.*, your body) in order to make the change from a liquid to a gas (vapor). When the water absorbs heat from you and evaporates, you have less heat, which means you have cooled off.

## Calculating the Heat of Phase Changes

heat of fusion ( $\Delta H_{fus}$ ) (sometimes called “latent heat” or “latent heat of fusion”): the amount of heat required to melt one kilogram of a substance. This is also the heat released when one kilogram of a liquid substance freezes. For example, the heat of fusion of water is  $334 \frac{\text{kJ}}{\text{kg}}$  (which also equals  $334 \frac{\text{J}}{\text{g}}$ ).

The heat required to melt a sample of water is therefore:

$$Q = m\Delta H_{fus} = m(334 \frac{\text{kJ}}{\text{kg}})$$

(Note that by convention, heat is in kilojoules rather than Joules.)

heat of vaporization ( $\Delta H_{vap}$ ): the amount of heat required to vaporize (boil) one kilogram of a substance. This is also the heat released when one kilogram of a gas condenses. For example, the heat of vaporization of water is  $2260 \frac{\text{kJ}}{\text{kg}}$ .

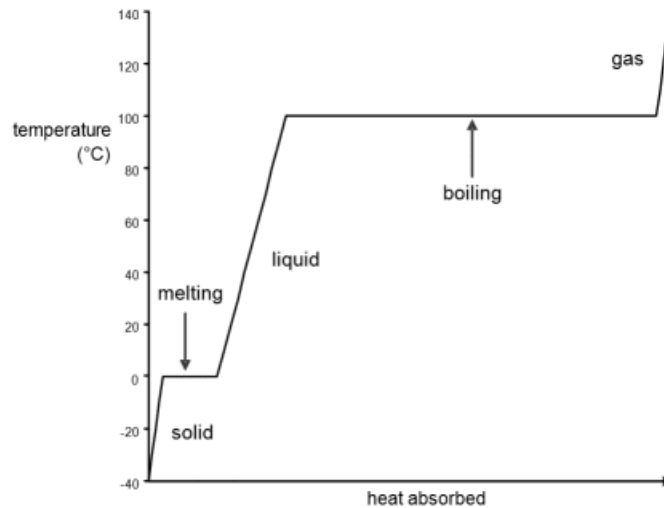
The heat required to boil a sample of water is therefore:

$$Q = m\Delta H_{vap} = m(2260 \frac{\text{kJ}}{\text{kg}})$$

(Again, note that heat is in kilojoules rather than Joules.)

Use this space for summary and/or additional notes.

heating curve: a graph of temperature vs. heat added. The following is a heating curve for water:



In the “solid” portion of the curve, the sample is solid water (ice). As heat is added, the temperature increases. The specific heat capacity of ice is  $2.11 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$ , so the heat required is:

$$Q_{\text{solid}} = mC\Delta T = m(2.11 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}})(\Delta T)$$

In the “melting” portion of the curve, the sample is a mixture of ice and water. As heat is added, the ice melts, but the temperature remains at  $0^{\circ}\text{C}$  until all of the ice is melted. The heat of fusion of ice is  $334 \frac{\text{kJ}}{\text{kg}}$ , so the heat required is:

$$Q_{\text{melt}} = m\Delta H_{\text{fus}} = m(334 \frac{\text{kJ}}{\text{kg}})$$

In the “liquid” portion of the curve, the sample is liquid water. As heat is added, the temperature increases. The specific heat capacity of liquid water is  $4.184 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$ , so the heat required is:

$$Q_{\text{liquid}} = mC\Delta T = m(4.184 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}})(\Delta T)$$

Use this space for summary and/or additional notes.

In the “boiling” portion of the curve, the sample is a mixture of water and water vapor (steam). As heat is added, the water boils, but the temperature remains at 100°C until all of the water has boiled. The heat of vaporization of water is  $2260 \frac{\text{kJ}}{\text{kg}}$ , so the heat required is:

$$Q_{\text{boil}} = m\Delta H_{\text{vap}} = m(2260 \frac{\text{kJ}}{\text{kg}})$$

In the “gas” portion of the curve, the sample is water vapor (steam). As heat is added, the temperature increases. The specific heat capacity of steam is approximately  $2.08 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$  (at 100°C; the specific heat capacity of steam decreases as the temperature increases), so the heat required is:

$$Q_{\text{gas}} = mC\Delta T = m(2.08 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}})(\Delta T)$$

Use this space for summary and/or additional notes.



### Steps for Solving Heating Curve Problems

A heating curve problem is a problem in which a substance is heated across a temperature range that passes through the melting and/or boiling point of the substance, which means the problem includes heating or cooling steps and melting/freezing or boiling/condensing steps.

1. Sketch the heating curve for the substance over the temperature range in question. Be sure to include the melting and boiling steps as well as the heating steps.
2. From your sketch, determine whether the temperature range in the problem passes through the melting and/or boiling point of the substance.
3. Split the problem into:
  - a. Heating (or cooling) steps within each temperature range.
  - b. Melting or boiling (or freezing or condensing) steps.
4. Find the heat required for each step.
  - a. For the heating/cooling steps, use the equation  $Q = mC\Delta T$ .
  - b. For melting/freezing steps, use the equation  $Q = m\Delta H_{fus}$ .
  - c. For boiling/condensing steps, use the equation  $Q = m\Delta H_{vap}$ .
5. Add the values of  $Q$  from each step to find the total.

Use this space for summary and/or additional notes.

**Sample Problem**

Q: How much heat would it take to raise the temperature of 0.0150 kg of H<sub>2</sub>O from -25.0°C to +125.0°C?

A: The H<sub>2</sub>O starts out as ice. We need to:

1. Heat the ice from -25.0°C to its melting point (0°C).
2. Melt it.
3. Heat the water up to its boiling point (from 0°C to 100°C).
4. Boil it.
5. Heat the steam from 100°C to 125°C.

$$Q_1 = mC\Delta T = (0.0150 \text{ kg})(2.11 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(25^\circ\text{C}) = 0.79125 \text{ kJ}$$

$$Q_2 = m\Delta H_{fus} = (0.0150 \text{ kg})(334 \frac{\text{kJ}}{\text{kg}}) = 5.010 \text{ kJ}$$

$$Q_3 = mC\Delta T = (0.0150 \text{ kg})(4.184 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(100^\circ\text{C}) = 6.276 \text{ kJ}$$

$$Q_4 = m\Delta H_{vap} = (0.0150 \text{ kg})(2260 \frac{\text{kJ}}{\text{kg}}) = 33.90 \text{ kJ}$$

$$Q_5 = mC\Delta T = (0.0150 \text{ kg})(2.08 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(25^\circ\text{C}) = 0.780 \text{ kJ}$$

$$Q_{total} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$Q_{total} = 0.791 + 5.010 + 6.276 + 33.90 + 0.780 = 46.757 \text{ kJ}$$

Use this space for summary and/or additional notes.

## Thermal Expansion

**Unit:** Thermal Physics (Heat)

**NGSS Standards:** HS-PS2-6

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- thermal expansion in solids, liquids and gases

**Skills:**

- calculate changes in length and volume for solids and liquids
- calculate changes in volume for gases

**Language Objectives:**

- Understand and correctly use the term “thermal expansion.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to thermal expansion of solids, liquids and gases.

**Notes:**

expand: to become larger

contract: to become smaller

thermal expansion: an increase in the length and/or volume of an object caused by a change in temperature.

When a substance is heated, the particles it is made of move farther and faster. This causes the particles to move farther apart, which causes the substance to expand.

Solids tend to keep their shape when they expand. (Liquids and gases do not have definite shape to begin with.)

Use this space for summary and/or additional notes:

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0°C to 4°C. (Water expands as the temperature increases above 4°C.)

### Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coefficient of thermal expansion. The changes in length or volume are given by the following equations:

Length:  $\Delta L = \alpha L_i \Delta T$

Volume:  $\Delta V = \beta V_i \Delta T$

where:

$\Delta L$  = change in length(m)

$\Delta V$  = change in volume(m<sup>3</sup>)

$L_i$  = initial length(m)

$V_i$  = initial volume(m<sup>3</sup>)

$\alpha$  = linear coefficient of thermal expansion(1/°C)

$\beta$  = volumetric coefficient of thermal expansion(1/°C)

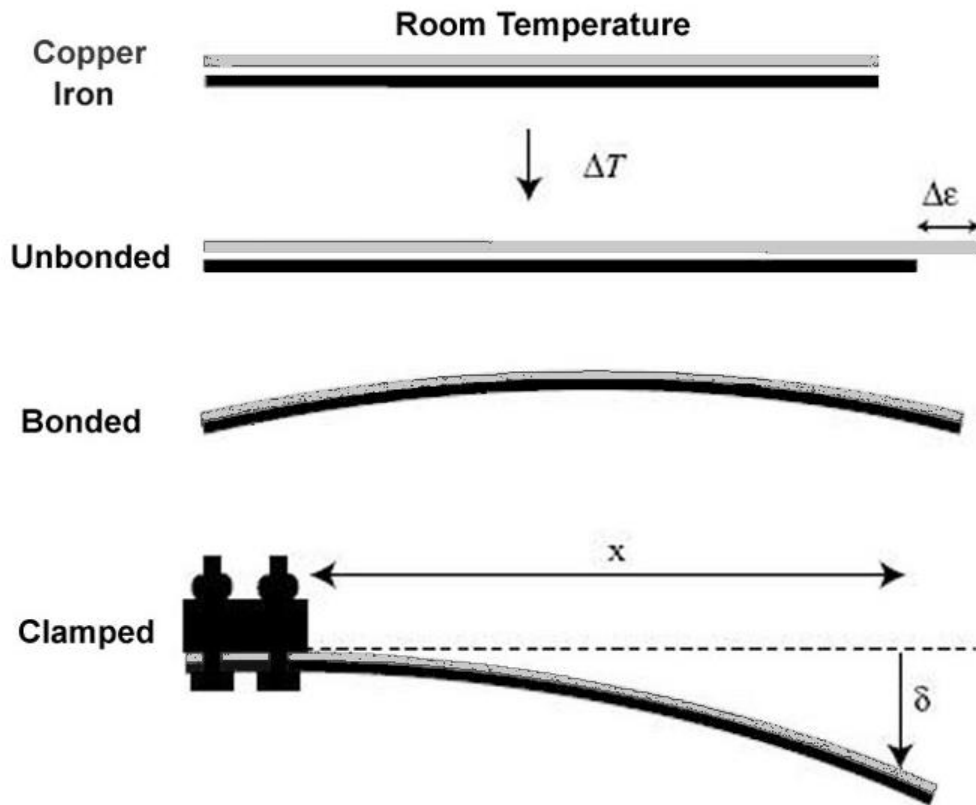
$\Delta T$  = temperature change

Values of  $\alpha$  and  $\beta$  at 20°C for some solids and liquids:

Substance	$\alpha$ (°C <sup>-1</sup> )	$\beta$ (°C <sup>-1</sup> )	Substance	$\alpha$ (°C <sup>-1</sup> )	$\beta$ (°C <sup>-1</sup> )
aluminum	$2.3 \times 10^{-5}$	$6.9 \times 10^{-5}$	gold	$1.4 \times 10^{-5}$	$4.2 \times 10^{-5}$
copper	$1.7 \times 10^{-5}$	$5.1 \times 10^{-5}$	iron	$1.18 \times 10^{-5}$	$3.33 \times 10^{-5}$
brass	$1.9 \times 10^{-5}$	$5.6 \times 10^{-5}$	lead	$2.9 \times 10^{-5}$	$8.7 \times 10^{-5}$
diamond	$1 \times 10^{-6}$	$3 \times 10^{-6}$	mercury	$6.1 \times 10^{-5}$	$1.82 \times 10^{-4}$
ethanol	$2.5 \times 10^{-4}$	$7.5 \times 10^{-4}$	silver	$1.8 \times 10^{-5}$	$5.4 \times 10^{-5}$
glass	$8.5 \times 10^{-6}$	$2.55 \times 10^{-5}$	water (liq.)	$6.9 \times 10^{-5}$	$2.07 \times 10^{-4}$

Use this space for summary and/or additional notes.

**bimetal strip:** a strip made from two metals with different coefficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.



Use this space for summary and/or additional notes.

**Sample Problems:**

Q: Find the change in length of an 0.40 m brass rod that is heated from 25°C to 980°C.

A: For brass,  $\alpha = 1.9 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ .

$$\Delta L = \alpha L_i \Delta T$$

$$\Delta L = (1.9 \times 10^{-5})(0.40)(955)$$

$$\Delta L = 0.0073 \text{ m}$$

Q: A typical mercury thermometer contains about 0.22 cm<sup>3</sup> (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25°C to 50.°C.

A: For mercury,  $\beta = 1.82 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ .

$$\Delta V = \beta V_i \Delta T$$

$$\Delta V = (1.82 \times 10^{-4})(0.22)(25)$$

$$\Delta V = 0.00091 \text{ cm}^3$$

If the distance from the 25°C to the 50°C mark is about 3.0 cm, we could use this information to figure out the bore of the thermometer:

$$V = \pi r^2 h$$

$$0.00091 = (3.14)r^2(3.0)$$

$$r^2 = 9.66 \times 10^{-5}$$

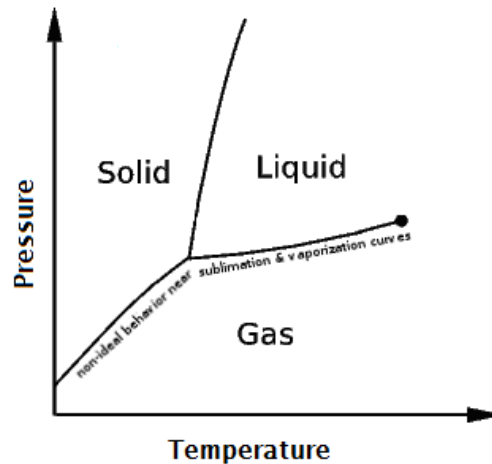
$$r = \sqrt{9.66 \times 10^{-5}} = 0.0098 \text{ cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is 0.020 cm or about 0.20 mm.

Use this space for summary and/or additional notes.

### Thermal Expansion of Gases

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*i.e.*, gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)



For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles' Law:

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

where volume can be any volume unit (as long as it is the same on both sides), but temperature must be in Kelvin.

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: If a 250 mL container of air is heated from 25°C to 95°C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

$$25^{\circ}\text{C} + 273 = 298\text{K}$$

$$95^{\circ}\text{C} + 273 = 368\text{K}$$

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

$$\frac{250}{298} = \frac{V_f}{368}$$

$$V_f = 308.7 = \boxed{310\text{ mL}}$$

Because we used mL for  $V_1$ , the value of  $V_2$  is therefore also in mL.

Use this space for summary and/or additional notes.



## Introduction: Electricity & Magnetism

**Unit:** Electricity & Magnetism

### Topics covered in this chapter:

Electric Charge .....	265
Coulomb's Law .....	271
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This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Electric Charge, Coulomb's Law, and Electric Fields* describe the behavior of individual charged particles and how to calculate the effects of these particles on each other.
- *Electric Current & Ohm's Law* describes equations and calculations involving the flow of charged particles (electric current).
- *Electrical Components, EMF & Internal Resistance of a Battery, Circuits, Series Circuits, Parallel Circuits, Mixed Series & Parallel Circuits, and Measuring Voltage, Current & Resistance* describe the behavior of electrical components in a circuit and how to calculate quantities relating to the individual components and the entire circuit, based on the way the components are arranged.

Use this space for summary and/or additional notes:

- *Magnetism* describes properties of magnets and what causes objects to be magnetic. *Electricity & Magnetism* describes how electricity and magnetism affect each other.

One of the new challenges encountered in this chapter is interpreting and simplifying circuit diagrams, in which different equations may apply to different parts of the circuit.

### Standards addressed in this chapter:

#### Next Generation Science Standards (NGSS):

**HS-PS2-4.** Use mathematical representations of ~~Newton's Law of Gravitation and Coulomb's Law~~ to describe and predict the ~~gravitational and electrostatic~~ forces between objects.

**HS-PS2-5.** Plan and conduct an investigation to provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.

**HS-PS2-6.** Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

**HS-PS3-2.** Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

**HS-PS3-3.** Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

**HS-PS3-5.** Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction.

#### Massachusetts Curriculum Frameworks (2006):

**5.1** Recognize that an electric charge tends to be static on insulators and can move on and in conductors. Explain that energy can produce a separation of charges.

**5.2** Develop qualitative and quantitative understandings of current, voltage, resistance, and the connections among them (Ohm's law).

Use this space for summary and/or additional notes.

**5.3** Analyze simple arrangements of electrical components in both series and parallel circuits. Recognize symbols and understand the functions of common circuit elements (battery, connecting wire, switch, fuse, resistance) in a schematic diagram.

**5.4** Describe conceptually the attractive or repulsive forces between objects relative to their charges and the distance between them (Coulomb's law).

**5.5** Explain how electric current is a flow of charge caused by a potential difference (voltage), and how power is equal to current multiplied by voltage.

**5.6** Recognize that moving electric charges produce magnetic forces and moving magnets produce electric forces. Recognize that the interplay of electric and magnetic forces is the basis for electric motors, generators, and other technologies.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Electric Fields, Forces, and Potentials**, such as Coulomb's law, induced charge, field and potential of groups of point charges, and charged particles in electric fields
- **Capacitance**, such as parallel-plate capacitors and time-varying behavior in charging/ discharging
- **Circuit Elements and DC Circuits**, such as resistors, light bulbs, series and parallel networks, Ohm's law, and Joule's law
- **Magnetism**, such as permanent magnets, fields caused by currents, particles in magnetic fields, Faraday's law, and Lenz's law.

1. Electric Charge
2. Electric Force
3. Electric Field
4. Electric Potential
5. Conductors and Insulators
6. Voltage
7. Current
8. Resistance
9. Energy, Power, and Heat
10. Circuits

Use this space for summary and/or additional notes.

11. Capacitors
12. Permanent Magnets
13. Magnetic Force on Charges
14. Magnetic Force on Current-Carrying Wires
15. The Magnetic Field Due to a current
16. Motional EMF
17. Faraday's Law

**Skills learned & applied in this chapter:**

- Working with material-specific constants from a table.
- Identifying electric circuit components.
- Simplifying circuit diagrams.

Use this space for summary and/or additional notes.

## Electric Charge

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.1, 5.4

### Knowledge/Understanding Goals:

- electric charge
- properties of electric charges
- conductors vs. insulators

### Language Objectives:

- Understand and correctly use the terms “electricity,” “charge,” “current,” “conductor,” “insulator,” and “induction.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

### Notes:

electric charge: a physical property of matter which causes it to experience a force when near other electrically charged matter.

positive charge: the charge of a proton. Originally defined as the charge left on a piece of glass when rubbed with silk. The glass becomes positively charged because the silk pulls electrons off the glass.

negative charge: the charge of an electron. Originally defined as the charge left on a piece of amber (or rubber) when rubbed with fur (or wool). The amber becomes negatively charged because the amber pulls the electrons off the fur.

static electricity: stationary electric charge, such as the charge left on silk or amber in the above definitions.

Use this space for summary and/or additional notes:

electric current (sometimes called electricity): the movement of electrons through a medium (substance) from one location to another. Note, however, that electric current is defined as the direction a positively charged particle would move. Thus electric current “flows” in the opposite direction from the actual electrons.



WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

### Devices that Produce, Use or Store Charge

capacitor: a device that stores electric charge.

battery: a device that uses chemical reactions to produce an electric current.

generator: a device that converts mechanical energy (motion) into an electric current.

motor: a device that converts an electric current into mechanical energy.

Use this space for summary and/or additional notes.

### Conductors vs. Insulators

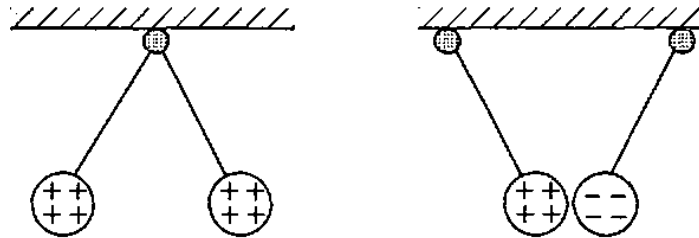
**conductor:** a material that allows charges to move freely through it. Examples of conductors include metals and liquids with positive and negative ions dissolved in them (such as salt water). When charges are transferred to a conductor, the charges distribute themselves evenly throughout the substance.

**insulator:** a material that does not allow charges to move freely through it. Examples of insulators include nonmetals and most pure chemical compounds (such as glass or plastic). When charges are transferred to an insulator, they cannot move, and remain where they are placed.

### Behavior of Charged Particles

**Like charges repel.** A pair of the same type of charge (two positive charges or two negative charges) exert a force that pushes the charges away from each other.

**Opposite charges attract.** A pair of opposite types of charge (a positive charge and a negative charge) exert a force that pulls the charges toward each other.



**Charge is conserved.** Electric charges cannot be created or destroyed, but can be transferred from one location or medium to another. (This is analogous to the laws of conservation of mass and energy.)

Use this space for summary and/or additional notes.

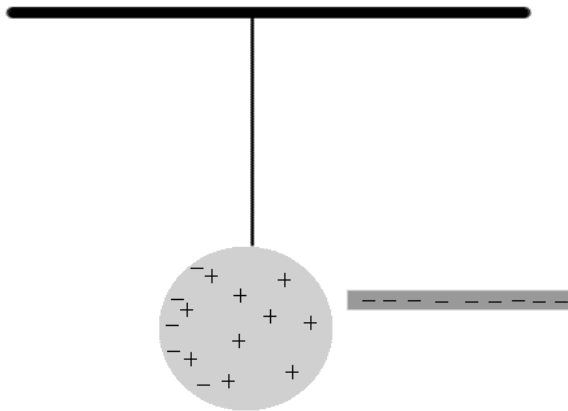
### Charge Density

The amount of electric charge on a surface is called the charge density. As with density (in the mass/volume sense), the variable used is usually the Greek letter rho, with a subscript q indicating charge ( $\rho_q$ ). Charge density can be expressed in terms of length, area, or volume, which means, the units for charge density can be  $\frac{C}{m}$ ,  $\frac{C}{m^2}$ , or  $\frac{C}{m^3}$ .

### Charging by Induction

induction: when an electrical charge on one object causes a charge in a second object.

When a charged rod is brought near a neutral object, the charge on the rod attracts opposite charges and repels like charges that are near it. The diagram below shows a negatively-charged rod repelling negative charges.

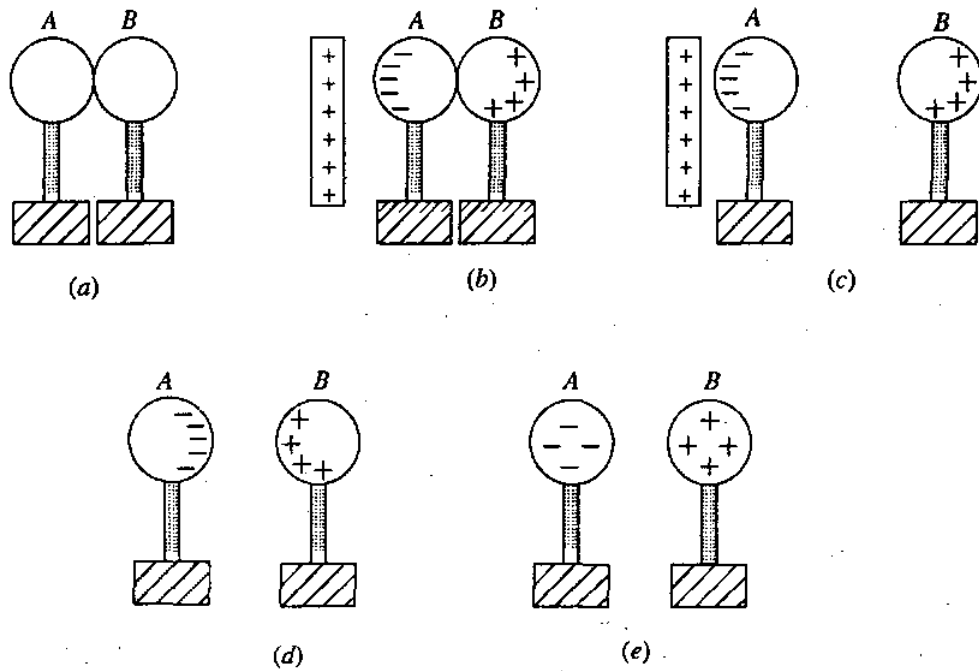


If the negatively-charged rod above were touched to the sphere, some of the charges from the rod would be transferred to the sphere at the point of contact, and the sphere would acquire an overall negative charge.

Use this space for summary and/or additional notes.



A process for inducing charges in a pair of metal spheres is shown below:



- (a) Metal spheres *A* and *B* are brought into contact.
- (b) A positively charged object is placed near (but not in contact with) sphere *A*. This induces a negative charge in sphere *A*, which in turn induces a positive charge in sphere *B*.
- (c) Sphere *B* (which is now positively charged) is moved away.
- (d) The positively charged object is removed.
- (e) The charges distribute themselves throughout the metal spheres.

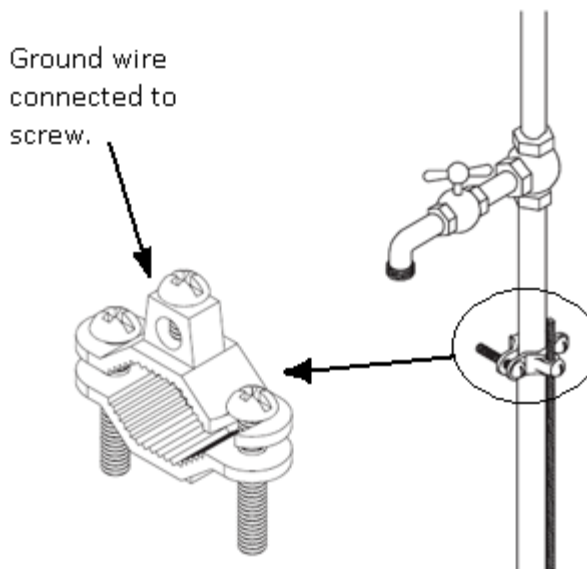
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## Grounding

For the purposes of our use of electric charges, the ground (Earth) is effectively an endless supply of both positive and negative charges. Under normal circumstances, if a charged object is touched to the ground, electrons will move to neutralize the charge, either by flowing from the object to the ground or from the ground to the object.

Grounding a charged object or circuit means neutralizing the electrical charge on an object or portion of the circuit.

In buildings, the metal pipes that bring water into the building are often used to ground the electrical circuits. The metal pipe is a good conductor of electricity, and carries the unwanted charge out of the building and into the ground outside.



Use this space for summary and/or additional notes.

## Coulomb's Law

**Unit:** Electricity & Magnetism

**NGSS Standards:** HS-PS2-4, HS-PS3-5

**MA Curriculum Frameworks (2006):** 5.4

**Skills:**

- understand & solve problems using Coulomb's Law

**Language Objectives:**

- Accurately describe Coulomb's Law using appropriate academic language.
- Set up and solve word problems relating to Coulomb's Law.

**Notes:**

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

1 C is the charge of  $6.2415 \times 10^{18}$  protons.

-1 C is the charge of  $6.2415 \times 10^{18}$  electrons.

One proton or electron (elementary charge) therefore has a charge of  $1.6022 \times 10^{-19}$  C.

Use this space for summary and/or additional notes:

Because charged particles exert a force on each other, that force can be measured and quantified. The force is directly proportional to the strengths of the charges and inversely proportional to the square of the distance. The formula is therefore:

$$\vec{F}_e = \frac{kq_1q_2}{d^2} \hat{d}_{12}$$

(vector form)

$$F_e = \frac{kq_1q_2}{d^2}$$

(scalar form)

where:

$\vec{F}_e$  = electrostatic force of repulsion between electric charges. A positive value of  $\vec{F}_e$  denotes that the charges are repelling (pushing away from) each other; a negative value of  $\vec{F}_e$  denotes that the charges are attracting (pulling towards) each other.

$k$  = electrostatic constant =  $8.9876 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$

$q_1$  and  $q_2$  = charges 1 and 2 respectively

$d$  = distance between the centers of the two charges

$\hat{d}_{12}$  = unit vector from  $q_1$  towards  $q_2$ .

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Use this space for summary and/or additional notes.

**Sample problem:**

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is  $1.60 \times 10^{-19}$  C, and the charge of a single electron is  $-1.60 \times 10^{-19}$  C.

$$37.1 \text{ pm} = 3.71 \times 10^{-11} \text{ m}$$

$$F_e = \frac{kq_1q_2}{d^2}$$

$$F_e = \frac{(8.99 \times 10^9)(1.60 \times 10^{-19})(-1.60 \times 10^{-19})}{(3.71 \times 10^{-11})^2}$$

$$F_e = -1.67 \times 10^{-7} \text{ N}$$

The value of the force is negative, which signifies that the force is attractive.

Use this space for summary and/or additional notes.

## Electric Fields

**Unit:** Electricity & Magnetism

**NGSS Standards:** HS-PS3-2

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

- drawing electric field lines
- calculating electric field strength

**Language Objectives:**

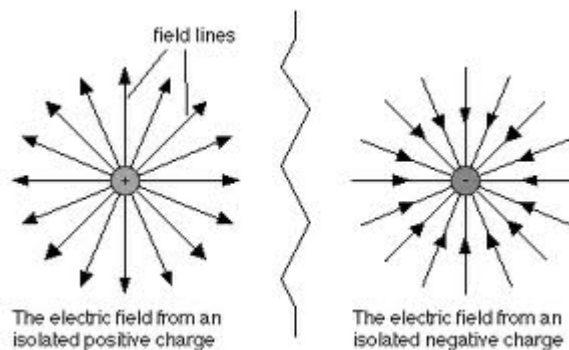
- Understand and correctly use the term “electric field.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to electric fields.

**Notes:**

force field: a region in which an object experiences a force because of some intrinsic property of the object that enables the force to act on it. One example is a gravitational field. Objects that mass (the intrinsic property) experience a force (gravity) when they are within a gravitational field (near the surface of the Earth). Force fields are vectors, which means they have both a magnitude and a direction.

electric field: an electrically charged region that exerts a force on any charged particle within the region.

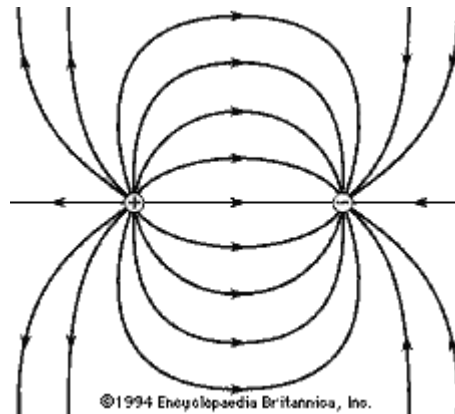
The simplest electric field is the region around a single charged particle:



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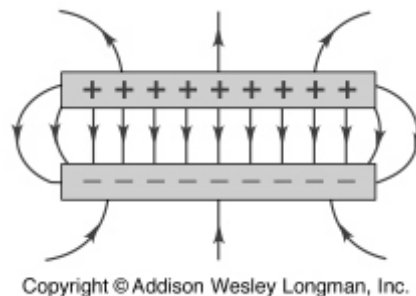
Field lines are vectors that show the directions of force on an object. In an electric field, the object is a positively-charged particle. This means that *the direction of the electric field is from positive to negative, i.e.*, field lines go outward in all directions from a positively-charged particle, and inward from all directions toward a negatively-charged particle.

If a positive and a negative charge are near each other, the field lines go from the positive charge toward the negative charge:



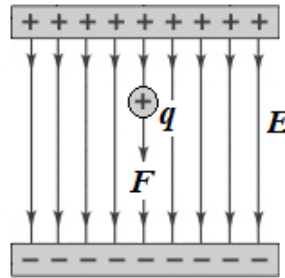
(Note that even though this is a two-dimensional drawing, the field itself is three-dimensional. Some field lines come out of the paper from the positive charge and go into the paper toward the negative charge, and some go behind the paper from the positive charge and come back into the paper from behind toward the negative charge.

In the case of two charged plates (flat surfaces), the field lines would look like the following:



Use this space for summary and/or additional notes.

We can measure the strength of an electric field by placing a particle with a positive charge ( $q$ ) in the field, and measuring the force ( $\vec{F}$ ) on the particle.



Coulomb's Law tells us that the force on the charge is due to the charges from the electric field:

$$F_e = \frac{kq_1q_2}{d^2}$$

If the positive and negative charges on the two surfaces that make the electric field are equal, *the force is the same everywhere in between the two surfaces.* (This is because as the particle gets farther from one surface, it gets closer to the other.) This means that the force on the particle is related only to the charges that make up the electric field and the charge of the particle.

We can therefore describe the electric field ( $\vec{E}$ ) as the force between the electric field and our particle, divided by the charge of our particle:

$$\vec{E} = \frac{\vec{F}}{q}$$

This is more commonly written:

$$\vec{F} = q\vec{E}$$

Use this space for summary and/or additional notes.



## Work Done by Electric Fields

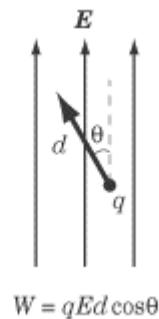
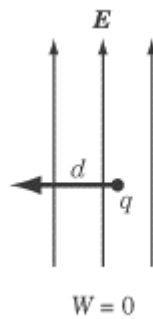
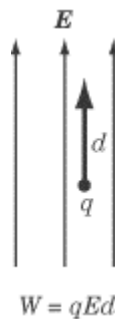
Recall that work is the dot product of force and displacement:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

Substituting  $q\vec{E}$  for  $\vec{F}$  (because  $\vec{F} = q\vec{E}$ ) gives:

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta$$

- $W$  = work (N · m)
- $q$  = charge (C)
- $\vec{E}$  = electric field strength ( $\frac{N}{C}$  or  $\frac{V}{m}$ )
- $E$  = magnitude of  $\vec{E}$  ( $\frac{N}{C}$  or  $\frac{V}{m}$ )
- $\vec{d}$  = displacement vector (m)
- $d$  = distance = magnitude of  $\vec{d}$  in the direction of force applied (m)



Use this space for summary and/or additional notes.

## Electric Potential

Recall from the work-energy theorem that work equals a change in energy. Because an electric field can do work on a charged particle, an electric field must therefore apply energy to the particle.

The ability of an electric field to apply energy to a charged particle over a distance is the electric *potential difference*. Potential difference is more commonly called *voltage* ( $V$ ), and is defined as the potential energy difference per unit of charge:

$$V = \frac{\Delta U}{q} = \frac{W}{q} = \frac{\vec{F} \cdot \vec{d}}{q} = \frac{Fd \cos \theta}{q} = \vec{E} \cdot \vec{d} = Ed \cos \theta$$

Voltage is a scalar quantity and is measured in volts (V). (This is perhaps the only instance in physics where the variable and unit symbol for the quantity are the same.)

$$1 \text{ V} \equiv 1 \frac{\text{N}\cdot\text{m}}{\text{C}} \equiv 1 \frac{\text{J}}{\text{C}}$$

Note that voltage is always measured between two points; there is no such thing as a voltage at one point. When we say that a battery supplies 1.5 V, we mean that this is the electric potential difference between the battery's positive and negative terminals.

Use this space for summary and/or additional notes.

## Electric Current & Ohm's Law

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.2, 5.5

**Knowledge/Understanding Goals:**

- electric potential (voltage)
- electric current
- resistance

**Skills:**

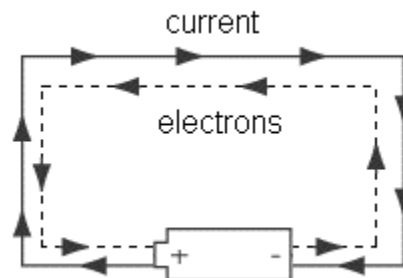
- perform calculations involving voltage, current, resistance and power

**Language Objectives:**

- Understand and correctly use the terms "current," "direct current," "alternating current," "potential difference," "voltage," "resistance," "power," and "work."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Ohm's Law.

**Notes:**

electric current: the flow of charged particles from one place to another, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged. This means that electric current "travels" in the opposite direction from the electrons.



Use this space for summary and/or additional notes:

Electric current ( $\vec{I}$ ) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta q}{t}$$

voltage (potential difference): the difference in electric potential energy between two locations, per unit of charge.

Potential difference is the work ( $W$ ) done on a charge per unit of charge ( $q$ ). Potential difference ( $V$ ) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

$$V = \frac{W}{q} = \vec{E} \cdot \vec{d}$$

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

resistance: the amount of electromotive force (electric potential) needed to force a given amount of current through an object.

Resistance ( $R$ ) is a scalar quantity and is measured in ohms ( $\Omega$ ). One ohm is one volt per ampere.

$$R = \frac{V}{I}$$

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{V}{R} \quad \text{or} \quad V = IR$$

Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.

Resistance is an intrinsic property of a substance. In the situations we will study in this course, the resistance of an object does not change.

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use resistors in circuits to reduce the amount of current that flows through the components.

Use this space for summary and/or additional notes.

resistivity: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance ( $\rho$ ), and of the length ( $L$ ) and cross-sectional area ( $A$ ) of the object. In MKS units, resistivity is measured in ohm-meters ( $\Omega\cdot\text{m}$ ).

$$R = \frac{\rho L}{A}$$

conductivity: the innate ability of a substance to conduct electricity. Conductivity ( $\sigma$ ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol  $\text{Ⓜ}$ ). (Note that "mho" is "ohm" spelled backwards.)

$$\sigma = \frac{1}{\rho}$$

capacitance: the ability of an object to hold an electric charge. Capacitance ( $C$ ) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt.

$$C = \frac{q}{V}$$

power: as discussed in the mechanics section of this course, power ( $P$ ) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$$

work: recall from mechanics that work ( $W$ ) equals power times time, and is measured in either newton-meters (N·m) or joules (J):

$$W = Pt = VIt = I^2Rt = \frac{V^2t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1 \text{ kW}\cdot\text{h} \equiv 3.6 \times 10^6 \text{ J} \equiv 3.6 \text{ MJ}$$

Use this space for summary and/or additional notes.

### Summary of Terms and Variables

Term	Variable	Unit	Term	Variable	Unit
charge	$q$ or $Q$	coulomb (C)	resistance	$R$	ohm ( $\Omega$ )
current	$I$	ampere (A)	capacitance	$C$	farad (F)
voltage	$V$	volt (V)	power	$P$	watt (W)
			work	$W$	joule (J)

### Alternating Current vs. Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

alternating current: electric current flows back and forth in one direction and then the other, like a sine wave. The current alternates at a particular frequency. In the U.S., household current is 110 volts AC with a frequency of 60 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Use this space for summary and/or additional notes.

## Electrical Components

**Unit:** Electricity & Magnetism

**NGSS Standards:** HS-PS2-6

**MA Curriculum Frameworks (2006):** 5.3

**Knowledge/Understanding Goals:**

- recognize common components of electrical circuits

**Language Objectives:**

- Recognize and be able to name and draw symbols for each of the electrical components described in this section.

**Notes:**














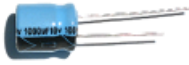




electrical component: an object that performs a specific task in an electric circuit.

A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

Use this space for summary and/or additional notes:

The following table describes some of the common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

Component	Symbol	Picture	Description
wire			Carries current in a circuit.
junction			Connection between two or more wires.
unconnected wires			Wires cross but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor			Stores charge.
diode			Allows current to flow in only one direction (from + to -).
light-emitting diode (LED)			Diode that gives off light.

Use this space for summary and/or additional notes.



Component	Symbol	Picture	Description
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).
transformer			Increases or decreases voltage.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground			Neutralizes charge.  (clamps to water pipe)

Use this space for summary and/or additional notes.

## EMF & Internal Resistance of a Battery

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.2, 5.5

**Knowledge/Understanding Goals:**

- resistance and resistivity

**Skills:**

- calculate the internal resistance of a battery

**Language Objectives:**

- Understand and correctly use the terms “battery,” “resistance,” and “internal resistance.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Ohm’s Law.

**Notes:**

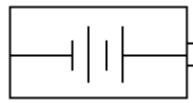
An ideal battery always supplies current at the voltage of the electrochemical cells inside of it. In a real battery, the voltage is a little less when the battery is “under load” (supplying current to a circuit) than when it is tested with no load. This difference is caused by real-world limitations of the chemical and physical processes that occur inside of the battery.

electromotive force (emf): the potential difference (voltage) supplied by a battery without load. This is the stated voltage of the battery. The term “electromotive force” literally means “electron-moving force”. EMF is often represented by the variable  $\epsilon$ .

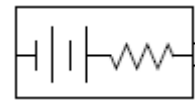
voltage: the observed potential difference between two points in a circuit. The voltage of a battery usually means the voltage under load.

Use this space for summary and/or additional notes:

To account for internal resistance, we model a battery as if it were a power supply with the ideal voltage, plus a resistor that is physically inside of the battery.



ideal



model

Note that this is a model; the actual situation is more complex, because in addition to the resistivity of the battery's component materials, the difference between the internal voltage and the supplied voltage also depends on factors such as electrolyte conductivity, ion mobility, and electrode surface area.

The following table shows the nominal voltage and internal resistance of common Duracell (coppertop) dry cell batteries of different sizes. These numbers are given by the manufacturer for a new battery at room temperature (25°C):

Size	AAA	AA	C	D	9V
$V_{NL}$ (V)	1.5	1.5	1.5	15	9
$R_{int}$ (mΩ)	250	120	150	137	1700

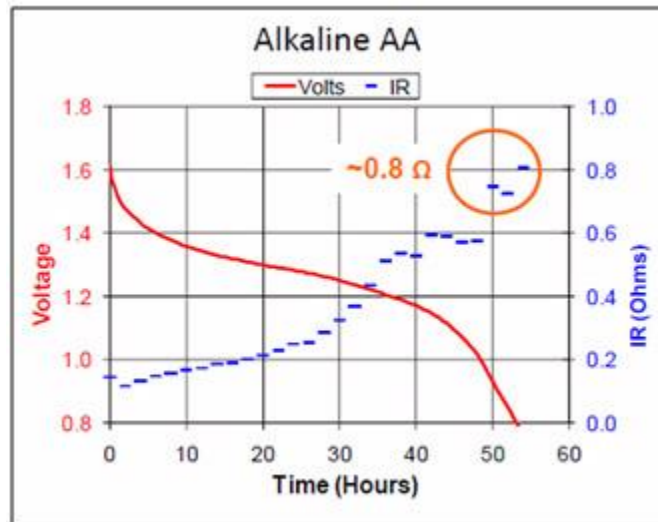
The internal resistance can be used to calculate the maximum current that a battery could theoretically supply. If you were to connect a wire from the positive terminal of a battery to the negative terminal, the only resistance in the circuit should be the battery's internal resistance.

The theoretical maximum current that the battery can supply is therefore the current that would be supplied when the only resistance is the battery's internal resistance, and can be calculated from Ohm's Law:

$$I_{\max} = \frac{V}{R_{\text{int}}}$$

Use this space for summary and/or additional notes.

Note also that the factors that affect a battery's internal resistance change as the battery ages. The following graph shows the changes in voltage and internal resistance of an alkaline AA battery as it supplied a current of 50 mA.



Internal resistance can be calculated by measuring the voltage with no “load” on the battery (*i.e.*, the voltmeter is connected directly to the battery with nothing else in the circuit) and the voltage with “load” (*i.e.*, the battery is connected to a circuit with measurable resistance):

$$R_{\text{int}} = \left( \frac{V_{NL}}{V_{FL}} - 1 \right) R_L$$

where:

$R_{\text{int}}$  = internal resistance of battery

$V_{FL}$  = voltage measured with full load (resistor with resistance  $R_L$  in circuit)

$V_{NL}$  = voltage measured with no load (voltmeter connected directly to battery)

$R_L$  = resistance of the load (resistor) that is used to experimentally determine the internal resistance of the battery

Use this space for summary and/or additional notes.

## Circuits

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.3

**Knowledge/Understanding Goals:**

- how resistance limits current in a circuit
- the difference between series and parallel circuits

**Language Objectives:**

- Explain why resistors are necessary in electric circuits.
- Understand and correctly use the terms “series” and “parallel” as applied to electric circuits.

**Notes:**

circuit: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

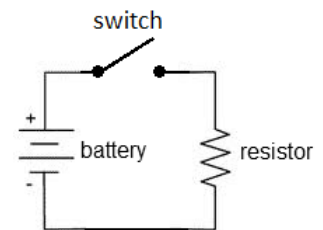
closed circuit: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

open circuit: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

short circuit: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

A diagram of a simple electric circuit might look like the diagram to the right.

When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and into the negative terminal of the battery.



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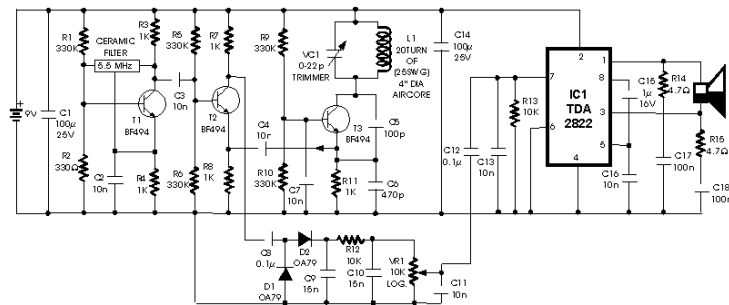
An electric circuit needs a power supply (often a battery) that provides current at a specific difference in electric potential (voltage), and one or more components that use the energy provided by the battery.

The battery continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and
2. The total resistance of the circuit is small enough to allow the current to flow.

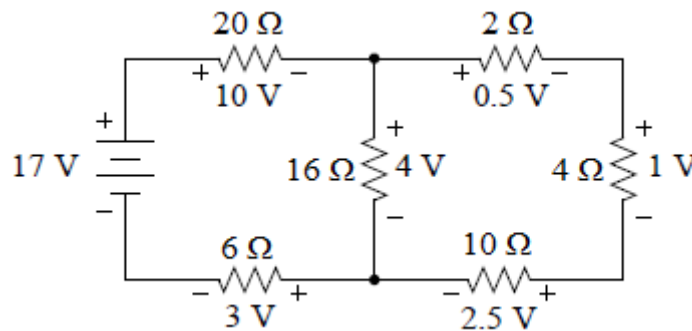
If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

Of course, as circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:



Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

The following is an example of a circuit with one 17 V battery and six resistors. Notice that the voltages and resistances are all labeled.

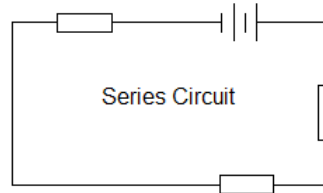


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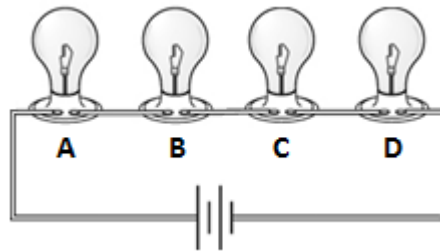
## Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

series: Components in **series** lie along the same path, one after the other.



In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of light bulbs A, B, C, or D is removed, no current can flow and none of the light bulbs will be illuminated.

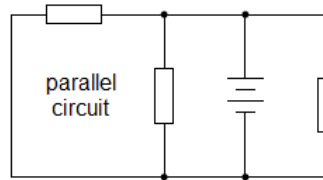


Because some voltage is “used up” by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

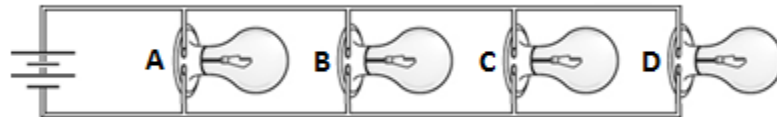
Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

Use this space for summary and/or additional notes.

parallel: Components in **parallel** lie in separate paths.

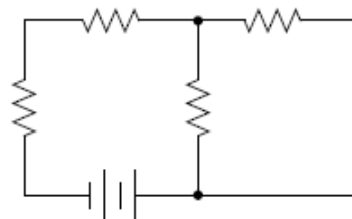


In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths. For example, in the following parallel circuit, if any of light bulbs A, B, C, or D is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, the total current will increase with each additional branch.)

Note that complex circuits may have some components that are in series with each other and other components that are in parallel.



Use this space for summary and/or additional notes.



## Series Circuits

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.3

**Knowledge/Understanding Goals:**

- the difference between series and parallel circuits

**Skills:**

- calculate voltage, current, resistance, and power in series circuits.

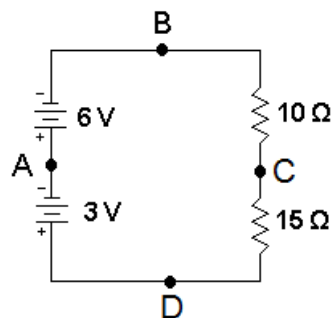
**Language Objectives:**

- Understand and correctly use the term “series circuit.”
- Set up and solve word problems relating to electrical circuits with components in series.

**Notes:**

### Analyzing Series Circuits

The following circuit shows two batteries and two resistors in series:



**Current**

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \dots$$

Use this space for summary and/or additional notes:

**Voltage**

In a series circuit, if there are multiple voltage sources (*e.g.*, batteries), the voltages add:

$$V_{total} = V_1 + V_2 + V_3 + \dots$$

In the above circuit, there are two batteries, one that supplies 6 V and one that supplies 3 V. The voltage from A to B is +6 V, the voltage from A to D is -3 V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3 V) + (+6 V) = +9 V.

**Resistance**

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \dots$$

In the above circuit, the resistance between points B and D is  $10\Omega + 15\Omega = 25\Omega$ .

**Power**

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \dots$$

**Calculations**

You can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

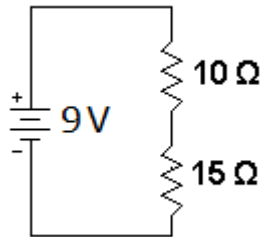
$$V = IR \qquad P = VI = I^2R = \frac{V^2}{R}$$

“Solving” the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Use this space for summary and/or additional notes.

**Sample Problem:**

Suppose we are given the following circuit:



and we are asked to fill in the table:

	Unit	R <sub>1</sub>	R <sub>2</sub>	Total
Voltage ( <i>V</i> )	V			9 V
Current ( <i>I</i> )	A			
Resistance ( <i>R</i> )	Ω	10 Ω	15 Ω	
Power ( <i>P</i> )	W			

First, we recognize that resistances in series add, so we have:

	Unit	R <sub>1</sub>	R <sub>2</sub>	Total
Voltage ( <i>V</i> )	V			9 V
Current ( <i>I</i> )	A			
Resistance ( <i>R</i> )	Ω	10 Ω	15 Ω	25 Ω
Power ( <i>P</i> )	W			

Now, we know two variables in the “Total” column, so we use  $V = IR$  to find the current. Because this is a series circuit, the total current is also the current through R<sub>1</sub> and R<sub>2</sub>.

$$V = IR$$

$$9 = (I)(25)$$

$$I = \frac{9}{25} = 0.36 \text{ A}$$

	R <sub>1</sub>	R <sub>2</sub>	Total
Voltage ( <i>V</i> )			9 V
Current ( <i>I</i> )	0.36 A	0.36 A	0.36 A
Resistance ( <i>R</i> )	10 Ω	15 Ω	25 Ω
Power ( <i>P</i> )			

Use this space for summary and/or additional notes.

As soon as we know the current, we can find the voltage across  $R_1$  and  $R_2$ , again using  $V = IR$ .

	$R_1$	$R_2$	Total
Voltage ( $V$ )	3.6 V	5.4 V	9 V
Current ( $I$ )	0.36 A	0.36 A	0.36 A
Resistance ( $R$ )	10 $\Omega$	15 $\Omega$	25 $\Omega$
Power ( $P$ )			

Finally, we can fill in the power, using  $P = VI$ ,  $P = I^2R$ , or  $P = \frac{V^2}{R}$ :

	$R_1$	$R_2$	Total
Voltage ( $V$ )	3.6 V	5.4 V	9 V
Current ( $I$ )	0.36 A	0.36 A	0.36 A
Resistance ( $R$ )	10 $\Omega$	15 $\Omega$	25 $\Omega$
Power ( $P$ )	1.30 W	1.94 W	3.24 W

Use this space for summary and/or additional notes.

## Parallel Circuits

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.3

**Skills:**

- calculate voltage, current, resistance, and power in parallel circuits.

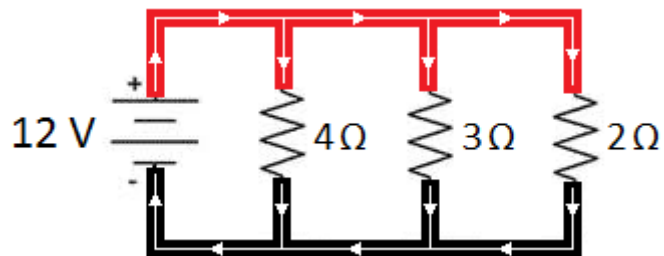
**Language Objectives:**

- Understand and correctly use the term “parallel circuit.”
- Set up and solve word problems relating to electrical circuits with components in parallel.

**Notes:**

### Parallel Circuits

The following circuit shows a battery and three resistors in parallel:



**Current**

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \dots$$

Use this space for summary and/or additional notes:

### Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \dots$$

### Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \dots$$

### Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Use this space for summary and/or additional notes.

Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.



The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

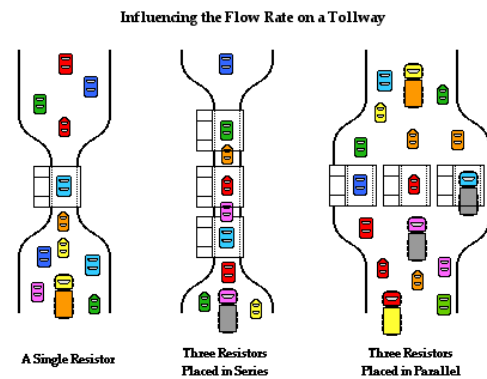
The water will flow through the set of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.

One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in a row would slow traffic down more. This is analogous to resistors in series.

Multiple toll booths next to each other (in parallel) make traffic flow faster because there are more paths for the cars to follow. Each toll booth further reduces the resistance to the flow of traffic.



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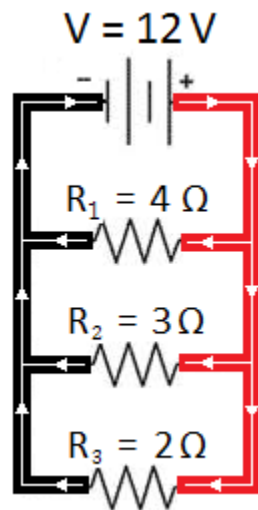
**Calculations**

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$V = IR \qquad P = VI = I^2R = \frac{V^2}{R}$$

**Sample Problem**

Suppose we are given the following circuit:



and we are asked to fill in the table:

	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	Total
Voltage ( <i>V</i> )				12 V
Current ( <i>I</i> )				
Resistance ( <i>R</i> )	4 Ω	3 Ω	2 Ω	
Power ( <i>P</i> )				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches.

The first thing we can do is use  $V = IR$  to find the current through each resistor:

	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	Total
Voltage ( <i>V</i> )	12 V	12 V	12 V	12 V
Current ( <i>I</i> )	3 A	4 A	6 A	13 A
Resistance ( <i>R</i> )	4 Ω	3 Ω	2 Ω	
Power ( <i>P</i> )				

In a parallel circuit, the current adds, so the total current is  $3 + 4 + 6 = 13$  A.

Use this space for summary and/or additional notes.



Now, we have two ways of finding the total resistance.

We can use  $V = IR$  for the voltage and total current:

$$V = IR$$

$$12 = 13 R$$

$$R = \frac{12}{13} = 0.923 \Omega$$

Or we can use the formula for resistances in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$

$$\frac{1}{R_{total}} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$

$$R_{total} = \frac{12}{13} = 0.923 \Omega$$

Now we have:

	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	Total
Voltage ( <i>V</i> )	12 V	12 V	12 V	12 V
Current ( <i>I</i> )	3 A	4 A	6 A	13 A
Resistance ( <i>R</i> )	4 Ω	3 Ω	2 Ω	0.923 Ω
Power ( <i>P</i> )				

Use this space for summary and/or additional notes.

As we did with series circuits, we can calculate the power, using  $P = VI$ ,  $P = I^2R$ ,  
or  $P = \frac{V^2}{R}$  :

	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	36 W	48 W	72 W	156 W

### Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts.

Use this space for summary and/or additional notes.

## Mixed Series & Parallel Circuits

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.3

**Skills:**

- analyze circuits by replacing networks of resistors with a single resistor of equivalent resistance.

**Language Objectives:**

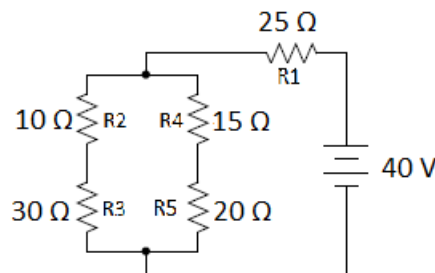
- Set up and solve word problems involving electrical circuits with some components in series and others in parallel with each other.

**Notes:**

### Mixed Series and Parallel Circuits

If a circuit has mixed series and parallel sections, you can simplify the circuit in order to determine the voltage, current, and/or resistance. The circuit can be simplified by replacing resistors in series or parallel with a single resistor of equivalent resistance, as in the following example:

Suppose we need to find the total current in the following circuit:



We need to simplify the resistor network to find the total resistance. Then we can use the total voltage and total resistance to find the total current.

Step 1: If we follow the current through the circuit, we see that it goes through resistor R1 first. Then it splits into two parallel pathways. One path goes through R2 and R3, and the other goes through R4 and R5.

Use this space for summary and/or additional notes:

There is no simple shorthand for representing series and parallel components, so let's define the symbols “—” to show resistors in series, and “||” to show resistors in parallel. The above network of resistors could be represented as:

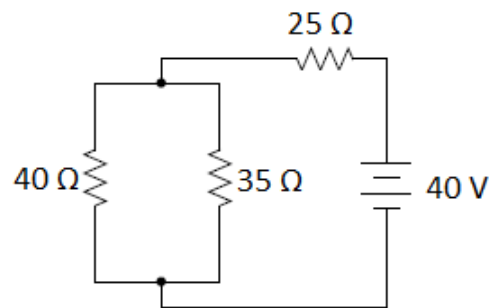
$$R_1 - [(R_2 - R_3) || (R_4 - R_5)]$$

Now, we simplify the network just like a math problem—start with the inside parentheses and work your way out.

Step 2: Replace  $R_2 - R_3$  with a single resistor. These are in series, so we just add  $10\Omega + 30\Omega = 40\Omega$ .

Step 3: Replace  $R_4 - R_5$  with a single resistor. These are also in series, so we just add  $15\Omega + 20\Omega = 35\Omega$ .

Now we have the following slightly simpler circuit:

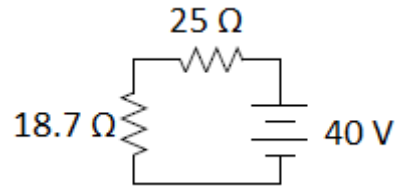


Step 4: Now we can combine the parallel  $40\Omega$  and  $35\Omega$  resistors into a single equivalent resistance:

$$\frac{1}{R_{total}} = \frac{1}{40} + \frac{1}{35}$$
$$\frac{1}{R_{total}} = 0.0250 + 0.0286 = 0.0536$$
$$R_{total} = \frac{1}{0.0536} = 18.7\Omega$$

Use this space for summary and/or additional notes.

Now, our circuit is equivalent to:



Step 5: Add the two resistances in series to get the total combined resistance of the circuit:

$$18.7 + 25 = 43.7 \Omega$$

Step 6: Now that we know the total voltage and resistance, we can use Ohm's Law to find the current:

$$\begin{aligned} V &= IR \\ 40 &= I(43.7) \\ I &= \frac{40}{43.7} = 0.92 \text{ A} \end{aligned}$$

Once we know the total current, voltage, and resistance in the circuit, we can use Ohm's Law ( $V = IR$ ) along with the equations for series and parallel circuits (and sub-circuits) to find the current and voltage for each resistor as we did in the previous section.

Use this space for summary and/or additional notes.

## Measuring Voltage, Current & Resistance

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 5.3

### Knowledge & Understanding:

- how voltage, current and resistance are measured.
- how positive and negative numbers for voltage and current correspond with the direction of current flow.

### Skills:

- measure voltage, current and resistance.

### Language Objectives:

- Accurately describe how to measure voltage, current and resistance in an electric circuit, using appropriate academic language.

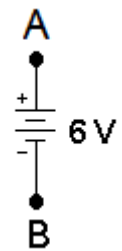
### Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

### Measuring Voltage

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:

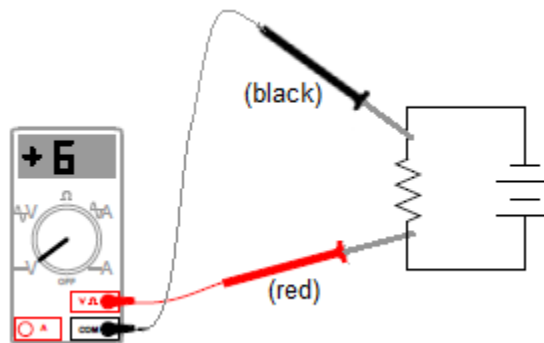
The voltage between points A and B is either +6V or -6V, depending on the direction. The voltage from A to B (positive to negative) is +6V, and the voltage from B to A (negative to positive) is -6V.



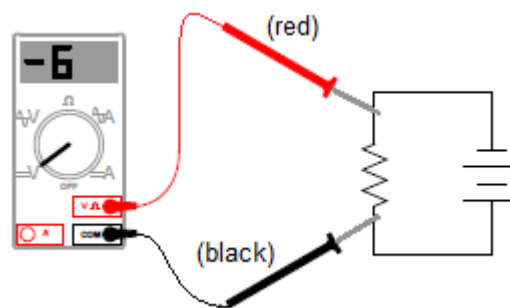
Use this space for summary and/or additional notes:

When measuring voltage, the circuit needs to be powered up with current flowing through it. Make sure that the voltmeter is set for volts (DC or AC, as appropriate) and that the red lead is plugged into the  $V\Omega$  socket (for measuring volts or ohms). Then touch the two leads *in parallel* with the two points you want to measure the voltage across. (Remember that voltage is the same across all branches of a parallel circuit. You want the voltmeter to be one of the branches, and the circuit to be the other branch.)

On a voltmeter (a meter that measures volts or voltage), the voltage is measured assuming the current is going from the red (+) lead to the black (-) lead. In the following circuit, if you put the red (+) lead on the more positive end of a resistor and the black (-) lead on the more negative end, the voltage reading would be positive. In this circuit, the voltmeter reads +6V:



If you switch the leads, so the black (-) lead is on the more positive end and the red (+) lead is on the more negative end, the voltage reading would be negative. In this circuit, the voltmeter reads -6V:



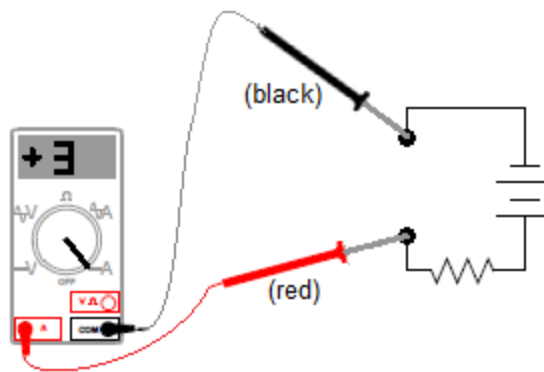
The reading of -6V indicates that the current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (-) lead to the red (+) lead.

Use this space for summary and/or additional notes.

### Measuring Current

When measuring current, the circuit needs to be open between two points. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes ( $\mu\text{A}$ ) AC or DC, depending on what you expect the current in the circuit to be. Make sure the red lead is plugged into appropriate socket (A if the current is expected to be 0.5 A or greater, or mA/ $\mu\text{A}$  if the current is expected to be less than 0.5 A). Then touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (Remember that current is the same through all components in a series circuit. You want all of the current to flow through the ammeter.)

On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (-) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (-) lead on the end that is connected to the negative terminal, the current reading would be positive. In this circuit, the current is +3A:



As with the voltage example above, if you switched the leads, the reading would be -3 A instead of +3 A.

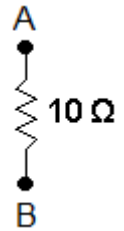
Use this space for summary and/or additional notes.



### Measuring Resistance

Resistance does not have a direction. If you placed an ohmmeter (a meter that measures resistance) across points A and B, it would read  $10\ \Omega$  regardless of which lead is on which point.

However, because an ohmmeter needs to supply a small amount of current across the component and measure the resistance, the reading is more susceptible to measurement problems, such as the resistance of the wire itself, how well the probes are making contact with the circuit, etc. It is often more reliable to measure the voltage and current and calculate resistance using Ohm's Law ( $V = IR$ ).



Use this space for summary and/or additional notes.

## Magnetism

**Unit:** Electricity & Magnetism

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- what magnetism is and how it occurs
- properties of magnets

**Language Objectives:**

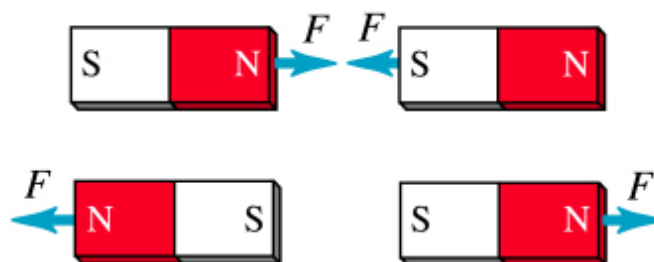
- Understand and correctly use the terms “magnet” and “magnetic field.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

magnet: a material with electrons that can align in a manner that attracts or repels other magnets.

A magnet has two ends or “poles”, called “north” and “south”. If a magnet is allowed to spin freely, the end that points toward the north on Earth is called the north end of the magnet. The end that points toward the south on Earth is called the south end of the magnet. (The Earth’s magnetic poles are near, but not in exactly the same place as its geographic poles.)

All magnets have a north and south pole. As with charges, opposite poles attract, and like poles repel.



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Use this space for summary and/or additional notes:

If you were to cut a magnet in half, each piece would be a magnet with its own north and south pole:



Use this space for summary and/or additional notes.

### Electrons and Magnetism

Magnetism is caused by unpaired electrons in atoms. Electrons within atoms reside in energy regions called “orbitals”. Each orbital can hold up to two electrons.

If two electrons share an orbital, they have opposite spins. (Note that the electrons are not actually spinning. “Spin” is the term for an intrinsic property of certain subatomic particles.) This means that if one electron aligns itself with a magnetic field, the other electron in the same orbital becomes aligned to oppose the magnetic field, and there is no net force.

However, if an orbital has only one electron, that electron is free to align with the magnetic field, which causes an attractive force between the magnet and the magnetic material. For example, as you may remember from chemistry, the electron configuration for iron is:



The inner electrons are paired up, but four of the electrons in the 3d sublevel are unpaired, and are free to align with an external magnetic field.

Use this space for summary and/or additional notes.

Magnetic measurements and calculations involve fields that act over 3-dimensional space and change continuously with position. This means that most calculations relating to magnetic fields need to be represented using multivariable calculus, which is beyond the scope of this course.

magnetic permeability (magnetic permittivity): the ability of a material to support the formation of a magnetic field. Magnetic permeability is represented by the variable  $\mu$ . The magnetic permeability of empty space is  $\mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}$ .

diamagnetic: a material whose electrons cannot align with a magnetic field. Diamagnetic materials have very low magnetic permeabilities.

paramagnetic: a material that has electrons that can align with a magnetic field. Paramagnetic materials have relatively high magnetic permeabilities.

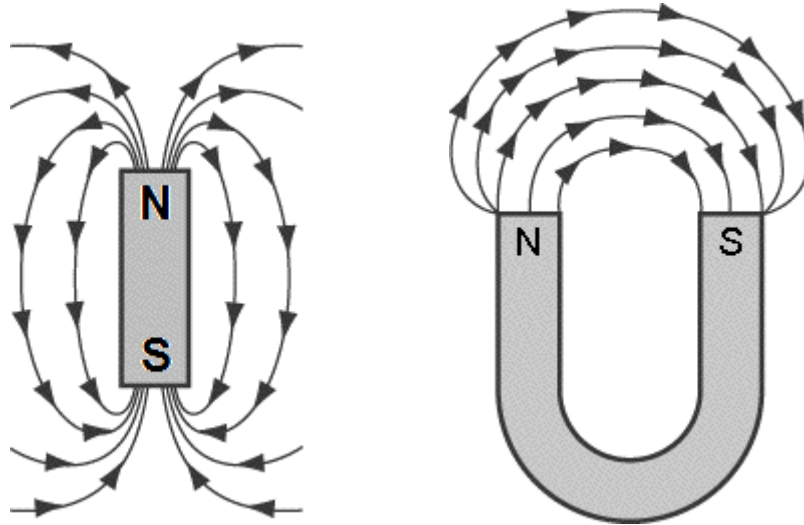
ferromagnetic: a material that can form crystals with permanently-aligned electrons, resulting in a permanent magnet. Ferromagnetic materials can have very high magnetic permeabilities. Some naturally-occurring materials that exhibit ferromagnetism include iron, cobalt, nickel, gadolinium, dysprosium, and magnetite ( $\text{Fe}_3\text{O}_4$ ).

Use this space for summary and/or additional notes.

### Magnetic Fields

magnetic field: a region in which magnetic attraction and repulsion are occurring.

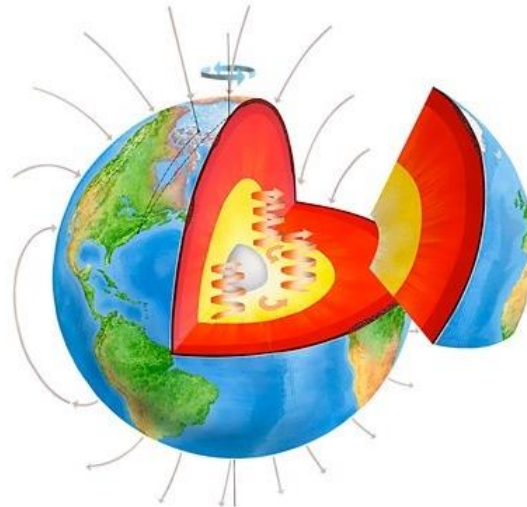
Similar to an electric field, we represent a magnetic field by drawing field lines. Magnetic field lines point from the north pole of a magnet toward the south pole, and they show the direction that the north end of a compass or magnet would be deflected if it was placed near the magnet:



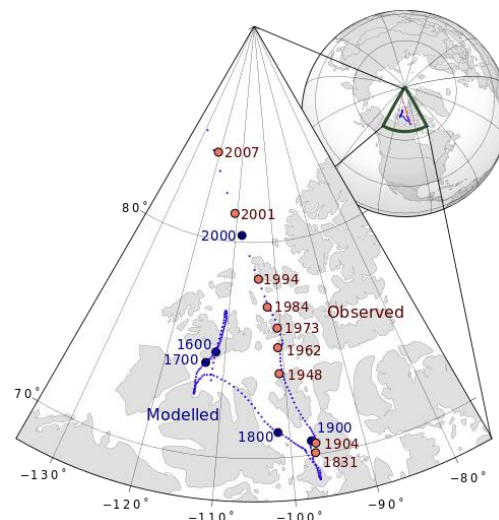
Use this space for summary and/or additional notes.

### The Earth's Magnetic Field

The molten iron outer core and very hot inner core of the Earth causes a magnetic field over the entire planet:

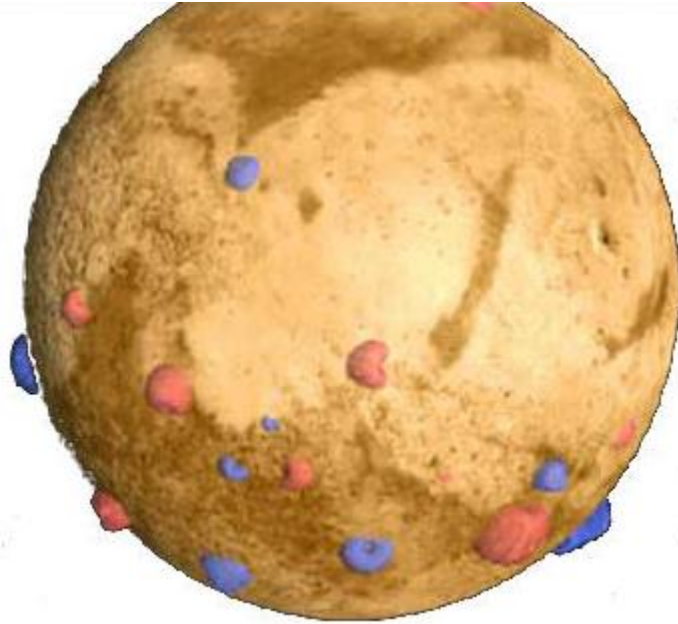


Because the core of the Earth is in constant motion, the Earth's magnetic field is constantly changing. The exact location of the Earth's magnetic north and south poles varies by about 80 km over the course of a day because of the rotation of the Earth. Its average location (shown on the map of Northern Canada below) drifts by about 50 km each year:



Use this space for summary and/or additional notes.

Not all planets have a planetary magnetic field. Mars, for example, is believed to have once had a planetary magnetic field, but the planet cooled off enough to disrupt the processes that caused it. Instead, Mars has some very strong localized magnetic fields that were formed when minerals cooled down in the presence of the planetary magnetic field:



In this picture, the blue and red areas represent regions with strong localized magnetic fields. On Mars, a compass could not be used in the ways that we use a compass on Earth; if you took a compass to Mars, the needle would point either toward or away from each these regions.

Jupiter, on the other hand, has a planetary magnetic field twenty times as strong as that of Earth. This field may be caused by water with dissolved electrolytes or by liquid hydrogen.

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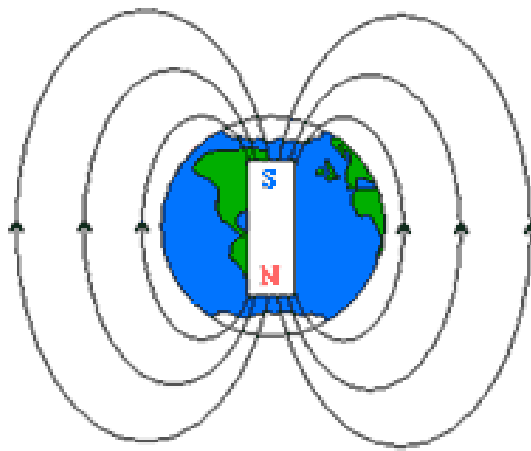


Recall that the north pole of a magnet is the end that points toward the north on Earth. This must mean that if the Earth is a giant magnet, one of its magnetic poles must be near the geographic north pole, and the other magnetic pole must be near the geographic south pole.

For obvious reasons, the Earth's magnetic pole near the north pole is called the Earth's "north magnetic pole" or "magnetic north pole". Similarly, the Earth's magnetic pole near the south pole is called the Earth's "south magnetic pole" or "magnetic south pole".

However, because the north pole of a magnet points toward the north, the Earth's north magnetic pole (meaning its location) must therefore be the south pole of the giant magnet that is the Earth.

Similarly, because the south pole of a magnet points toward the south, the Earth's south magnetic pole (meaning its location) must therefore be the north pole of the giant Earth-magnet.



Unfortunately, the term "magnetic north pole," "north magnetic pole" or any other similar term almost always means the magnetic pole that is in the north part of the Earth. There is no universally-accepted way to name the poles of the Earth-magnet.

Use this space for summary and/or additional notes.

## Electricity & Magnetism

**Unit:** Electricity & Magnetism

**NGSS Standards:** HS-PS2-5

**MA Curriculum Frameworks (2006):** 5.6

### Knowledge/Understanding Goals:

- how a charge moving through a magnetic field produces a force
- how this property can be used to make electric motors, generators, and transformers

### Skills:

- calculate the force on a wire produced by a current moving through a magnetic field
- calculate the voltage and current changes in a step-up or step-down transformer

### Language Objectives:

- Understand and correctly use the terms “electromagnet,” “motor,” “generator,” and “transformer.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the behavior of electric current in a magnetic field.

### Notes:

#### Magnetic Fields and Moving Charges

Like gravitational and electric fields, a magnetic field is a force field. (Recall that force fields are vector quantities, meaning that they have both magnitude and direction.) Magnetic field is represented by the variable  $\vec{B}$  and is measured in Teslas (T).

$$1 \text{ T} = 1 \frac{\text{N}}{\text{A}\cdot\text{m}}$$

Use this space for summary and/or additional notes:

The force,  $\vec{F}$  on a charge  $q$  moving through a magnetic field  $\vec{B}$  with a velocity  $\vec{v}$  is given by the equation:

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{and} \quad F = qvB \sin\theta$$

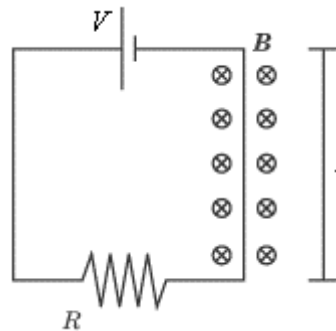
Recall that current is just a flow of charges, which means that an electric current moving through a magnetic field creates a force on the wire carrying the current.

Recall that  $\vec{I} = \frac{\Delta q}{t}$  and  $\vec{v} = \frac{\vec{d}}{t} = \frac{\ell}{t}$ , where  $\ell$  is the length (distance) of the wire that passes through the magnetic field. This means that  $q\vec{v} = \ell\vec{I}$ , which we can use to create an equivalent equation:

$$\vec{F} = \ell(\vec{I} \times \vec{B}) \quad \text{and} \quad F = \ell IB \sin\theta$$

Note that the direction of the cross products  $\vec{v} \times \vec{B}$  and  $\vec{I} \times \vec{B}$  can be determined using the right-hand rule.

A current passing through a magnetic field would be represented like this:



In the above diagram, the battery has voltage  $V$ , the resistor has resistance  $R$ , and the length of wire passing through the magnetic field is  $\ell$ .

The magnetic field strength is  $B$ , and the field itself is denoted by the symbols  $\otimes \otimes \otimes \otimes \otimes$  which denote a magnetic field going into the page. (A field coming out of the page would be denoted by  $\odot \odot \odot \odot \odot$  instead. Think of the circle as an arrow inside a tube. The dot represents the tip of the arrow facing toward you, and the "X" represents the fletches (feathers) on the tail of the arrow facing away from you.)

Use this space for summary and/or additional notes.

For example, suppose we were given the following for the above diagram:

$$\vec{B} = 4.0 \times 10^{-5} \text{ T}$$

$$V = 30 \text{ V}$$

$$R = 5 \Omega$$

$$\ell = 2 \text{ m}$$

Current (from Ohm's Law):

$$V = IR$$

$$30 = I(5)$$

$$I = 6 \text{ A}$$

Force on the wire:

$$\vec{F} = \ell(\vec{I} \times \vec{B})$$

$$F = \ell IB \sin\theta$$

$$F = (2)(6)(4.0 \times 10^{-5}) \sin 90^\circ$$

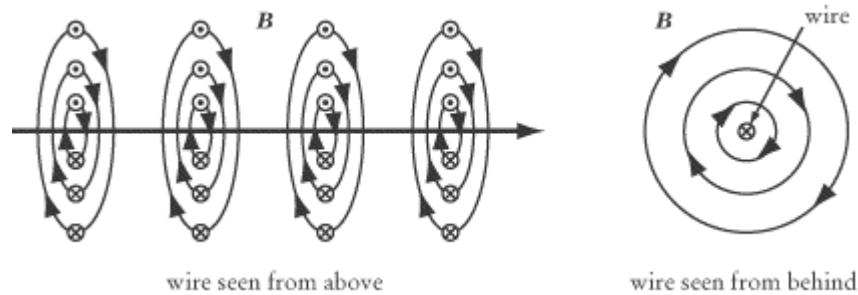
$$F = (2)(6)(4.0 \times 10^{-5})(1)$$

$$F = 4.8 \times 10^{-4} \text{ N}$$

If the current is going upward through the magnetic field, and the magnetic field is pointing into the paper, then the right-hand rule tells us that the force would be directed to the left.

### Magnetic Field Produced by Electric Current

An electric current moving through a wire also produces a magnetic field around the wire:

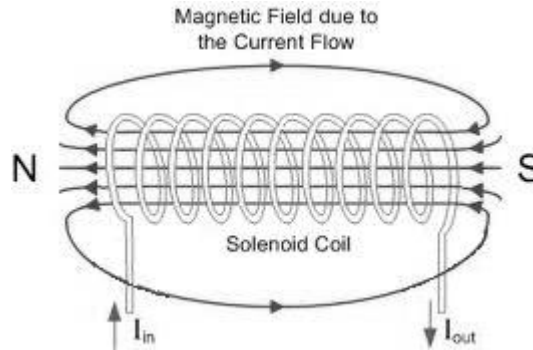


This time, we use the right-hand rule with our thumb pointing in the direction of the current, and our fingers curl in the direction of the magnetic field.

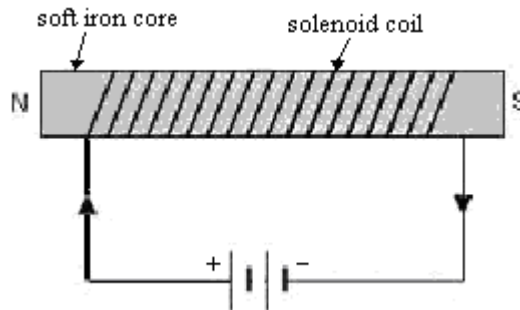
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## Electromagnets

An electromagnet is a magnet that takes advantage of this principle. The wire is wrapped tightly in a coil (called a solenoid), which causes the magnetic field lines to reinforce each other.



A soft iron core is placed in the center of the coil. The very high magnetic permeability of iron causes the resulting magnetic field to become thousands of times stronger:

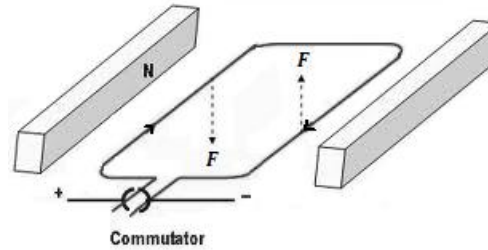


Of course, the electromagnet only works when current is flowing through the circuit.

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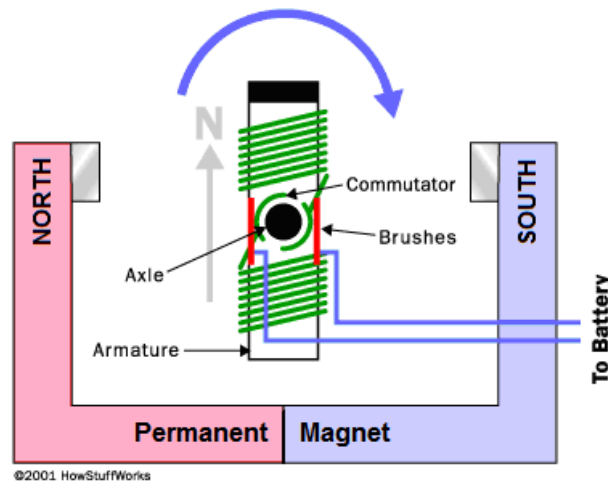
## Electric Motors

The force produced by a moving current in a magnetic field can be used to cause a loop of wire to spin:



A commutator is used to reverse the direction of the current as the loop turns, so that the combination of attraction and repulsion always applies force in the same direction.

If we replace the loop of wire with an electromagnet (a coil of wire wrapped around a material such as iron that has both a high electrical conductivity and a high magnetic permeability), the electromagnet will spin with a strong force.

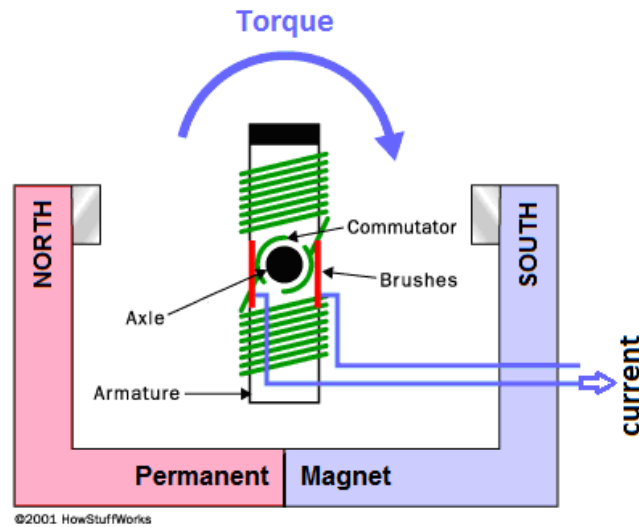


An electromagnet that spins because of its continuously switching attraction and repulsion to the magnetic field produced by a separate set of permanent magnets is called an electric motor. An electric motor turns electric current into rotational motion, which can be used to do work.

Use this space for summary and/or additional notes.

## Generators

A *generator* uses the same components and operates under the same principle, except that a mechanical force is used to spin the coil. When the coil moves through the magnetic field, it produces an electric current. Thus a generator is a device that turns rotational motion (work) into an electric current.

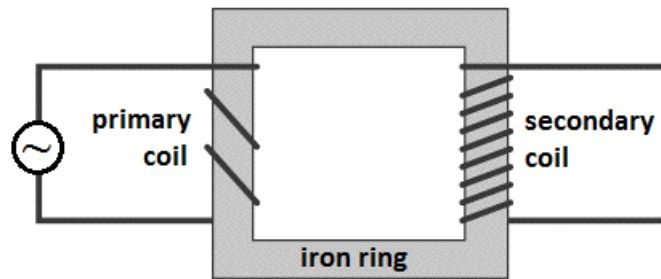


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## Inductors (Transformers)

Because electric current produces a magnetic field, a ring made of a ferromagnetic material can be used to move an electric current. An inductor (transformer) is a device that takes advantage of this phenomenon in order to increase or decrease the voltage in an AC circuit.

The diagram below shows an inductor or transformer.



The current on the input side (primary) generates a magnetic field in the iron ring. The magnetic field in the ring generates a current on the output side (secondary).

In this particular transformer, the coil wraps around the output side more times than the input. This means that each time the current goes through the coil, the magnetic field adds to the electromotive force (voltage). This means the voltage will increase in proportion to the increased number of coils on the output side. However, the magnetic field on the output side will produce less current with each turn, which means the current will decrease in the same proportion:

$$\frac{\#turns_{in}}{\#turns_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$$

$$P_{in} = P_{out}$$

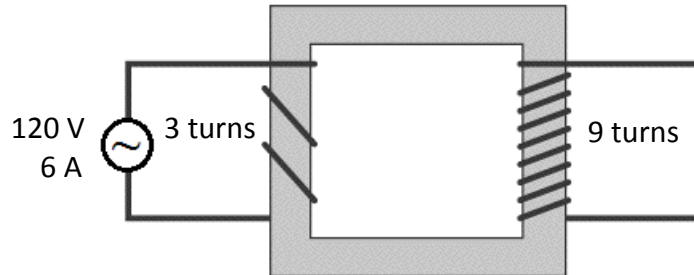
A transformer like this one, which produces an increase in voltage, is called a step-up transformer; a transformer that produces a decrease in voltage is called a step-down transformer.

Use this space for summary and/or additional notes.



**Sample Problem:**

If the input voltage to the following transformer is 120 V, and the input current is 6 A, what are the output voltage and current?



The voltage on either side of a transformer is proportional to the number of turns in the coil on that side. In the above transformer, the primary has 3 turns, and the secondary coil has 9 turns. This means the voltage on the right side will be  $\frac{9}{3} = 3$  times as much as the voltage on the left, or 360 V. The current will be  $\frac{3}{9} = \frac{1}{3}$  as much, or 2 A.

We can also use:

$$\frac{\# \text{turns}_{in}}{\# \text{turns}_{out}} = \frac{V_{in}}{V_{out}}$$

$$\frac{3}{9} = \frac{120 \text{ V}}{V_{out}}$$

$$V_{out} = 360 \text{ V}$$

$$\frac{\# \text{turns}_{in}}{\# \text{turns}_{out}} = \frac{I_{out}}{I_{in}}$$

$$\frac{3}{9} = \frac{I_{out}}{6 \text{ A}}$$

$$I_{out} = 2 \text{ A}$$

Use this space for summary and/or additional notes.



## Introduction: SHM & Mechanical Waves

**Unit:** SHM & Mechanical Waves

### Topics covered in this chapter:

Simple Harmonic Motion .....	329
Springs.....	332
Pendulums .....	335
Waves.....	339
Reflection and Superposition.....	347
Sound & Music .....	352
The Doppler Effect .....	362

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Simple Harmonic Motion* (SHM) describes the concept of repetitive back-and-forth motion and situations that apply to it.
- *Springs* and *Pendulums* describe specific examples of SHM and the specific equations relating to each.
- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *The Doppler Effect* describes the effects of motion of the source or receiver (listener) on the perception of sound.

### Standards addressed in this chapter:

#### Next Generation Science Standards (NGSS):

- HS-PS4-1.** Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Use this space for summary and/or additional notes:

**Massachusetts Curriculum Frameworks (2006):**

- 4.1 Describe the measurable properties of waves (velocity, frequency, wavelength, amplitude, period) and explain the relationships among them. Recognize examples of simple harmonic motion.
- 4.2 Distinguish between mechanical and electromagnetic waves.
- 4.3 Distinguish between the two types of mechanical waves, transverse and longitudinal.
- 4.4 Describe qualitatively the basic principles of reflection and refraction of waves.
- 4.5 Recognize that mechanical waves generally move faster through a solid than through a liquid and faster through a liquid than through a gas.
- 4.6 Describe the apparent change in frequency of waves due to the motion of a source or a receiver (the Doppler effect).

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Simple Harmonic Motion**, such as mass on a spring and the pendulum
- **General Wave Properties**, such as wave speed, frequency, wavelength, superposition, standing wave diffraction, and the Doppler effect.
  1. Periodic Motion (Simple Harmonic Motion)
  2. Frequency and Period
  3. Springs
  4. Pendulums
  5. Wave Motion
  6. Transverse Waves and Longitudinal Waves
  7. Superposition
  8. Standing Waves and Resonance
  9. The Doppler Effect

**Skills learned & applied in this chapter:**

- Visualizing wave motion.

Use this space for summary and/or additional notes.

## Simple Harmonic Motion

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 4.1

**Knowledge/Understanding Goals:**

- what simple harmonic motion is
- examples of simple harmonic motion

**Language Objectives:**

- Understand and correctly use the term “simple harmonic motion” and be able to give examples.

**Notes:**

simple harmonic motion; motion consisting of regular, periodic back-and-forth oscillation.

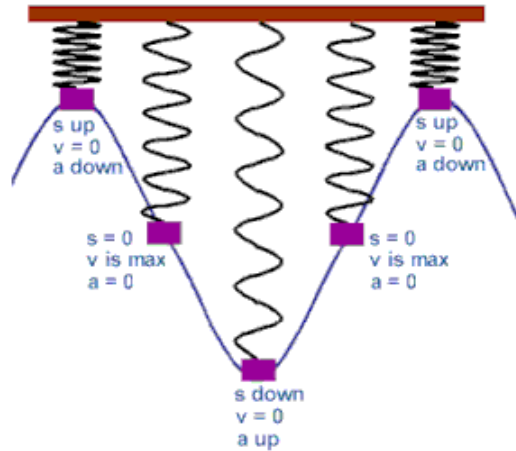
Requirements:

- The acceleration is always in the opposite direction from the displacement. This means the acceleration always slows down the motion and reverses the direction.
- In an ideal system (no friction), once simple harmonic motion is started, it would continue forever.
- A graph of displacement vs. time will result in the trigonometric function sine or cosine.

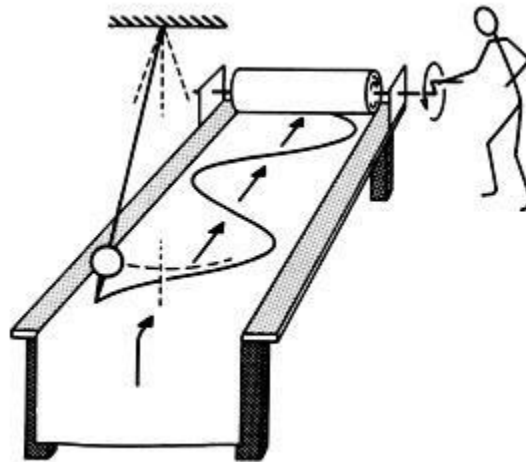
Use this space for summary and/or additional notes:

### Examples of Simple Harmonic Motion

- **Springs;** as the spring compresses or stretches, the spring force accelerates it back toward its equilibrium position.

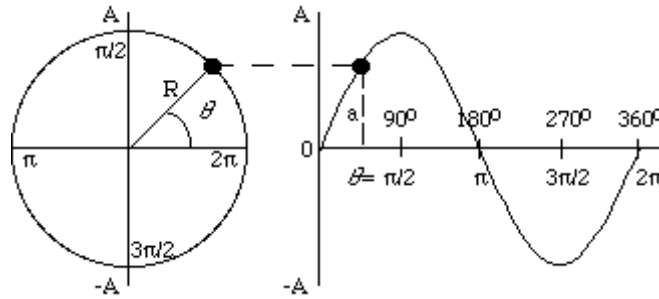


- **Pendulums:** as the pendulum swings, gravity accelerates it back toward its equilibrium position.

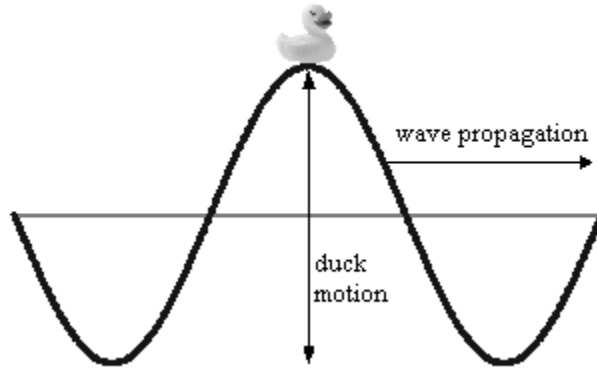


Use this space for summary and/or additional notes.

- **Uniform circular motion:** as an object moves around a circle, its vertical position ( $y$ -position) is continuously oscillating between  $+r$  and  $-r$ .



- **Waves:** waves passing through some medium (such as water or air) cause the medium to oscillate up and down, like a duck sitting on the water as waves pass by.



Use this space for summary and/or additional notes.

## Springs

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- springs and spring constants
- spring force as a vector quantity

**Skills:**

- calculate the force and potential energy of a spring

**Language Objectives:**

- Understand and correctly use the term “spring.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

spring: a coiled object that resists motion parallel with the direction of propagation of the coil.

### Spring Force

The equation for the force (vector) from a spring is given by Hooke’s Law, named for the British physicist Robert Hooke:

$$\vec{F}_s = -k\vec{x}$$

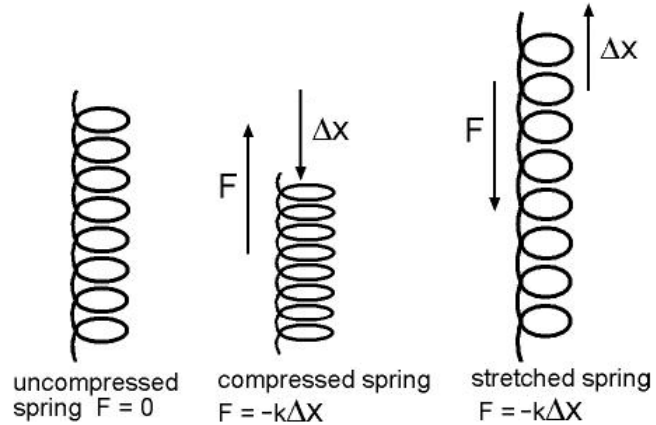
Where  $\vec{F}_s$  is the spring force (vector quantity representing the force exerted by the spring),  $\vec{x}$  is the displacement of the end of the spring (also a vector quantity), and  $k$  is the spring constant, an intrinsic property of the spring based on its mass, thickness, and the elasticity of the material that it is made of.

The negative sign in the equation is because the force is always in the opposite (negative) direction from the displacement.

A Slinky has a spring constant of  $0.5 \frac{\text{N}}{\text{m}}$ , while a heavy garage door spring might have a spring constant of  $500 \frac{\text{N}}{\text{m}}$ .

Use this space for summary and/or additional notes:





### Potential Energy

The potential energy stored in a spring is given by the equation:

$$U = \frac{1}{2}kx^2$$

Where  $U$  is the potential energy (measured in joules),  $k$  is the spring constant, and  $x$  is the displacement. Note that the potential energy is always positive (or zero); this is because energy is a scalar quantity. A stretched spring and a compressed spring both have potential energy.

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: A spring with a spring constant of  $2.7 \frac{\text{N}}{\text{m}}$  is compressed 0.3 m. Find the force needed to compress the spring and the potential energy stored in the spring when it is compressed.

A: The force is given by Hooke's Law.

Substituting these values gives:

$$\vec{F} = -k\vec{x}$$

$$\vec{F} = -(2.7 \frac{\text{N}}{\text{m}})(+0.3 \text{ m})$$

$$\vec{F} = -0.81 \text{ N}$$

The potential energy is:

$$U = \frac{1}{2}kx^2$$

$$U = (0.5)(2.7 \frac{\text{N}}{\text{m}})(0.3 \text{ m})^2$$

$$U = 0.12 \text{ J}$$

Use this space for summary and/or additional notes.

## Pendulums

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- factors that affect the period and motion of a pendulum

**Skills:**

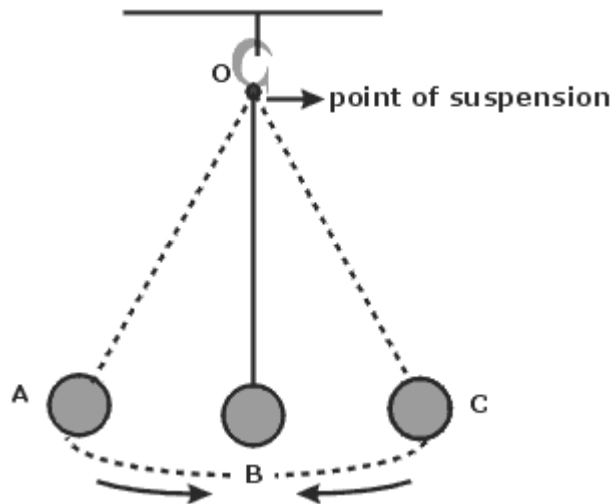
- calculate the period of a pendulum

**Language Objectives:**

- Understand and correctly use the terms “pendulum” and “period.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

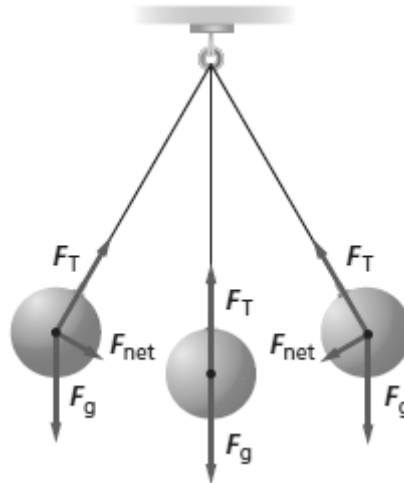
pendulum: a lever that is suspended from a point such that it can swing back and forth.



Use this space for summary and/or additional notes:

### The Forces on a Pendulum

As the pendulum swings, its mass remains constant, which means the force of gravity pulling it down remains constant. The tension on the pendulum (which we can think of as a rope or string, though the pendulum can also be solid) also remains constant as it swings.



However, as the pendulum swings, the angle of the tension force changes. When the pendulum is not in the center (bottom), the vertical component of the tension is  $F_T \cos \theta$ , and the horizontal component is  $F_T \sin \theta$ . Because the angle is between  $0^\circ$  and  $90^\circ$ ,  $\cos \theta < 1$ , which means  $F_g$  is greater than the upward component of  $F_T$ . This causes the pendulum to eventually stop. Also because the angle is between  $0^\circ$  and  $90^\circ$ ,  $\sin \theta > 0$ , This causes the pendulum to start swinging in the opposite direction.

Use this space for summary and/or additional notes.

### The Period of a Pendulum

period or period of oscillation: the time it takes a pendulum to travel from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is  $T$ , and the unit is usually seconds.

Note that the time between pendulum “beats” (such as the tick-tock of a pendulum clock) are  $\frac{1}{2}$  of the period of the pendulum. Thus a “grandfather” clock with a pendulum that beats seconds has a period  $T = 2$  s.

The period of a pendulum depends on the force of gravity, the length of the pendulum, and the maximum angle of displacement. For small angles ( $\theta < 15^\circ$ ), the period is given by the equation:

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

where  $T$  is the period of oscillation,  $\ell$  is the length of the pendulum in meters, and  $g$  is the acceleration due to gravity ( $9.8 \frac{m}{s^2}$  on Earth).

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: An antique clock has a pendulum that is 0.20 m long. What is its period?

A: The period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T = 2(3.14)\sqrt{\frac{0.20}{9.8}}$$

$$T = 6.28\sqrt{0.0204}$$

$$T = (6.28)(0.142)$$

$$T = 0.898 \text{ s}$$

Use this space for summary and/or additional notes.

## Waves

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** HS-PS4-1

**MA Curriculum Frameworks (2006):** 4.1, 4.3

**Knowledge/Understanding:**

- what waves are & how they move/propagate
- transverse vs. longitudinal waves
- mechanical vs. electromagnetic waves

**Skills:**

- calculate wavelength, frequency, period, and velocity of a wave

**Language Objectives:**

- Understand and correctly use the terms “wave,” “medium,” “propagation,” “mechanical wave,” “electromagnetic wave,” “transverse,” “longitudinal,” “torsional,” “crest,” “trough,” “amplitude,” “frequency,” “wavelength,” and “velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the wavelength, frequency and velocity of a wave.

**Notes:**

wave: a disturbance that travels from one place to another.

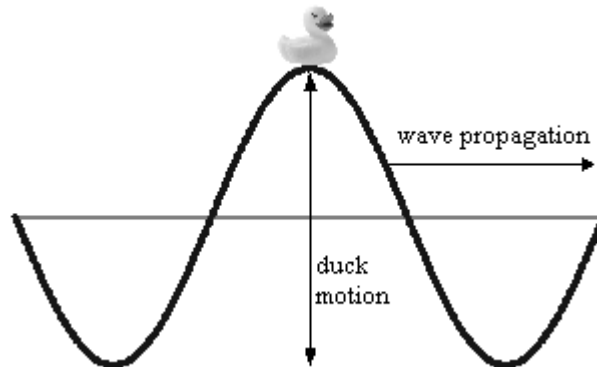
medium: a substance that a wave travels through.

propagation: the process of a wave traveling through space.

Use this space for summary and/or additional notes:

**mechanical wave:** a wave that propagates through a medium via contact between particles of the medium. Some examples of mechanical waves include ocean waves and sound waves.

- The energy of the wave is transmitted via the particles of the medium as the wave passes through it.
- The wave travels through the medium. The particles of the medium are moved by the wave passing through, and then return to their original position. (The duck sitting on top of the wave below is an example.)



- The denser the medium, the more frequently the particles come in contact, and therefore the faster the wave propagates. For example,

medium	density ( $kg/m^3$ )	velocity of sound waves
air (20°C and 1 atm)	1.2	$343 \frac{m}{s}$ ( $768 \frac{mi}{hr}$ )
water (20°C)	998	$1\,481 \frac{m}{s}$ ( $3\,317 \frac{mi}{hr}$ )
steel (longitudinal wave)	7\,800	$6\,000 \frac{m}{s}$ ( $13\,000 \frac{mi}{hr}$ )

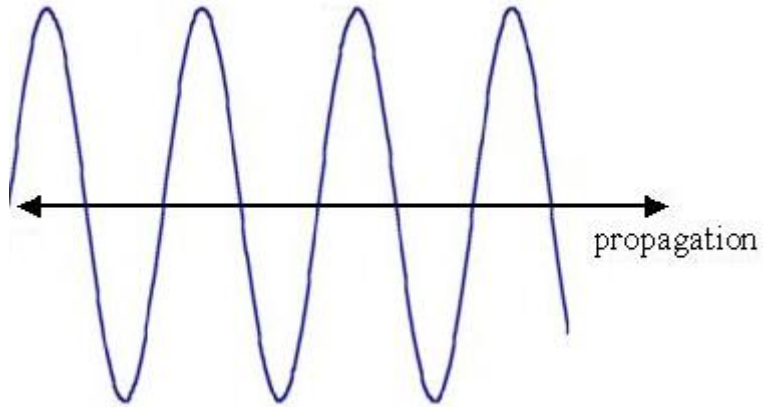
**electromagnetic wave:** a wave of electricity and magnetism interacting with each other. Electromagnetic waves can propagate through empty space.

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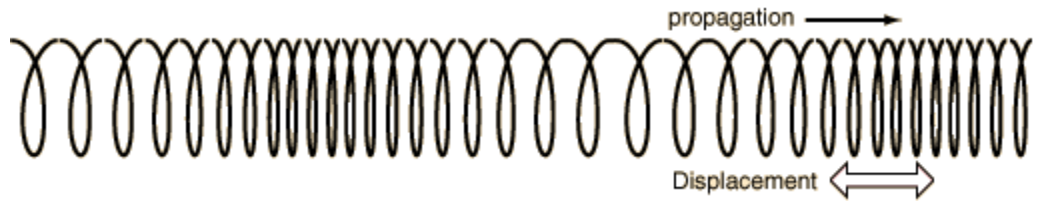


### Types of Waves

transverse wave: moves its medium up & down (or back & forth) as it travels through. Examples: light, ocean waves



longitudinal wave (or compressional wave): compresses and decompresses the medium as it travels through. Example: sound.



Use this space for summary and/or additional notes.

torsional wave: a type of transverse wave that propagates by twisting about its direction of propagation.



The most famous example of the destructive power of a torsional wave was the Tacoma Narrows Bridge, which collapsed on November 7, 1940. On that day, strong winds caused the bridge to vibrate torsionally. At first, the edges of the bridge swayed about eighteen inches. (This behavior had been observed previously, resulting in the bridge acquiring the nickname “Galloping Gertie”.) However, after a support cable snapped, the vibration increased significantly, with the edges of the bridge being displaced up to 28 feet! Eventually, the bridge started twisting in two halves, one half twisting clockwise and the other half twisting counterclockwise, and then back again. This opposing torsional motion eventually caused the bridge to twist apart and collapse.

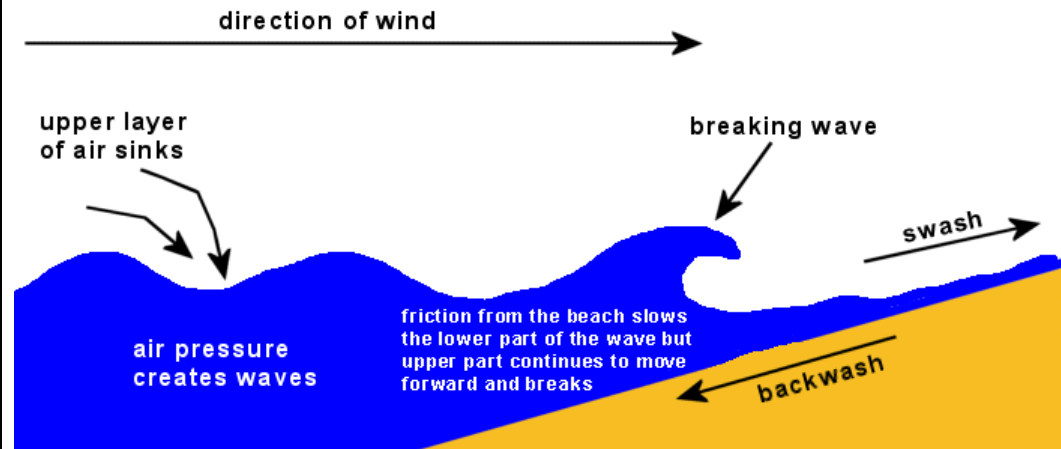


The bridge's collapse was captured on film. Video clips of the bridge twisting and collapsing are available on YouTube. There is a detailed analysis of the bridge's collapse at <http://www.vibrationdata.com/Tacoma.htm>

Use this space for summary and/or additional notes.

surface wave: a transverse wave that travels at the interface between two mediums.

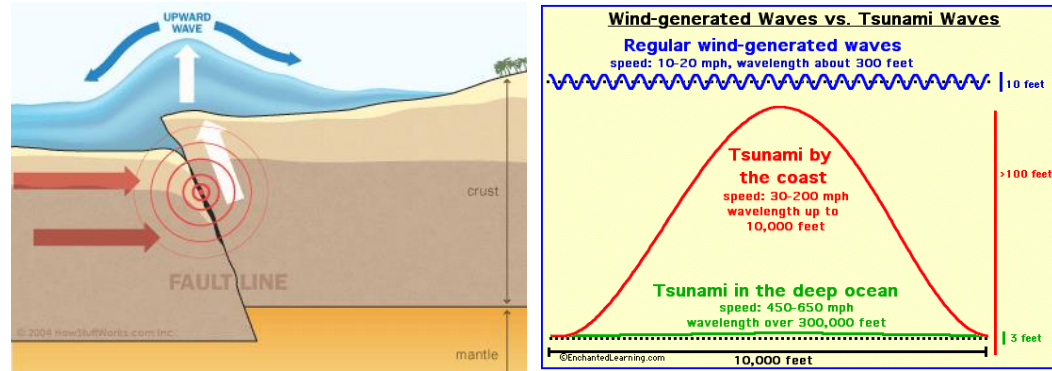
Ocean waves are an example of surface waves, because they travel at the interface between the air and the water. Surface waves on the ocean are caused by wind disturbing the surface of the water. Until the wave gets to the shore, surface waves have no effect on water molecules far below the surface.



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### Tsunamis

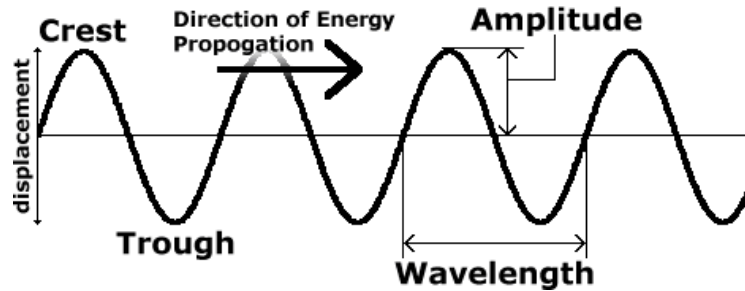
The reason tsunamis are much more dangerous than regular ocean waves is because tsunamis are created by earthquakes on the ocean floor. The tsunami wave propagates through the entire depth of the water, which means tsunamis carry many times more energy than surface waves.



This is why a 6–12 foot high surface wave breaks harmlessly on the beach; however, a tsunami that extends 6–12 feet above the surface of the water includes a significant amount of energy below the surface and can destroy an entire city.

Use this space for summary and/or additional notes.

### Properties of Waves



crest: the point of maximum positive displacement of a transverse wave. (The highest point.)

trough: the point of maximum negative displacement of a transverse wave. (The lowest point.)

amplitude: the distance of maximum displacement of a point in the medium as the wave passes through it.

wavelength: the length of the wave, measured from a specific point in the wave to the same point in the next wave. Symbol =  $\lambda$  (lambda); unit = distance (m, cm, nm, etc.)

frequency: the number of waves that travel past a point in a given time.  
Symbol =  $f$ ; unit =  $1/\text{time}$  ( $\text{Hz} = 1/\text{s}$ )

period or time period: the amount of time between two adjacent waves.  
Symbol =  $T$ ; unit = time (usually seconds)

$$T = 1/f$$

Use this space for summary and/or additional notes.

velocity: the velocity of a wave depends on its frequency ( $f$ ) and its wavelength ( $\lambda$ ):

$$v = \lambda f$$

The velocity of electromagnetic waves (such as light, radio waves, microwaves, X-rays, *etc.*) is called the speed of light, which is  $3.00 \times 10^8 \frac{\text{m}}{\text{s}}$  in a vacuum. The speed of light is slower in a medium that has an index of refraction greater than 1. (We will discuss index of refraction in more detail in the light and optics topic.)

The velocity of a wave traveling through string under tension (such as a piece of string, a rubber band, a violin/guitar string, *etc.*), the velocity depends on the tension and the ratio of the mass of the string to its length:

$$v_{\text{string}} = \sqrt{\frac{F_T L}{m}}$$

where  $F_T$  is the tension on the string,  $L$  is the length, and  $m$  is the mass.

### Sample Problem:

Q: The radio station WZLX broadcasts waves with a frequency of 100.7 MHz. If the waves travel at the speed of light, what is the wavelength?

A:  $f = 100.7 \text{ MHz} = 100\,700\,000 \text{ Hz} = 1.007 \times 10^8 \text{ Hz}$

$$v = c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$v = \lambda f$$

$$3.00 \times 10^8 = \lambda (1.007 \times 10^8)$$

$$\lambda = \frac{3.00 \times 10^8}{1.007 \times 10^8} = 2.98 \text{ m}$$

Use this space for summary and/or additional notes.

## Reflection and Superposition

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** N/A

MA Curriculum Frameworks (2006): 4.1, 4.3

**Knowledge/Understanding Goals:**

- what happens when a wave reflects (“bounces”) off an object or surface
- what happens when two or more waves occupy the same space

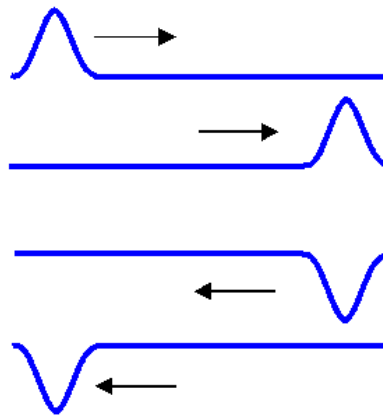
**Language Objectives:**

- Understand and correctly use the terms “reflection,” “superposition,” “constructive interference,” and “destructive interference.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

### Reflection of Waves

reflection: when a wave hits a fixed (stationary) point and “bounces” back.



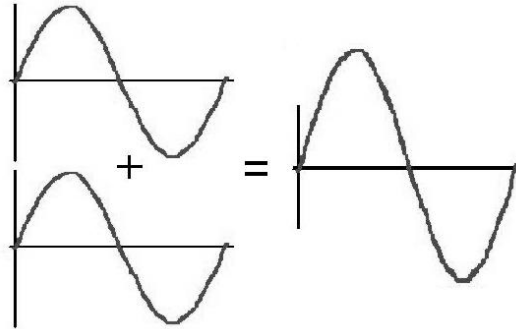
Notice that when the end of the rope is fixed, the reflected wave is inverted. (If the end of the rope was free, the wave would not invert.)

Use this space for summary and/or additional notes:

### Superposition of Waves

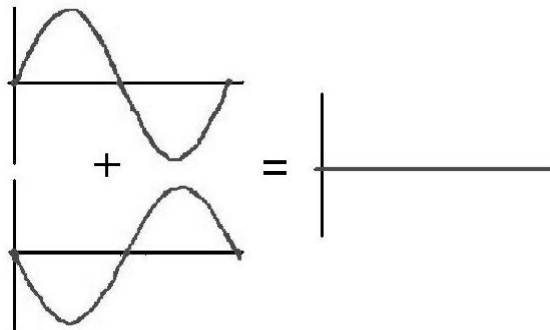
When waves are superimposed (occupy the same space), their amplitudes add:

constructive interference: when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.



Because the wavelengths are the same and the maximum, minimum, and zero points all coincide (line up), the two component waves are said to be “in phase” with each other.

destructive interference: when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves. (Sometimes we say that the waves “cancel” each other.)

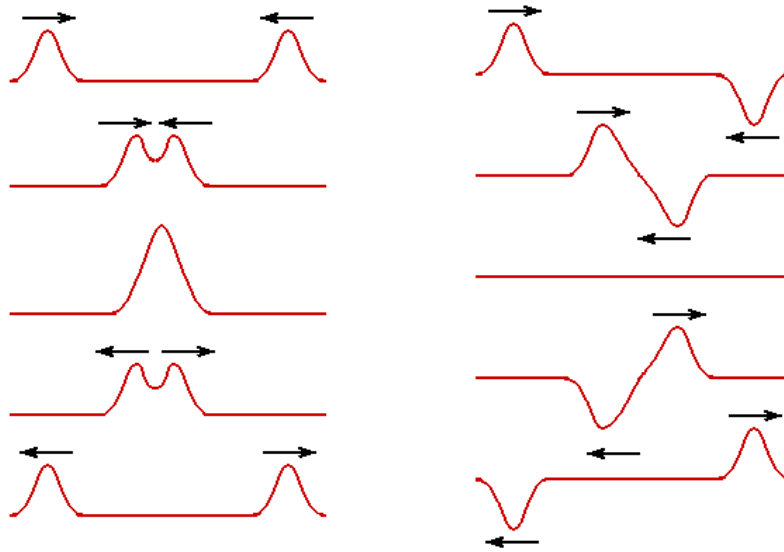


Because the wavelengths are the same but the maximum, minimum, and zero points do not coincide, the waves are said to be “out of phase” with each other.

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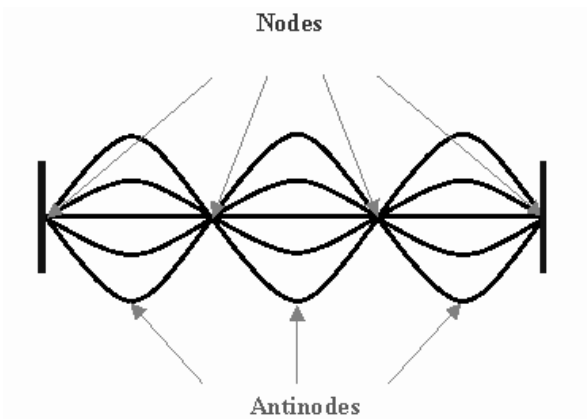


Note that waves can travel in two opposing directions at the same time. When this happens, the waves pass through each other, exhibiting constructive and/or destructive interference as they pass:



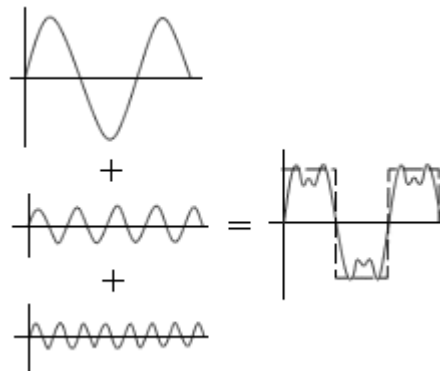
## Standing Waves

standing wave: when the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary. Points along the wave that are not moving are called “nodes”. Points of maximum displacement are called “antinodes”.



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When we add waves with different wavelengths and amplitudes, the result can be complex:

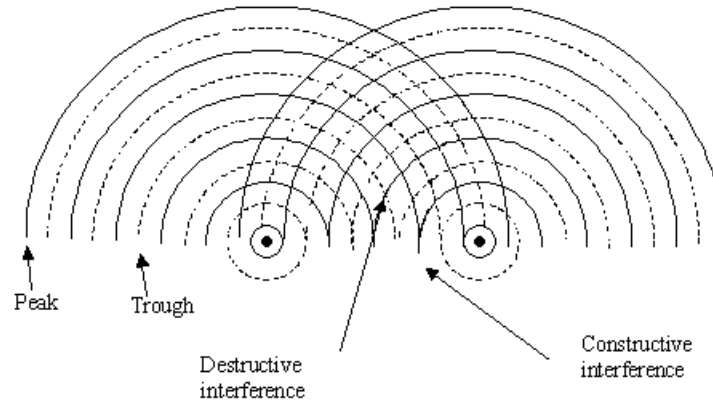


This is how radio waves encode a signal on top of a “carrier” wave. Your radio’s antenna receives (“picks up”) radio waves within a certain range of frequencies. Imagine that the bottom wave (the one with the shortest wavelength and highest frequency) is the “carrier” wave. If you tune your radio to its frequency, the radio will filter out other waves that don’t include the carrier frequency. Then your radio subtracts the carrier wave, and everything that is left is sent to the speakers.

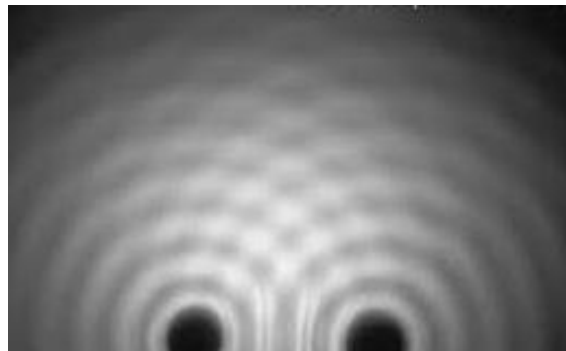
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## Interference Patterns

When two progressive waves propagate into each other's space, the waves produce interference patterns. This diagram shows how interference patterns form:



The resulting interference pattern looks like the following picture:



In this picture, the bright regions are wave peaks, and the dark regions are troughs. The brightest intersections are regions where the peaks interfere constructively, and the darkest intersections are regions where the troughs interfere constructively.

Use this space for summary and/or additional notes.

## Sound & Music

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how musical notes are produced and perceived

**Skills:**

- calculate the frequency of the pitch produced by a string or pipe

**Language Objectives:**

- Understand and correctly use the terms “resonance,” “frequency,” and “harmonic series.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the frequencies and pitches (notes) produced by musical instruments.

**Notes:**

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

pitch: how “high” or “low” a musical note is. The pitch is determined by the frequency of the sound wave.

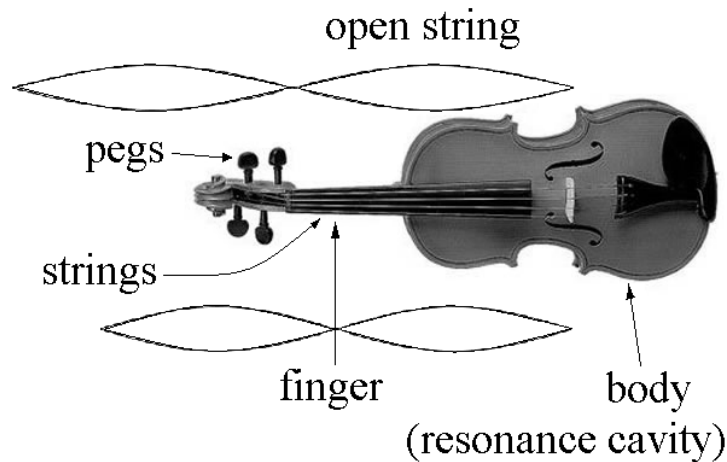


resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

Use this space for summary and/or additional notes:

### String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, *i.e.*,  $\lambda = 2L$ . The musician changes the half-wavelength by using a finger to shorten the length of the string that is able to vibrate. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string (such as a violin string) is given by the equation:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where:

$f$  = frequency (Hz)

$F_T$  = tension (N)

$m$  = mass of string (kg)

$L$  = length of string (m) =  $\frac{\lambda}{2}$

The frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

Use this space for summary and/or additional notes.

## Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. Something in the design of the mouthpiece causes the air inside the instrument to vibrate. When air is blown through the instrument, the air molecules compress and spread out at regular intervals that correspond with the length of the instrument, which determines the wavelength.

Most wind instruments use one of three ways of causing the air to vibrate:

### Brass Instruments

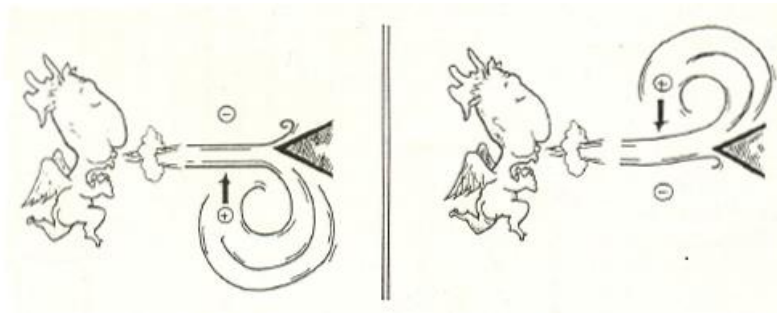
With brass instruments like trumpets, trombones, French horns, *etc.*, the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

### Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

### Fipples

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple causes a pressure difference on one side vs. the other. The pressure builds more on one side, which forces air past the sharp edge. Then the pressure builds on the other side and the air switches back:



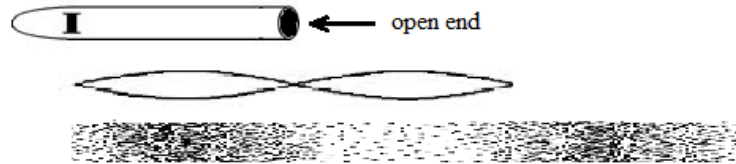
The frequency of this back-and-forth motion is what determines the pitch.

Use this space for summary and/or additional notes.

### Open vs. Closed-Pipe Instruments

An open pipe has an opening at each end. A closed pipe has an opening at one end, and is closed at the other.

Examples of open pipes include uncapped organ pipes, whistles, recorders and flutes;



Notice that the two openings determine where the nodes are—these are the regions where the air pressure must be equal to atmospheric pressure (*i.e.*, the air is neither compressed nor expanded). Notice also that as with strings, the wavelength of the sound produced is twice the length of the pipe, *i.e.*,  $\lambda = 2L$ .

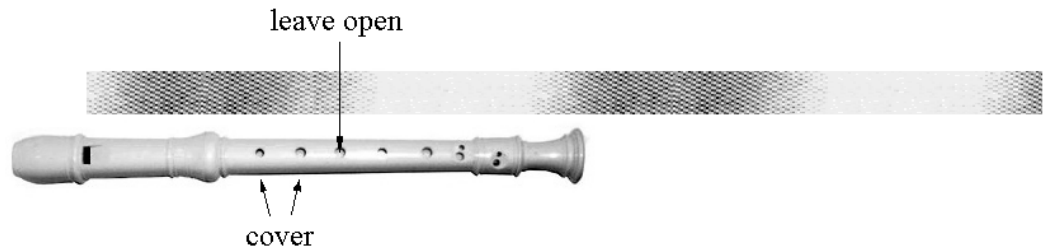
If the pipe is open to the atmosphere at only one end, such as a clarinet or brass instrument, there is only one node, at the mouthpiece. The opening, where the person is blowing into the instrument, is an antinode—a region of high pressure. This means that the body of the instrument is  $\frac{1}{4}$  of a wave instead of  $\frac{1}{2}$ , *i.e.*,  $\lambda = 4L$ .



This is why a closed-pipe instrument (*e.g.*, a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.*, a flute).

Use this space for summary and/or additional notes.

For an instrument with holes, like a flute or recorder, the first open hole creates a node at that point, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is  $v_s$  ( $343 \frac{m}{s}$  at  $20^\circ C$  and 1 atm), which means the frequency of the note (from the formula  $v_s = \lambda f$ ) will be:

$$f = \frac{v_s}{2L} \text{ for an open-pipe instrument (flute, recorder, whistle), and}$$

$$f = \frac{v_s}{4L} \text{ for an closed-pipe instrument (clarinet, brass instrument).}$$

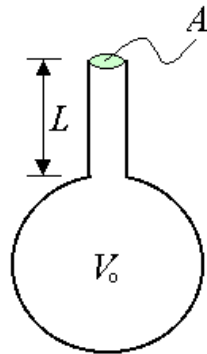
Note that the speed of sound in air increases as the temperature increases. This means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. When this happens, it's not the instrument that is going out of tune, but the speed of sound!

Use this space for summary and/or additional notes.



## Helmholtz Resonators (Bottles)

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate, because it depends on the resonance frequencies of the air in the large cavity, the air in the neck of the bottle, and the cross-sectional area of the opening.



The formula works out to be:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_0 L}}$$

where:

$f$  = resonant frequency

$v_s$  = speed of sound in air ( $343 \frac{\text{m}}{\text{s}}$  at  $20^\circ\text{C}$  and 1 atm)

$A$  = cross-sectional area of the neck of the bottle ( $\text{m}^2$ )

$V_0$  = volume of the main cavity of the bottle ( $\text{m}^3$ )

$L$  = length of the neck of the bottle (m)









(Note that it may be more convenient to use measurements in cm,  $\text{cm}^2$ , and  $\text{cm}^3$ , and use  $v_s = 34,300 \frac{\text{cm}}{\text{s}}$ .)

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You change the pitch by changing the size of the opening.

Use this space for summary and/or additional notes.

### Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch		frequency (Hz)	pitch		frequency (Hz)
	C	261.6		G	392.0
	D	293.7		A	440.0
	E	329.6		B	493.9
	F	349.2		C	523.2

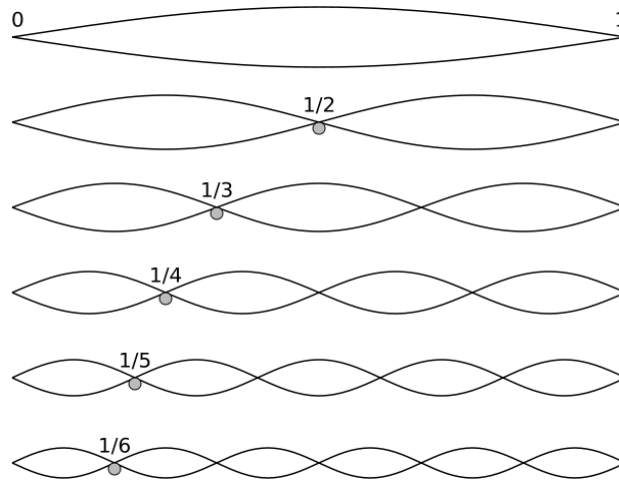
Note that a note that is an octave above another note has exactly twice the frequency of the lower note.

### Harmonic Series

harmonic series: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves. The natural frequency is called the fundamental frequency, and the harmonics above it are numbered—1<sup>st</sup> harmonic, 2<sup>nd</sup> harmonic, *etc.*) Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

Use this space for summary and/or additional notes.

The following diagram shows the fundamental frequency and the first five harmonics produced by a pipe or a vibrating string:

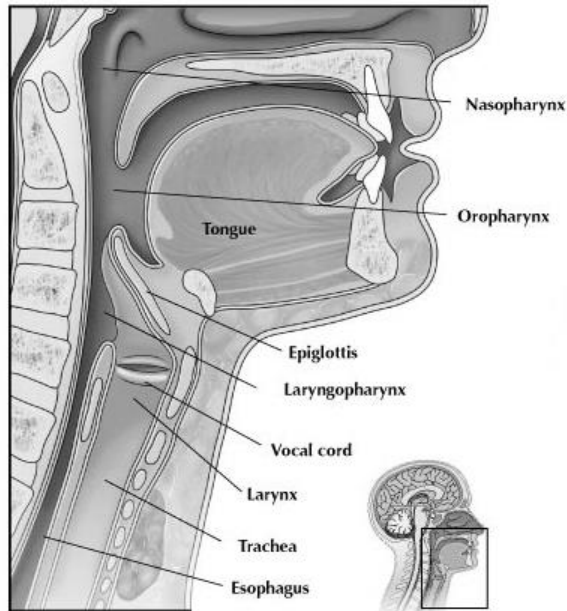


Fraction of String	Wave-length	Harmonic	Pitch (relative to fundamental)
1	$2L$	0	Fundamental.
$\frac{1}{2}$	$L$	1 <sup>st</sup>	One octave above.
$\frac{1}{3}$	$\frac{2L}{3}$	2 <sup>nd</sup>	One octave + a fifth above.
$\frac{1}{4}$	$\frac{2L}{4}$	3 <sup>rd</sup>	Two octaves above.
$\frac{1}{5}$	$\frac{2L}{5}$	4 <sup>th</sup>	Two octaves + approximately a major third above.
$\frac{1}{6}$	$\frac{2L}{6}$	5 <sup>th</sup>	Two octaves + a fifth above.
$\frac{1}{n}$	$\frac{2L}{n}$	$(n-1)^{\text{th}}$	<i>etc.</i>

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## The Biophysics of Sound

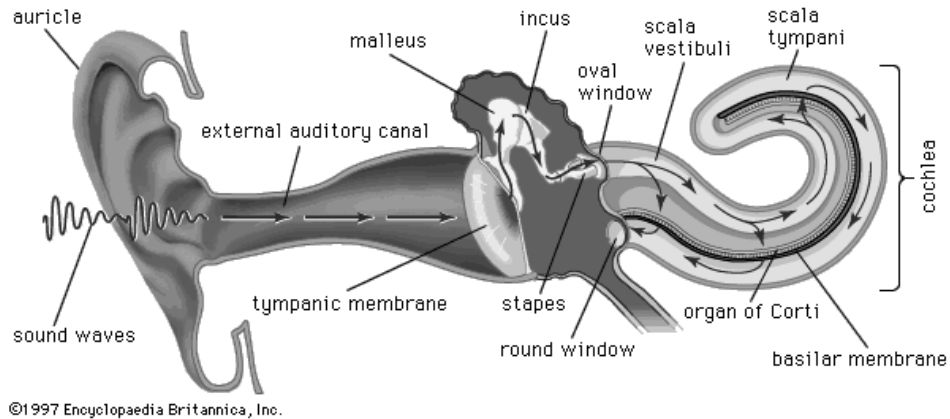
When a person speaks, stomach muscles force air from the lungs through the larynx.



The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency with which it vibrates. Just like with a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

Use this space for summary and/or additional notes.

When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. The membrane is thickest close to the oval window and it gets thinner as the distance from the oval window increases. These differences in thickness cause sounds of different frequencies (pitches) to resonate in different regions of the cochlea. The brain uses the precise locations of these resonant frequencies to determine the pitch of a sound.

Use this space for summary and/or additional notes.

## The Doppler Effect

**Unit:** SHM & Mechanical Waves

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 4.6

**Knowledge/Understanding Goals:**

- the Doppler effect

**Skills:**

- calculate the apparent shift in wavelength/frequency due to a difference in velocity between the source and receiver

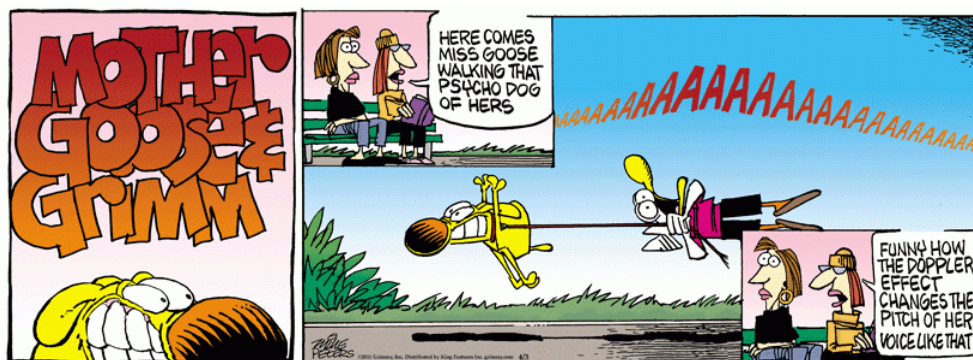
**Language Objectives:**

- Understand and correctly use the terms “Doppler effect” and “mach number.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the Doppler effect.

**Notes:**

Doppler effect or Doppler shift: the apparent change in frequency/wavelength of a wave due to a difference in velocity between the source of the wave and the observer. The effect is named for the Austrian physicist Christian Doppler.

You have probably noticed the Doppler effect when an emergency vehicle with a siren drives by.



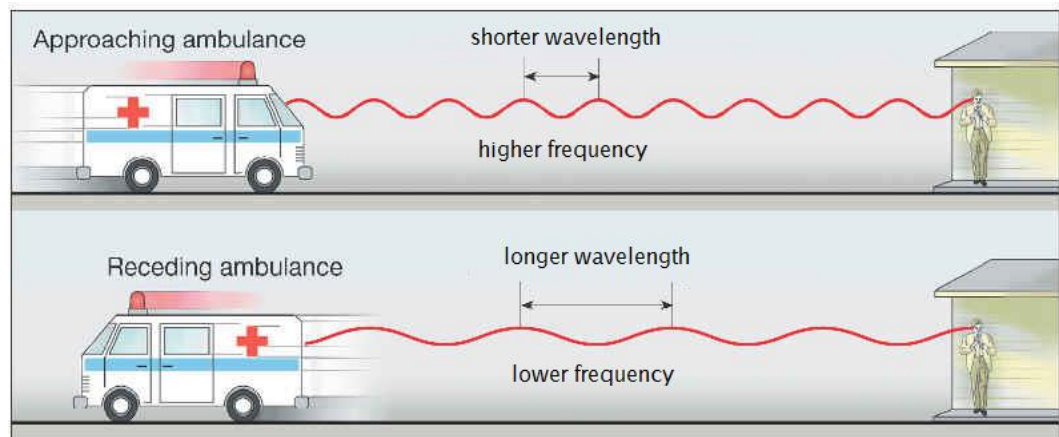
Use this space for summary and/or additional notes:

### Why the Doppler Shift Happens

The Doppler shift occurs because a wave is created by a series of pulses at regular intervals, and the wave moves at a particular speed.

If the source is approaching, each pulse arrives sooner than it would have if the source had been stationary. Because frequency is the number of pulses that arrive in one second, the moving source results in an increase in the frequency observed by the receiver.

Similarly, if the source is moving away from the observer, each pulse arrives later, and the observed frequency is lower.



Use this space for summary and/or additional notes.

### Calculating the Doppler Shift

The change in frequency is given by the equation:

$$f = f_o \left( \frac{v_w \pm v_r}{v_w \pm v_s} \right)$$

where:

$f$  = observed frequency

$f_o$  = frequency of the original wave

$v_w$  = velocity of the wave

$v_r$  = velocity of the receiver (you)

$v_s$  = velocity of the source

The rule for adding or subtracting velocities is:

- The receiver's (your) velocity is in the numerator. If you are moving toward the sound, this makes the pulses arrive sooner, which makes the frequency higher. So if you are moving **toward** the sound, **add** your velocity. If you are moving **away** from the sound, **subtract** your velocity.
- The source's velocity is in the denominator. If the source is moving toward you, this makes the frequency higher, which means the denominator needs to be smaller. This means that if the source is moving **toward** you, **subtract** its velocity. If the source is moving **away** from you, **add** its velocity.

*Don't try to memorize a rule for this—you will just confuse yourself. It's safer to reason through the equation. If something that's moving would make the frequency higher, that means you need to make the numerator larger or the denominator smaller. If it would make the frequency lower, that means you need to make the numerator smaller or the denominator larger.*

Use this space for summary and/or additional notes.



**Sample Problem:**

Q: The horn on a fire truck sounds at a pitch of 350 Hz. What is the perceived frequency when the fire truck is moving toward you at  $20 \frac{\text{m}}{\text{s}}$ ? What is the perceived frequency when the fire truck is moving away from you at  $20 \frac{\text{m}}{\text{s}}$ ? Assume the speed of sound in air is  $343 \frac{\text{m}}{\text{s}}$ .

A: The observer is not moving, so  $v_r = 0$ .

The fire truck is the source, so its velocity appears in the denominator.

When the fire truck is moving toward you, that makes the frequency higher. This means we need to make the denominator smaller, which means we need to **subtract**  $v_s$ :

$$f = f_o \left( \frac{v_w}{v_w - v_s} \right)$$
$$f = 350 \left( \frac{343}{343 - 20} \right) = 350 (1.062) = 372 \text{ Hz}$$

When the fire truck is moving away, the frequency will be lower, which mean we need to make the denominator larger. This means we need to **add**  $v_s$ :

$$f = f_o \left( \frac{v_w}{v_w + v_s} \right)$$
$$f = 350 \left( \frac{343}{343 + 20} \right) = 350 (0.9449) = 331 \text{ Hz}$$

Note that the pitch shift in each direction corresponds with about one half-step on the musical scale.

Use this space for summary and/or additional notes.

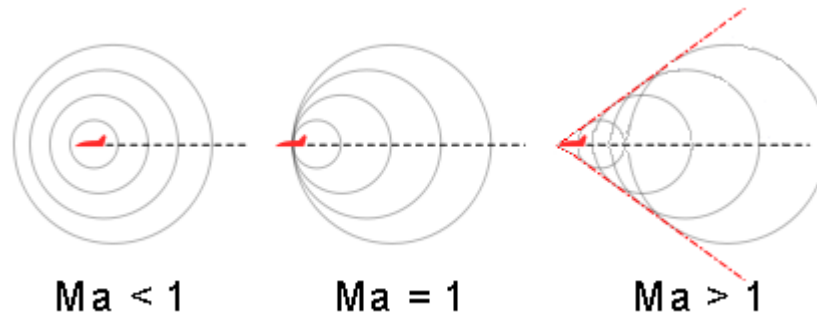
### Exceeding the Speed of Sound

The speed of an object relative to the speed of sound in the same medium is called the Mach number (abbreviation  $Ma$ ), named after the Austrian physicist Ernst Mach.

$$Ma = \frac{v_{object}}{v_{sound}}$$

Thus “Mach 1” or a speed of  $Ma = 1$  is the speed of sound. An object such as an airplane that is moving at 1.5 times the speed of sound would be traveling at “Mach 1.5” or  $Ma = 1.5$ .

When an object such as an airplane is traveling slower than the speed of sound, the jet engine noise is Doppler shifted just like any other sound wave.



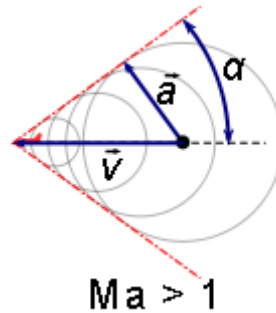
When the airplane’s velocity reaches the speed of sound ( $Ma = 1$ ), the leading edge of all of the sound waves produced by the plane coincides. These waves amplify each other, producing a loud shock wave called a “sonic boom”.

(The “crack” of a bullwhip is a sonic boom—when a bullwhip is snapped sharply, the end of the bullwhip travels faster than sound.)

Use this space for summary and/or additional notes.



When an airplane is traveling faster than sound, the sound waves coincide at points behind the airplane at a specific angle,  $\alpha$ :



The angle  $\alpha$  is given by the equation:

$$\sin(\alpha) = \frac{1}{Ma}$$

*i.e.*, the faster the airplane is traveling, the smaller the angle  $\alpha$ , and the narrower the cone.

Use this space for summary and/or additional notes.



## Introduction: Light & Optics

**Unit:** Light & Optics

### Topics covered in this chapter:

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Polarization .....	401
Lenses.....	403
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This chapter discusses the behavior and our perception of light.

- *Electromagnetic Waves* discusses properties and equations that are specific to electromagnetic waves (including light).
- *Color* discusses properties of visible light and how we perceive it.
- *Reflection, Refraction, Polarization, and Diffraction* discuss specific properties of light, and the equations that relate these properties.
- *Mirrors* and *Lenses* discuss what happens when the direction of a ray of light is changed by a mirror or a lens.

One of the new skills learned in this chapter is visualizing and drawing representations of how light is affected as it is reflected off a mirror or refracted by a lens. This can be challenging because the behavior of the light rays and the size and location of the image changes depending on the location of the object relative to the focal point of the mirror or lens. Another challenge is in drawing precise, to-scale ray tracing drawings such that you can use the drawings to accurately determine properties of the image, or of the mirror or lens.

Use this space for summary and/or additional notes:

**Standards addressed in this chapter:**

**Next Generation Science Standards (NGSS):**

**HS-PS4-2.** Evaluate questions about the advantages of using a digital transmission and storage of information.

**HS-PS4-4.** Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.

**HS-PS4-5.** Communicate technical information about how some technological devices use the principles of wave behavior and wave interactions with matter to transmit and capture information and energy.

**Massachusetts Curriculum Frameworks (2006):**

**4.2** Distinguish between mechanical and electromagnetic waves.

**6.1** Recognize that electromagnetic waves are transverse waves and travel at the speed of light through a vacuum.

**6.2** Describe the electromagnetic spectrum in terms of frequency and wavelength, and identify the locations of radio waves, microwaves, infrared radiation, visible light (red, orange, yellow, green, blue, indigo, and violet), ultraviolet rays, x-rays, and gamma rays on the spectrum.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Reflection and Refraction**, such as Snell's law and changes in wavelength and speed.
- **Ray Optics**, such as image formation using pinholes, mirrors, and lenses.
- **Physical Optics**, such as single-slit diffraction, double-slit interference, polarization, and color.

1. The Electromagnetic Spectrum
2. Classical Optics
3. Optical Instruments
4. Wave Optics

**Skills learned & applied in this chapter:**

- Drawing images from mirrors and through lenses.

Use this space for summary and/or additional notes.

## Electromagnetic Waves

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 6.1, 6.2

**Knowledge/Understanding Goals:**

- know the regions of the electromagnetic spectrum

**Skills:**

- calculate frequency & wavelength for electromagnetic waves

**Language Objectives:**

- Understand and correctly use the terms “electromagnetic spectrum” and “visible spectrum.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to energy produced by a wave of a given frequency.

**Notes:**

electromagnetic wave: a transverse, traveling wave that is caused by oscillating electric and magnetic fields.

Electromagnetic waves travel through space and do not require a medium.

Electromagnetic waves (such as light, radio waves, etc.) travel at a constant speed—the speed of light. The speed of light is a constant, and is denoted by the letter “ $c$ ” in equations. In a vacuum, the speed of light is:

$$c = 3.00 \times 10^8 \text{ m/s} = 186,000 \text{ miles per second}$$

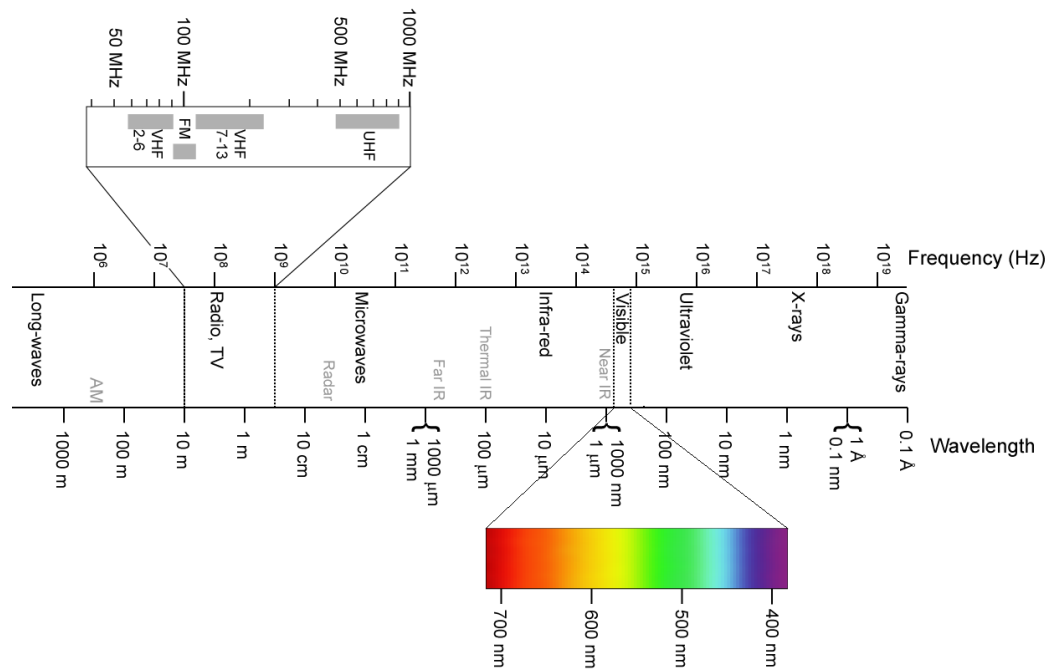
Recall that the speed of a wave equals its frequency times its wavelength:

$$c = \lambda f$$

Use this space for summary and/or additional notes:

electromagnetic spectrum: the entire range of possible frequencies and wavelengths for electromagnetic waves.

The waves that make up the electromagnetic spectrum are shown in the diagram below:



In this diagram, energy increases from left to right, *i.e.*, higher frequency waves carry more energy.

visible spectrum: wavelengths of light that can be detected by the human eye.

This corresponds with wavelengths from roughly 400 nm to 700 nm.

An antenna is a piece of metal that is affected by electromagnetic waves and amplifies waves of specific wavelengths. The optimum length for an antenna is either the desired wavelength, or some fraction of the wavelength such that one wave is an exact multiple of the length of the antenna. (In other words, good lengths for an antenna could be  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ , *etc.* of the wavelength.)

Use this space for summary and/or additional notes.



The energy ( $E$ ) that a wave carries equals a constant times the frequency. (Think of it as the number of bursts of energy that travel through the wave every second.) For electromagnetic waves (including light), the constant is Planck's constant (named after the physicist Max Planck), which is denoted by a script  $h$  in equations.

The energy of a wave is given by the Planck-Einstein equation:

$$E = hf = \frac{hc}{\lambda}$$

where  $E$  is the energy of the wave in Joules,  $f$  is the frequency in Hz,  $h$  is Planck's constant, which is equal to  $6.626 \times 10^{-34}$  J·s,  $c$  is the speed of light, and  $\lambda$  is the wavelength in meters.

**Sample problem:**

Q: What is the wavelength of a radio station that broadcasts at 98.5 MHz?

A:

$$c = \lambda f$$
$$3.00 \times 10^8 = \lambda(9.85 \times 10^7)$$
$$\lambda = \frac{3.00 \times 10^8}{9.85 \times 10^7} = 3.05 \text{ m}$$

Q: What would be a good length for a car antenna that might be used to listen to this radio station?

A: 3.05 m (about 10 feet) is too long for a car antenna. Somewhere between half a meter and a meter is a good range.

$\frac{1}{4}$  wave would be 0.76 m (76 cm), which would be a good choice.

Use this space for summary and/or additional notes.

## Color

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** 6.1, 6.2

**Knowledge/Understanding Goals:**

- how different colors of light are reflected or absorbed

**Language Objectives:**

- Understand and correctly use the terms “color,” “pigment,” “primary color,” “secondary color,” “primary pigment,” and “secondary pigment.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

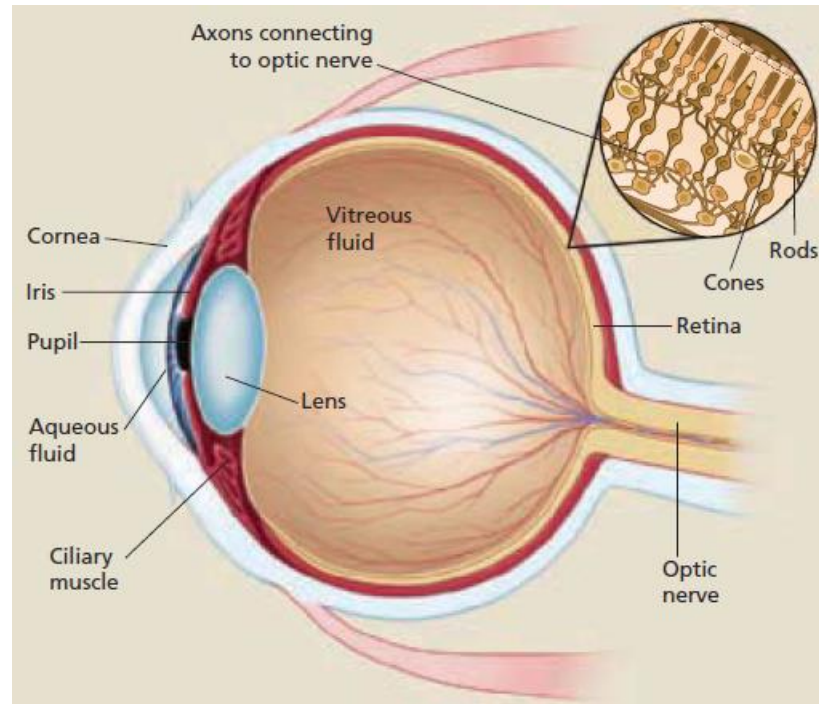
color: the perception by the human eye of how a light wave appears, based on its wavelength/frequency.

Light with frequencies/wavelengths in the part of the spectrum that the eye can detect is called visible light.

Use this space for summary and/or additional notes:

## How We See Color

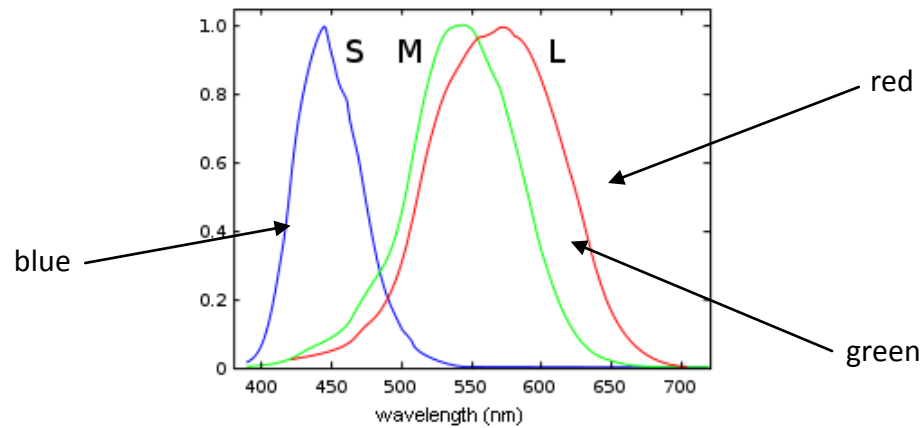
Humans (and other animals) have two types of cells in our retina that respond to light:



Rod cells resolve the physical details of images. Cone cells are responsible for distinguishing colors. Rod cells can operate in low light, but cone cells need much more light. This is why we cannot see colors in low light.

Use this space for summary and/or additional notes.

There are three different types of cone cells in our eyes, called “S”, “M”, and “L”, which stand for “short,” “medium,” and “long.” Each type of cone cells responds to different wavelengths of light, having a peak (maximum) absorbance in a different part of the visible spectrum:



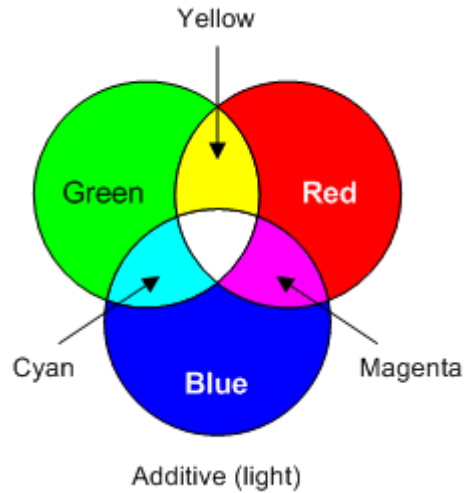
For example, light with a wavelength of 400–450 nm appears blue to us, because most of the response to this light is from the S cells, and our brains are wired to perceive this response as blue color. Light with a wavelength of around 500 nm would stimulate mostly the M cells and would appear green. Light with a wavelength of around 570 nm would stimulate the M and L cells approximately equally. When green and red receptors both respond, our brains perceive the color as yellow.

Colorblindness occurs when a genetic mutation causes a deficiency or absence of one or more types of cone cells. Most common is a deficiency in the expression of M cone cells, which causes red-green colorblindness. Because colorblindness is recessive and the relevant gene is on the x-chromosome, red-green colorblindness is much more common in men than in women.

Use this space for summary and/or additional notes.

### Light: Additive Mixing

Because our cone cells respond to red, green, and blue light, we call these colors the primary colors of light. Other colors can be made by mixing different amounts of these colors, thereby stimulating the different types of cone cells to different degrees. When all colors are mixed, the light appears white:



primary color: light that excites only one type of cone cell. The primary colors of light are red, green, and blue.

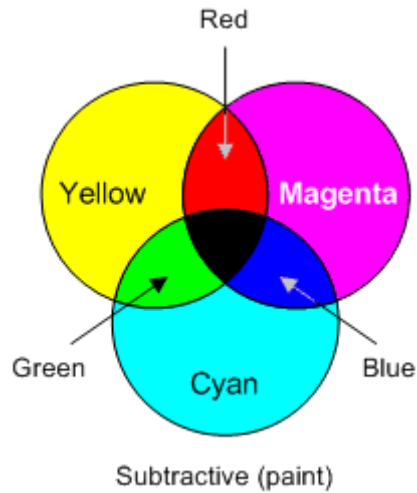
secondary color: light that is a combination of exactly two primary colors. The secondary colors of light are cyan, magenta, and yellow.

Use this space for summary and/or additional notes.

## Reflected Light: Subtractive Mixing

When light shines on an object, properties of that object cause it to absorb certain wavelengths of light and reflect others. The wavelengths that are reflected are the ones that make it to our eyes, causing the object to appear that color.

pigment: a material that changes the color of reflected light by absorbing light with specific wavelengths.



primary pigment: a material that absorbs light of only one primary color (and reflects the other two primary colors). The primary pigments are cyan, magenta, and yellow. Note that these are the secondary colors of light.

secondary pigment: a pigment that absorbs two primary colors (and reflects the other). The secondary pigments are red, green, and blue. Note that these are the primary colors of light.

Use this space for summary and/or additional notes.

## Reflection

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- parallelism of light waves

**Skills:**

- angle of incidence equals the angle of reflection

**Language Objectives:**

- Understand and correctly use the terms “reflection,” “specular reflection,” “diffuse reflection,” “real image,” “virtual image,” “angle of incidence,” and “angle of reflection.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving angles of incidence and reflection.

**Notes:**

reflection: when a wave “bounces” off an object and changes direction.

specular reflection: reflection from a smooth surface.

diffuse reflection: reflection from a rough surface.

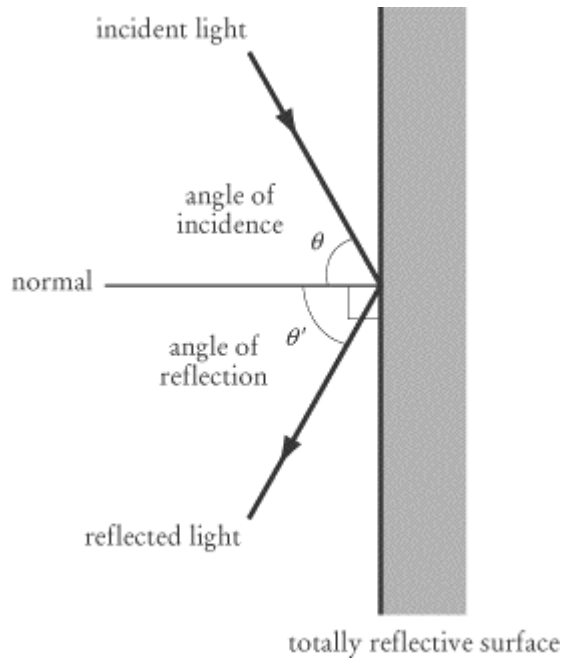
real image: a reflected image that is created by photons (rays of light) that converge. Because light is reflected back from a mirror (*i.e.*, light cannot pass through it), a **real image** is one that appears **in front of the mirror**.

virtual image: a perceived image that appears to be the point of origin of photons (rays of light) that diverge. Because light is reflected back from a mirror (*i.e.*, light cannot pass through it), a **virtual image** is one that appears **behind** (or “inside”) **the mirror**.

Use this space for summary and/or additional notes:

A rule of thumb that works for both mirrors and lenses is that a real image is produced by the convergence of actual rays of light. A virtual image is our perception of where the rays of light would have come from.

When light waves strike a surface at an angle (measured from the perpendicular or “normal” direction), they are reflected at the same angle away from the perpendicular. The most common statement of this concept is “The angle of incidence equals the angle of reflection.”

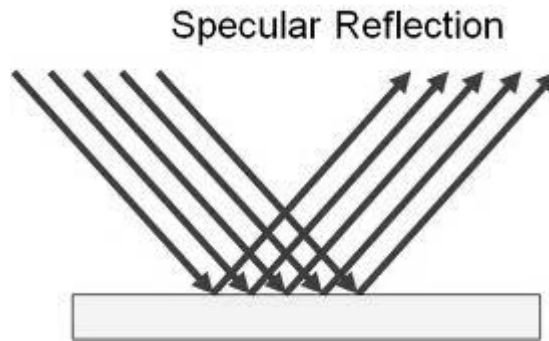


Which can be stated mathematically as either  $\theta = \theta'$  or  $\theta_i = \theta_r$ .

Use this space for summary and/or additional notes.

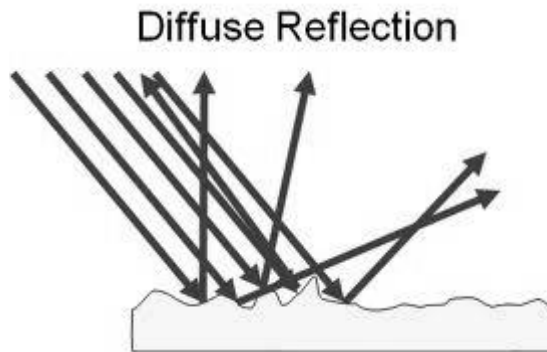


Specular reflection is reflection from a smooth surface, such as a mirror:



If the photons of light from the source are parallel when they strike the surface, they will also be parallel when they reflect from the surface. This results in a reflected image that appears to be the same size, shape, and distance from the surface as the original object.

Diffuse reflection is reflection from a rough surface:



Light striking a rough surface will illuminate the surface. However, because the reflected light rays are not parallel, the reflected light does not create a reflected image of the object.

Use this space for summary and/or additional notes.

## Mirrors

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how convex and concave mirrors reflect images

**Skills:**

- calculating the focal point and the height and location of images reflected by mirrors

**Language Objectives:**

- Understand and correctly use the terms “mirror,” “convex,” “concave,” “focal point,” “principal axis,” and “magnification.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to convex, concave, and flat mirrors.

**Notes:**

### Convex & Concave Mirrors

convex: an object that curves outward.

concave: an object that curves inward.

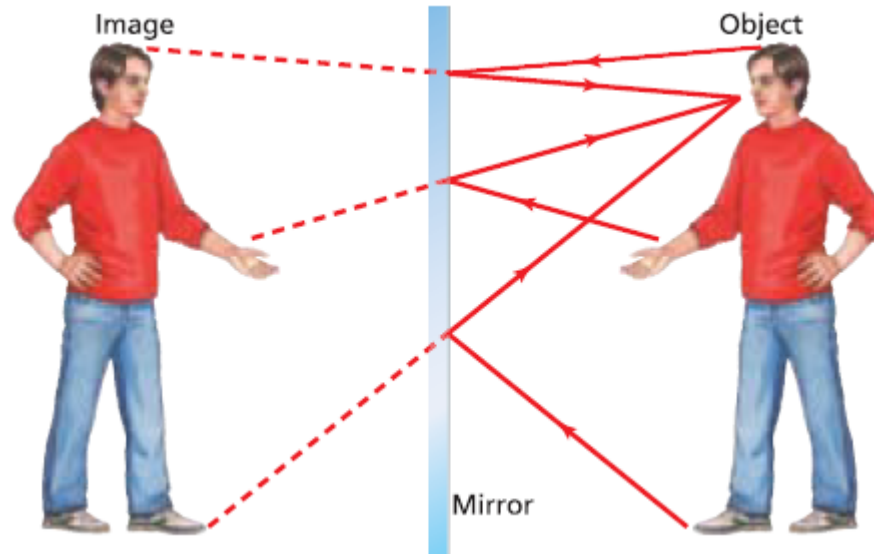
flat: an object that is neither convex nor concave.

focal point: the point at which parallel rays striking a mirror converge.

principal axis: the imaginary line where a ray of light is reflected back along its incident (incoming) path. Most physics problems represent the principal axis as horizontal, but it can actually be any direction.

Use this space for summary and/or additional notes:

In a flat mirror, the light reflected off the object (such as the person in the picture below) bounces off the mirror and is reflected back. Because our eyes and the part of our brains that decode visual images can't tell that the light has been reflected, we "see" the reflection of the object in the mirror.

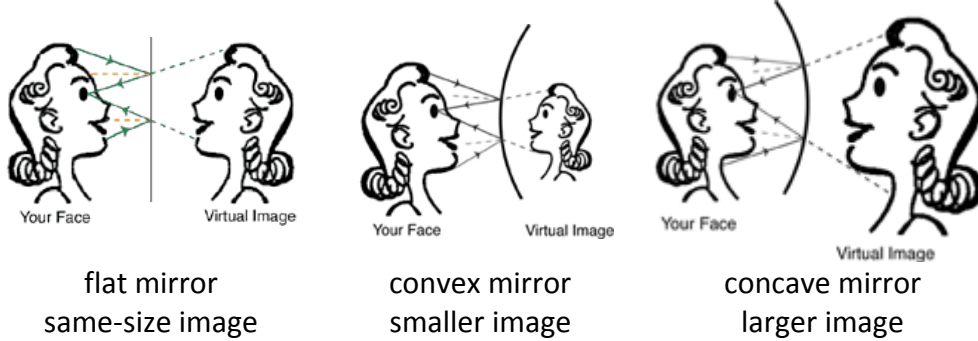


If the mirror is flat, the reflection is the same size and the same distance from the mirror as the actual object. However, the image appears reversed horizontally, but not vertically.

It would seem that the mirror "knows" to reverse the image horizontally but not vertically. (Of course this is not true. If you want the mirror to reverse the image vertically, all you need to do is put the mirror on the floor.) What is actually happening is that light is reflected straight back from the mirror. Anything that is on your right will also be on the right side of the image (from your point of view; if the image were actually a person, this would be the other person's left). Anything that is on top of you will also be on top of the image as you look at it.

What the mirror is doing is the same transformation as flipping a polygon over the  $y$ -axis. The reversal is actually front-to-back (where "front" means closer to the mirror and "back" means farther away from it).

Use this space for summary and/or additional notes.



In a convex mirror (curved outwards), the reflected rays diverge (get farther apart). When this happens, it makes the reflection appear smaller.

In a concave mirror (curved inwards), the reflected rays converge (get closer together). When this happens, it makes the reflection appear larger.

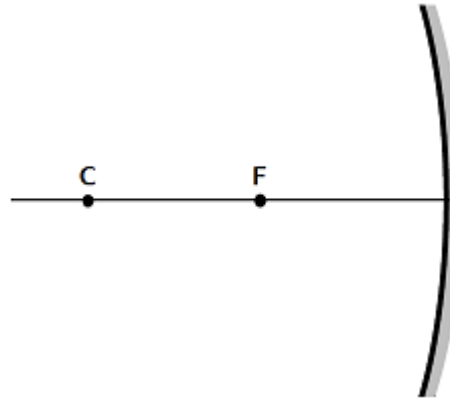
### Focal Points

The focal point or focus of a mirror is the point where the rays of reflected light converge. For a spherical mirror (one in which the shape of the mirror is the surface of a sphere), the distance from the mirror to the focus ( $d_f$ ) is always halfway between the mirror itself and the radius of curvature ( $r_c$ ). The radius of curvature can also be described as the distance from the center of the sphere to the mirror, which is part of its surface:

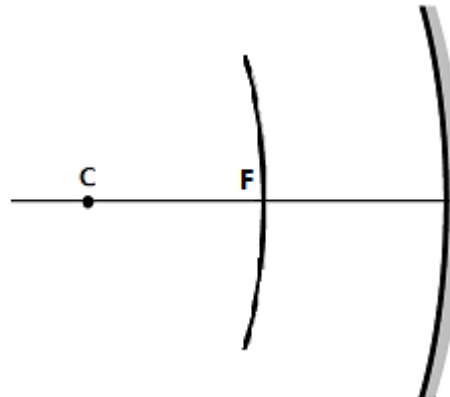
$$d_f = \frac{r_c}{2} \quad \text{or} \quad r_c = 2d_f$$

Use this space for summary and/or additional notes.

In an introductory physics class, the focus of a curved mirror is often described as a single point, as in the following diagram:



However, it is important to remember that a principal axis (a line perpendicular to the mirror) can be drawn from any point along the surface of the mirror. This means that the focus is not a single point, but rather the **set of all points** that are halfway between the center of the sphere and the surface of the mirror:



Use this space for summary and/or additional notes.

### Equations

The distance from the mirror to the focus ( $d_f$ ) can be calculated from the distance to the object ( $d_o$ ) and the distance to the image ( $d_i$ ), using the following equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_f}$$

Distances for the image ( $d_i$ ) and focus ( $d_f$ ) are positive in front of the mirror (where a real image would be), and negative behind the mirror (where a virtual image would be).

The height of the image ( $h_i$ ) can be calculated from the height of the object ( $h_o$ ) and the two distances ( $d_i$  and  $d_o$ ), using the following equation:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

A positive value for  $h_i$  means the image is upright (right-side-up), and a negative value for  $h_i$  means the image is inverted (upside-down).

magnification: the ratio of the size of the image to the size of the object.

If  $m > 1$ , the image is larger than the object. (For example, if  $m = 2$ , then the image is twice as large as the object.) If  $m = 1$ , the object and image are the same size. If  $m < 1$ , the image is smaller. Finally, note that in a mirror, virtual images are always upright, and real images are always inverted.

Use this space for summary and/or additional notes.

**Sample Problem:**

- Q: An object that is 5 cm high is placed 9 cm in front of a spherical convex mirror. The radius of curvature of the mirror is 10 cm. Find the height of the image and its distance from the mirror. State whether the image is real or virtual, and upright or inverted.
- A: The mirror is convex, which means the focus is behind the mirror. This is the side where a **virtual** image would be, so the distance to the focus is therefore negative. The distance to the focus is half the radius of curvature, which means  $d_f = -5$  cm. From this information, we can find the distance from the mirror to the image ( $d_i$ ):

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_f}$$
$$\frac{1}{9} + \frac{1}{d_i} = \frac{1}{-5}$$
$$\frac{5}{45} + \frac{1}{d_i} = -\frac{9}{45}$$
$$\frac{1}{d_i} = -\frac{14}{45}$$
$$d_i = -\frac{45}{14} = -3.2 \text{ cm}$$

The value of  $-3.2$  cm means the image is a virtual image located 3.2 cm behind the mirror.

Now that we know the distance from the mirror to the image, we can calculate the height of the image ( $h_i$ ):

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$
$$\frac{h_i}{5} = -\frac{-3.2}{9}$$
$$(5)(3.2) = 9 h_i$$
$$h_i = \frac{16}{9} = 1.8 \text{ cm}$$

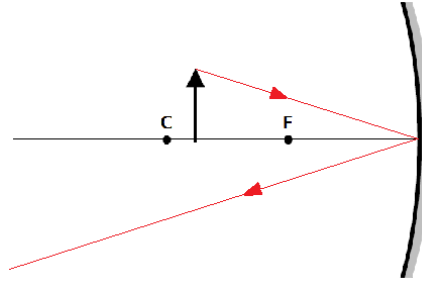
The image is 1.8 cm high. Because the height is a positive number, this means the image is upright (right-side-up).

Use this space for summary and/or additional notes.

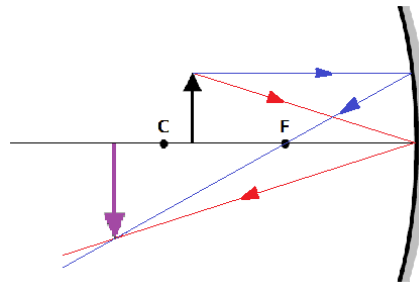
### Ray Tracing

A more intuitive way of finding the location, size and orientation of an image in a mirror is to draw (trace) the rays of light to see where they converge. A simple way to do this is:

1. A ray of light that hits the mirror on the principal axis is reflected back at the same angle relative to the principal axis. (The angle of incidence relative to the principal axis equals the angle of reflection.)



2. A ray of light that hits the mirror parallel to the principal axis is reflected directly toward or away from the focus.

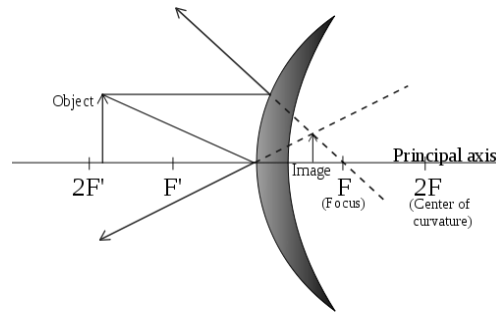


3. If you draw a pair of rays from the top of the object as described by #1 and #2 above, the intersection will be at the top of the image of the object.

Use this space for summary and/or additional notes.



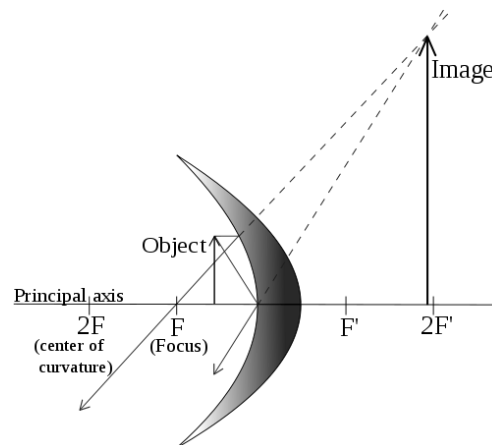
For a convex mirror, the image is always virtual (behind the mirror) and is always smaller than the object:



For a concave mirror, the equations are exactly the same as for a convex mirror.

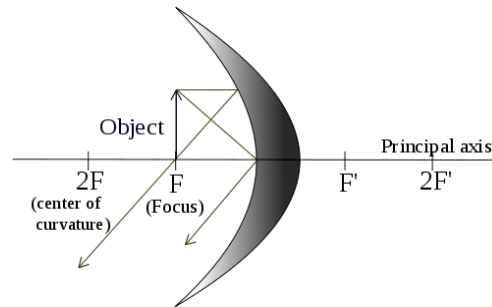
However, for a concave mirror, the image changes, depending on where the object is relative to the center of curvature and the focus, as is shown by ray tracing in each the following cases.

If the object is closer to the mirror than the focus, you see a virtual image (behind the mirror) that is upright (right-side-up), and larger than the original:

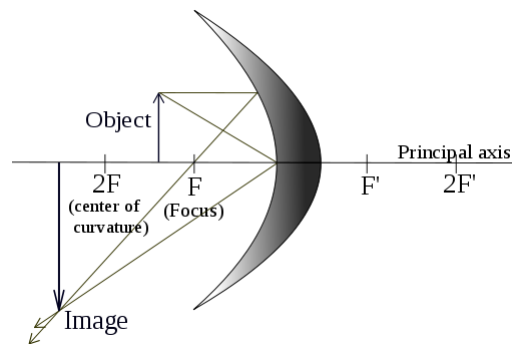


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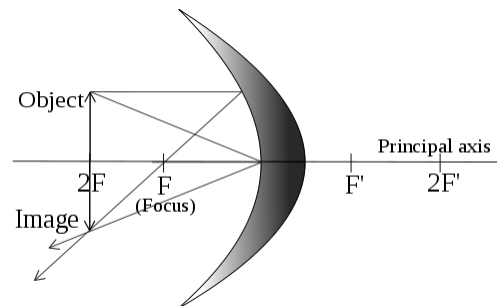
If the object is at the focus, it is not reflected because the rays do not converge:



If the object is between the focus and the center of curvature, you would see a real (in front of the mirror) image that is behind the object, inverted (upside-down), and larger:

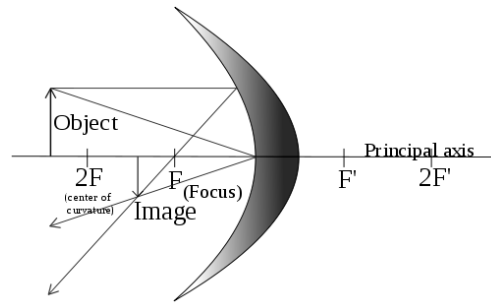


If the object is at the center of curvature, you would see a real, inverted image that is the same size and same distance from the mirror as the object:



Use this space for summary and/or additional notes.

If the object is farther from the mirror than the center of curvature, you would see a real, inverted image that is smaller and closer to the mirror than the object.



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## Refraction

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- what refraction is and why it happens
- higher index of refraction bends light toward the normal (perpendicular)
- how & why rainbows occur

**Skills:**

- solve problems using Snell's Law

**Language Objectives:**

- Understand and correctly use the terms "refraction," "index of refraction," "prism," and "critical angle."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Snell's Law.

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**Notes:**

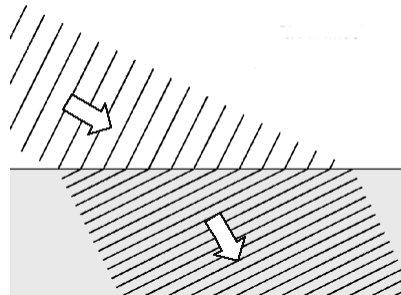
refraction: a change in the velocity and direction of a wave as it passes from one medium to another. The change in direction occurs because the wave travels at different velocities in the different media.

index of refraction: a number that relates the velocity of light in a medium to the velocity of light in a vacuum.

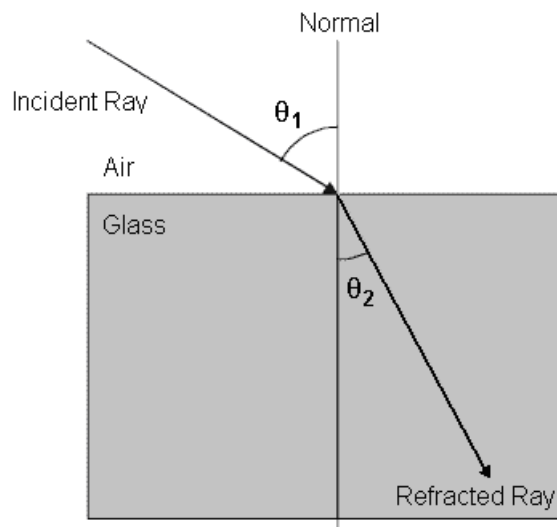
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Use this space for summary and/or additional notes:

When light crosses from one medium to another, the difference in velocity of the waves causes the wave to bend. For example, in the picture below, the waves are moving faster in the upper medium. As they enter the lower medium, they slow down. Because the part of the wave that enters the medium soonest slows down first, the angle of the wave changes as it crosses the boundary.



When the waves slow down, they are bent toward the normal (perpendicular), as in the following diagram:



Use this space for summary and/or additional notes.

The index of refraction of a medium is the velocity of light in a vacuum divided by the velocity of light in the medium:

$$n = \frac{c}{v}$$

Thus the larger the index of refraction, the more the medium slows down light as it passes through.

The index of refraction for different substances is given below.

Substance	Index of Refraction	Substance	Index of Refraction
vacuum	1.00000	quartz	1.46
air (0°C and 1 atm)	1.00029	glass (typical)	1.52
water (20°C)	1.333	NaCl (salt) crystals	1.54
acetone	1.357	polystyrene (#6 plastic)	1.55
ethyl alcohol	1.362	diamond	2.42

These values are for yellow light with a wavelength of 589 nm.

For light traveling from one medium into another, the ratio of the speeds of light is related to the ratio of the indices of refraction, as described by Snell's Law (named for the Dutch astronomer Willebrord Snellius):

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

The more familiar presentation of Snell's Law is:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

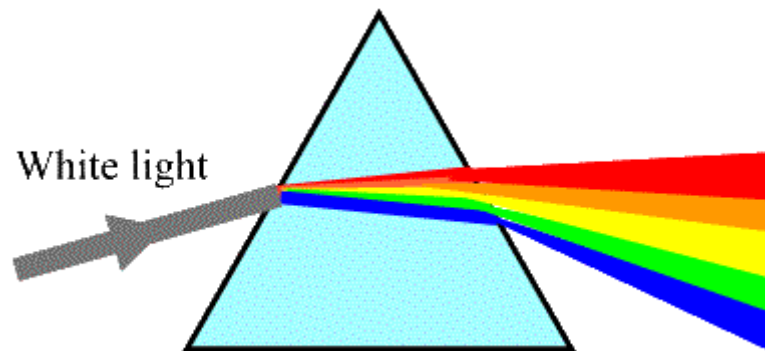
Use this space for summary and/or additional notes.

### Prisms

The index of refraction of a medium varies with the wavelength of light passing through it. The index of refraction is greater for shorter wavelengths (toward the violet end of the spectrum) and less (closer to 1) for longer wavelengths (toward the red end of the spectrum).

prism: an object that refracts light

If light passes through a prism (from air into the prism and back out) and the two interfaces are not parallel, the different indices of refraction for the different wavelengths will cause the light to spread out.



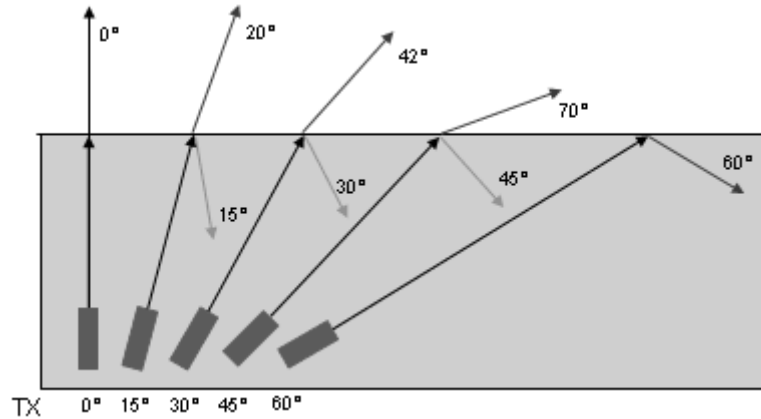
When light is bent by a prism, the ratio of indices of refraction is the inverse of the ratio of wavelengths. Thus we can expand Snell's Law as follows:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

Use this space for summary and/or additional notes.

### Total Internal Reflection

If a light wave is traveling from a slower medium to a faster one and the angle is so steep that the refracted angle would be 90° or greater, the boundary acts as a mirror and the light ray reflects off of it. This phenomenon is called total internal reflection:



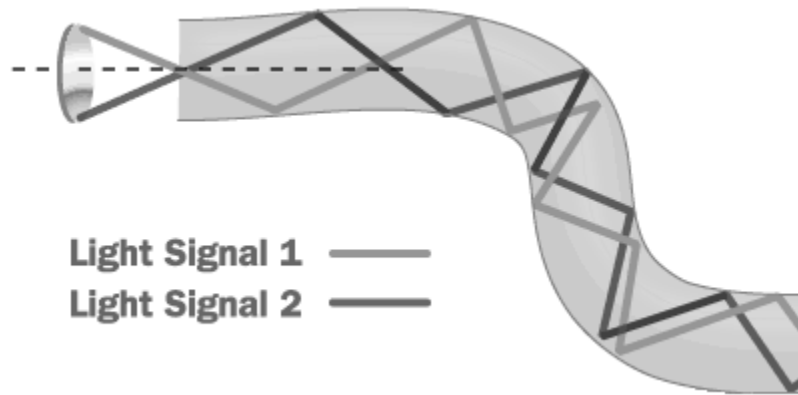
critical angle ( $\theta_c$ ): the angle beyond which total internal reflection occurs.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Use this space for summary and/or additional notes.



Total internal reflection is how optical fibers (long strands of optically pure glass with a high index of refraction) are used to transmit information over long distances, using pulses of light.



Total internal reflection is also the principle behind speech teleprompters:



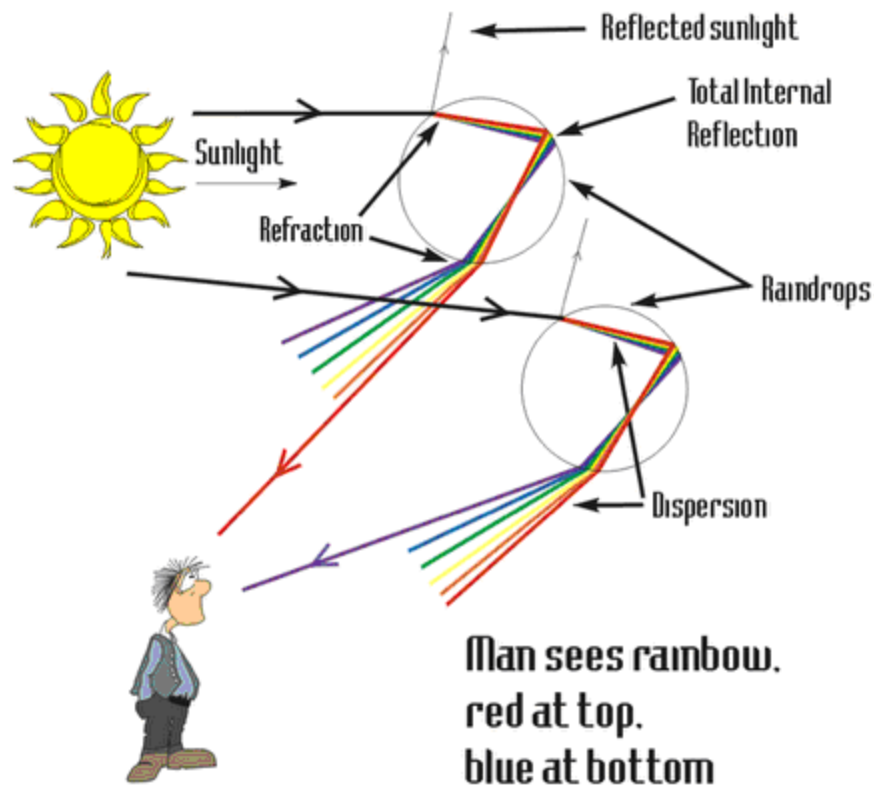
The speaker stands behind a clear piece of glass. The image of the speech is projected onto the glass. The text is visible to the speaker, but not to the audience.

Use this space for summary and/or additional notes.

## Rainbows

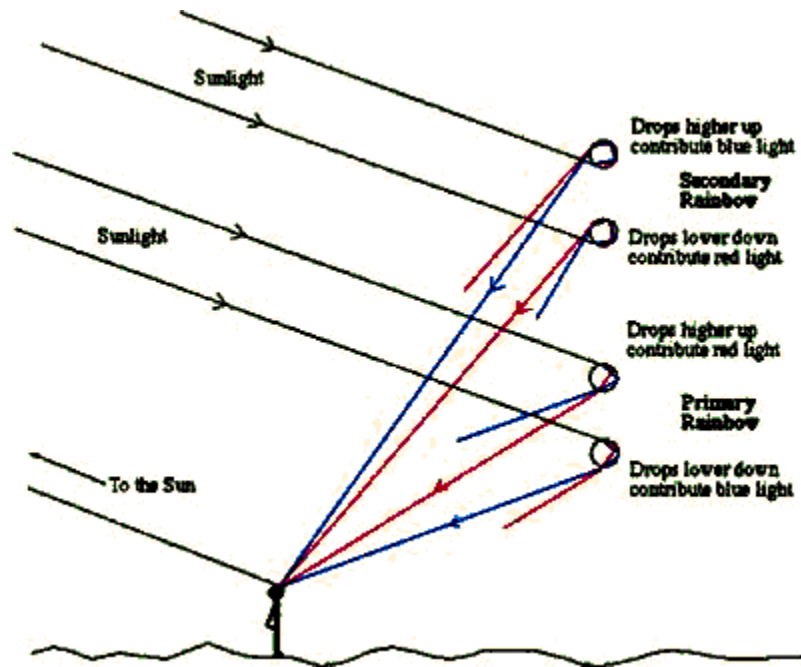
A rainbow occurs from a combination of refraction, total internal reflection, and a second refraction, with raindrops acting as the prisms.

When this process occurs, different wavelengths of are refracted at different angles. Because colors near the red end of the spectrum have a lower index of refraction, the critical angle is shallower for these wavelengths, and they are reflected starting at a shallower angle than colors closer to the violet end of the spectrum. The overall change in the direction of the light after this combination of refraction–reflection–refraction (including both refractions as well as the reflection) ranges from approximately  $40^\circ$  for violet light to approximately  $42^\circ$  for red light. This difference is what produces the spread of colors in a rainbow.



Use this space for summary and/or additional notes.

When internal reflection occurs twice on the inside of a raindrop, the result is a second rainbow.



The second rainbow appears above the first because the angle of light exiting the raindrop is greater—varying from  $50^\circ$  for red light to  $52.5^\circ$  for violet light.

Use this space for summary and/or additional notes.

Note that the order of the colors in the second rainbow is reversed.



Note also that the sky is brighter inside the rainbow. Because red light reflects at all angles greater than or equal to  $40^\circ$ , red light is therefore a component of all of the colors inside the red band of the rainbow. The same is true for each of the other colors; inside of the violet band, all wavelengths of visible light are present, and the result is white light. Outside of the red band, no visible light is refracted, which causes the sky outside the rainbow to appear darker.

Because the second rainbow is reversed, the sky will be brighter outside the second rainbow.

Use this space for summary and/or additional notes.

## Polarization

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

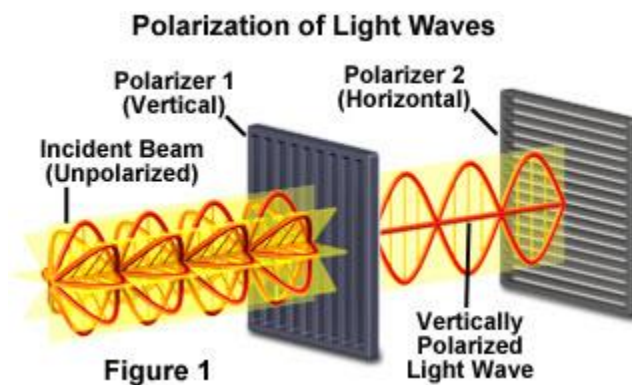
- how and under what circumstances light can be polarized

**Language Objectives:**

- Understand and correctly use the term “polarized” or “polarization.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

Normally, light (and other electromagnetic waves) propagate in all directions. When the light is passed through a special filter, called a polarizer, it blocks light waves in all but one plane (direction), as shown in the following diagram:



Light that is polarized in this manner is called plane-polarized light.

Use this space for summary and/or additional notes:

A flat surface can act as a polarizer at certain angles. The Scottish physicist Sir David Brewster derived a formula for the angle of maximum polarization based on the indices of refraction of the two substances:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where:

$\theta_B$  = Brewster's angle, the angle of incidence at which unpolarized light striking a surface is perfectly polarized when reflected.

$n_1$  &  $n_2$  = indices of refraction of the two substances

One example is light reflecting off a wet road. When the sun shines on a wet road at a low angle, the reflected light is polarized parallel to the surface (*i.e.*, horizontally). Sunglasses that are polarized vertically (*i.e.*, that allow only vertically polarized light to pass through) will effectively block most or all of the light reflected from the road.

Another example is the light of a rainbow. When sunlight reflects off the inside of a raindrop, the angle of incidence is very close to Brewster's angle. This causes the light that exits the raindrop to be polarized in the same direction as the bows of the rainbow (*i.e.*, horizontally at the top). This is why you cannot see a rainbow through polarized sunglasses!

Note that if you place two polarizers on top of each other and turn them so they polarize in different directions, no light can get through. This is called crossed polarization.

Use this space for summary and/or additional notes.

## Lenses

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how lenses work
- the difference between convex & concave lenses

**Skills:**

- calculate the focus, and the location and size of the image

**Language Objectives:**

- Understand and correctly use the terms “lens,” “convex,” “concave,” “principal axis,” “focus,” “vertex,” “real image,” “virtual image,” “upright,” and “inverted.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving convex and concave lenses.

**Notes:**

Lenses are similar to curved mirrors in that they change the direction of light rays to produce an image of an object that can have a different size, orientation and distance from the mirror relative to the object.

Lenses are different from mirrors in that light passes through them, which means they operate by refraction instead of reflection.

Use this space for summary and/or additional notes:

lens: a symmetrical optical device which refracts light in a way that makes the rays of light either converge or diverge.

convex lens: a lens that refracts light so that it converges as it passes through.

concave lens: a lens that refracts light so that it diverges as it passes through.

principal axis: the axis (line) that is normal to the surface of the lens. Light traveling along the principal axis is refracted at an angle of  $0^\circ$  (*i.e.*, the direction is not changed). Most physics problems represent the principal axis as horizontal, but it can actually be any direction. Note also that a lens may have a different principal axis on each side.

focus or focal point: the point at which light rays converge after passing through the lens.

vertex: the point where the principal axis passes through the center of the lens.

real image: an image produced by light rays that pass through the lens. A **real image** will appear on the **opposite side of the lens** from the object.

virtual image: an apparent image produced at the point where diverging rays appear to originate. A **virtual image** will appear on the **same side of the lens** as the object.

A rule of thumb that works for both mirrors and lenses is that a real image is produced by the convergence of actual rays of light. A virtual image is our perception of where the rays of light would have come from.

upright image: an image that is oriented in the same direction as the object. ("right-side-up")

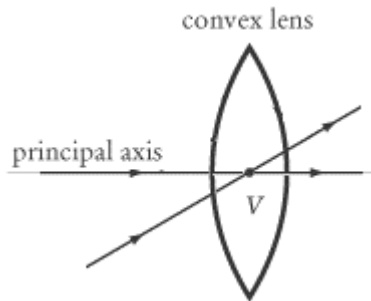
inverted image; an image that is oriented in the opposite direction from the object. ("upside-down")

Use this space for summary and/or additional notes.

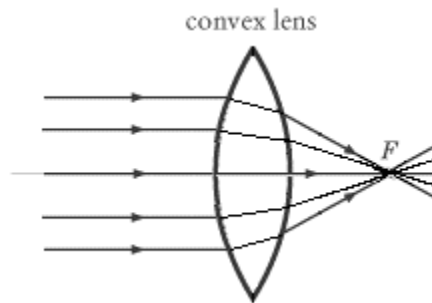


In all lenses:

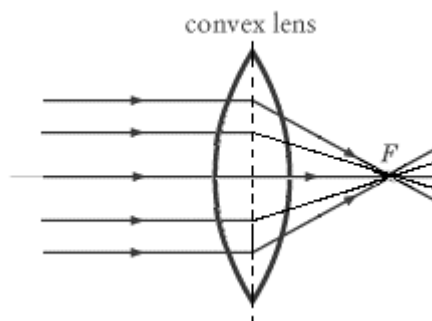
1. Light passing through the vertex of the lens comes out of the lens in the same direction as it entered, as if the lens were not there.



2. Light passing through any part of the lens other than the vertex is refracted through the focus:



Notice that the light is refracted twice, once upon entering the lens and a second time upon entering the air when it exits. For convenience, we usually draw the ray trace as if the light is refracted once when it crosses the center of the lens:



Use this space for summary and/or additional notes.

The equations for lenses are the same as the equations for curved mirrors. Distances are measured from the vertex.

The magnification ( $m$ ) is the ratio of the height of the image ( $h_i$ ) to the height of the object ( $h_o$ ), which is equal to the ratio of the distance of the image ( $d_i$ ) to distance of the object ( $d_o$ ).

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

As with mirrors, the distance to the image is defined to be positive for a real image, and negative for a virtual image. However, note that with lenses the real image is caused by the rays of light that pass through the lens, which means a real image is behind a lens, where as a real image is in front of a mirror.

As with mirrors, the distance from the vertex of the lens to the focus ( $d_f$ ) is defined by the equation:

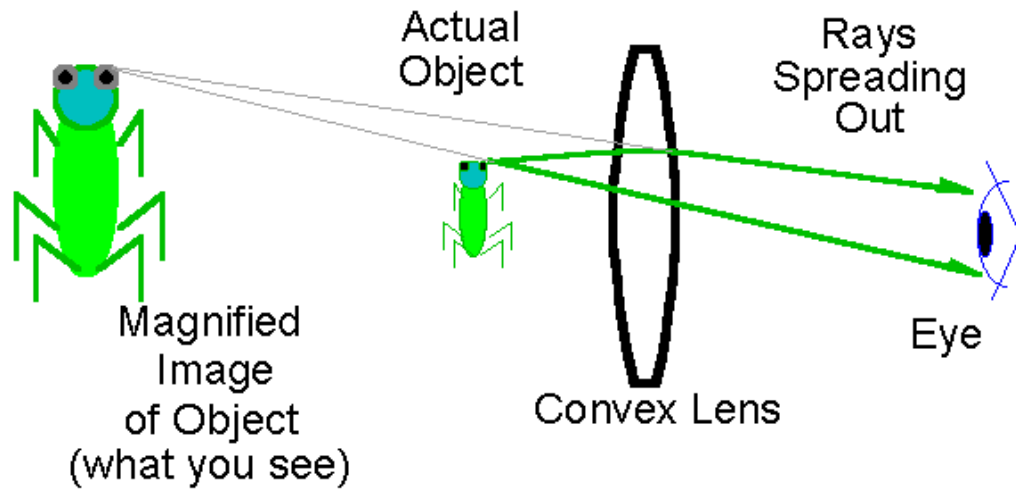
$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{d_f}$$

Use this space for summary and/or additional notes.

### Convex Lenses

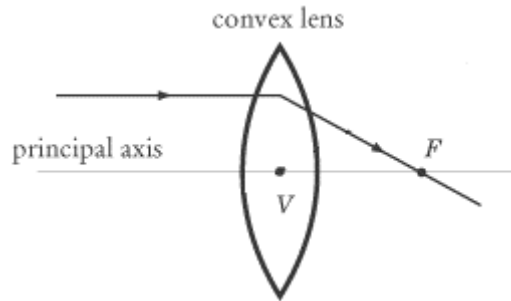
A convex lens causes light rays to converge (bend towards each other) as they pass through.

The most familiar use of convex lenses is as a magnifying glass. Note how the bending of the light rays makes the object appear larger

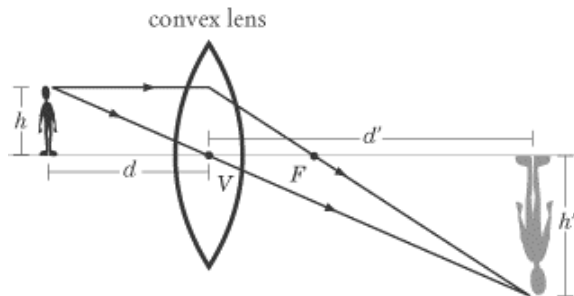


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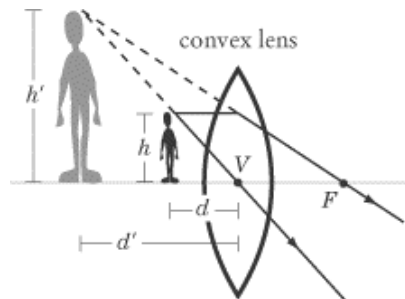
For a convex lens, the focus is always on the opposite side of the lens from the object:



If the object is farther away from the lens than the focus, the image is real (on the opposite side of the lens) and inverted (upside-down):



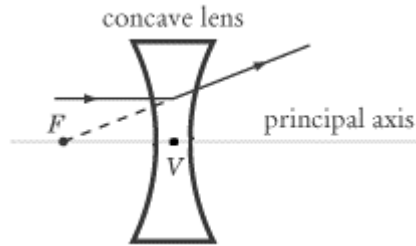
If the object is closer than the focus, the image is virtual (on the same side of the lens) and upright (right-side-up):



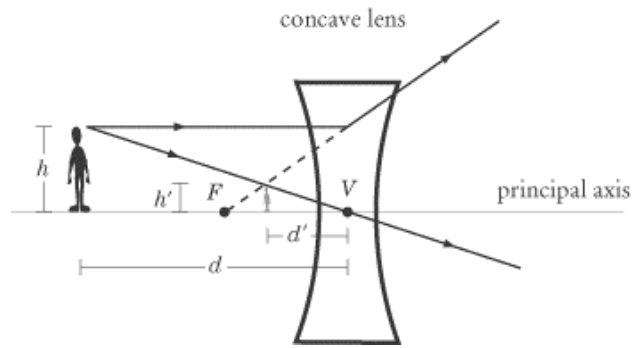
Use this space for summary and/or additional notes.

### Concave Lenses

A concave lens causes light rays to diverge (bend away from each other). For a concave lens, the focus is on the same side of the lens as the object.



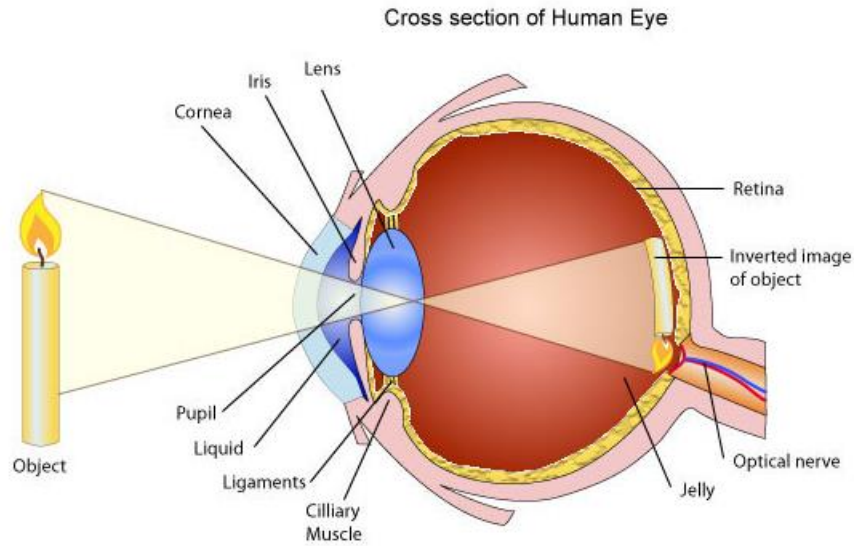
For a concave lens, the image is always virtual (on the same side of the lens) and upright (right-side-up):



Use this space for summary and/or additional notes.

### Physiology

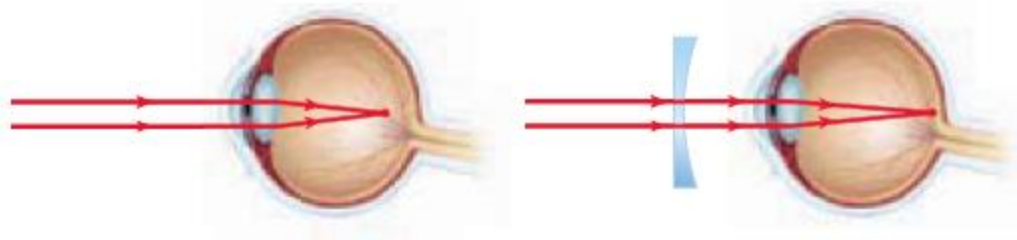
In the human eye, the cornea and lens both act as lenses. However, because the action of the ciliary muscles changes the shape of the lens, the lens is responsible for the exact focal point, which determines what we are focusing our eyes on. When the ciliary muscles relax, the images of distant objects are focused on the retina. When these muscles contract, the focal point moves and closer objects come into focus.



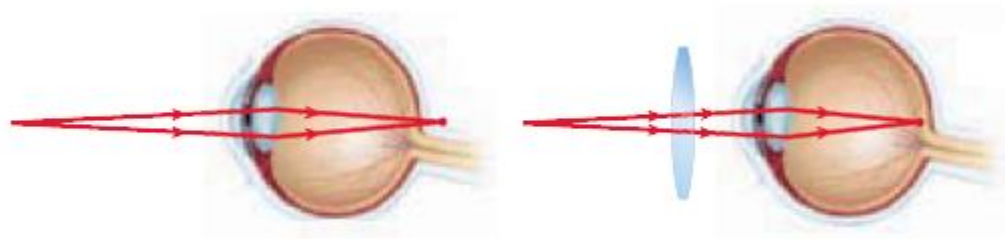
Use this space for summary and/or additional notes.

### Nearsightedness and Farsightedness

“Nearsighted” means only objects near the eye are in focus; the viewer is unable to focus on distant objects. This happens because the focus of the lens when the ciliary muscles are fully relaxed is in front of the retina. Nearsightedness is corrected by eyeglasses with concave lenses, which move the focal point back to the retina.



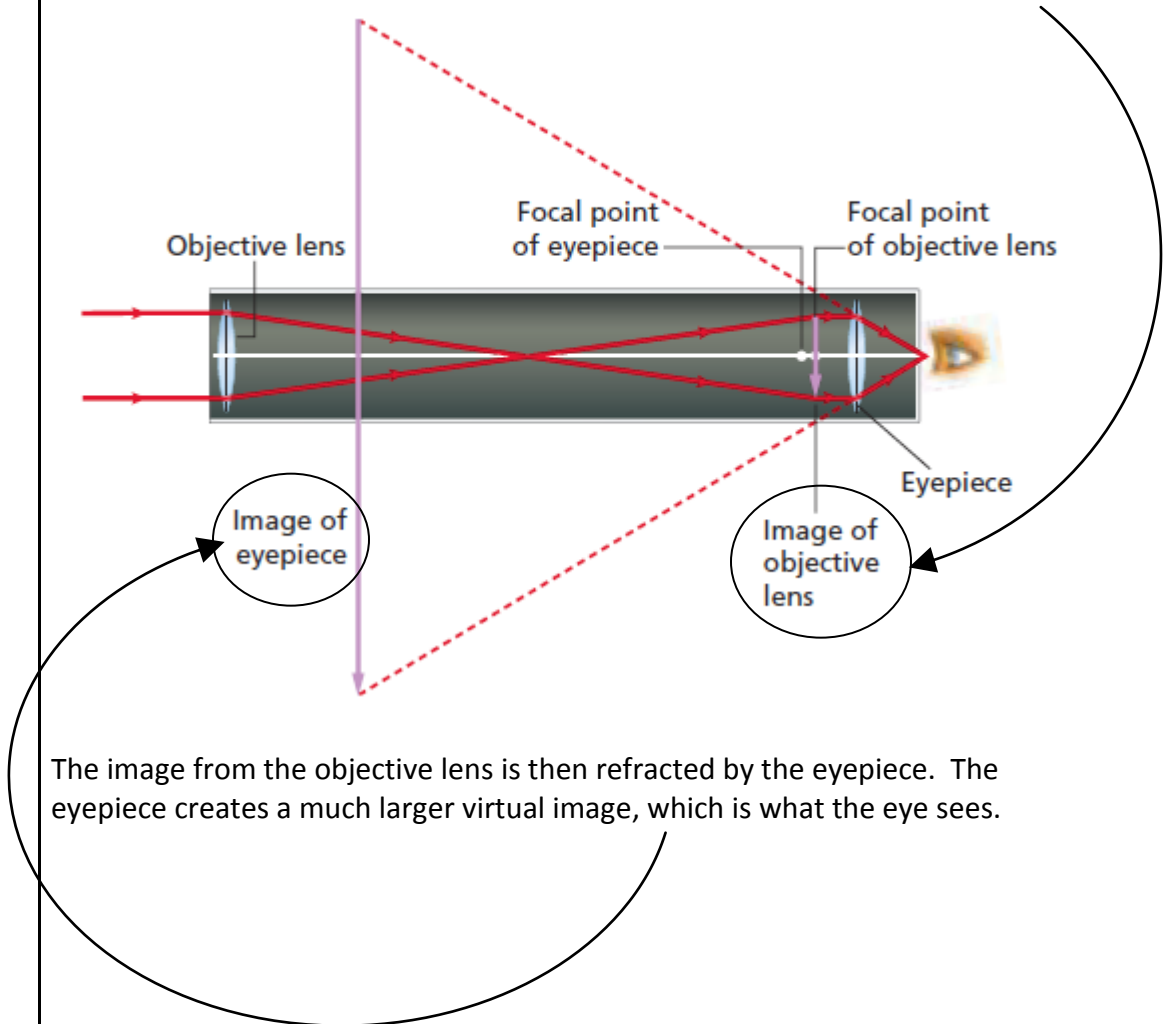
“Farsighted” means only objects far away from the eye are in focus; the viewer is unable to focus on close objects. This happens because the ciliary muscles cannot contract enough to bring the focal point of the lens for light coming from nearby objects onto the retina. Farsightedness is corrected by eyeglasses with convex lenses, which move the focal point forward to the retina.



Use this space for summary and/or additional notes.

### Telescopes

A telescope performs two tasks. The objective lens focuses light from a distant object and creates a virtual image in front of the eyepiece.



The image from the objective lens is then refracted by the eyepiece. The eyepiece creates a much larger virtual image, which is what the eye sees.

Use this space for summary and/or additional notes.



## Diffraction

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- how light spreads from a slit
- how a diffraction grating works

**Skills:**

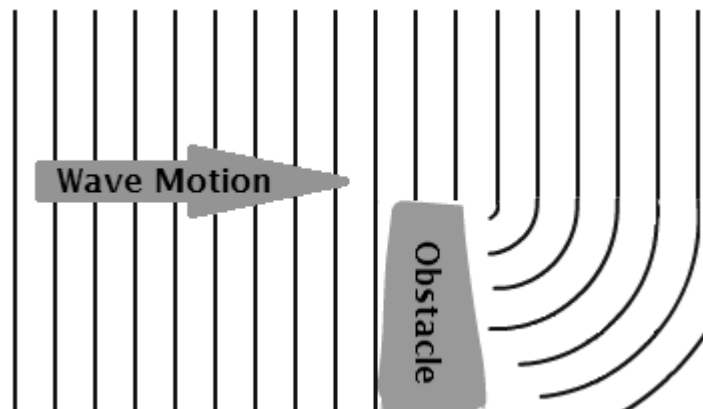
- calculate the distance between colored bands for a diffraction grating

**Language Objectives:**

- Understand and correctly use the terms “diffraction” and “diffraction grating.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to diffraction gratings.

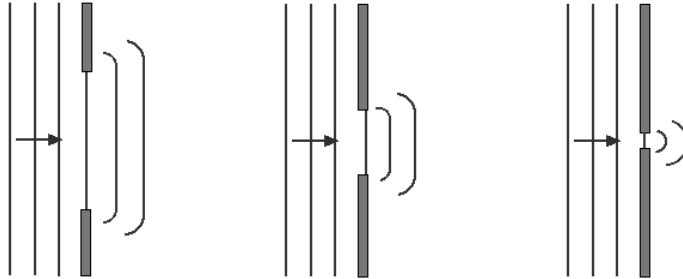
**Notes:**

diffraction: the slight bending of a wave as it passes around the edge of an object or through a slit:



Use this space for summary and/or additional notes:

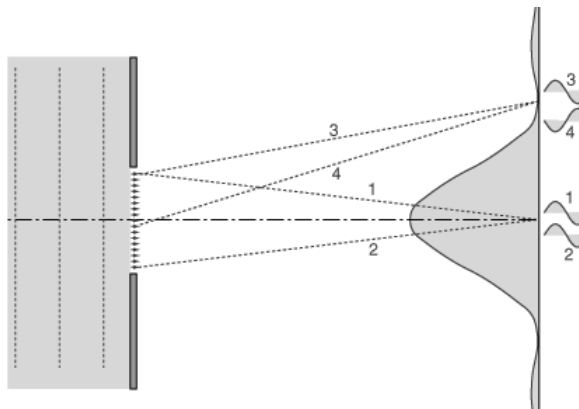
When light passes straight through a wide opening, the rays continue in a straight line. However, if we make the slit so narrow that the width is approximately equal to the wavelength, then the slit effectively becomes a point, and waves propagate in all directions from it:



If we shine light through a slit whose thickness is approximately the same order of magnitude as the wavelength, the light can only hit the wall in specific locations.

In the following diagram, light travels the same distance for paths 1 and 2—the same number of wavelengths. Light waves hitting this point will add constructively, which makes the light brighter.

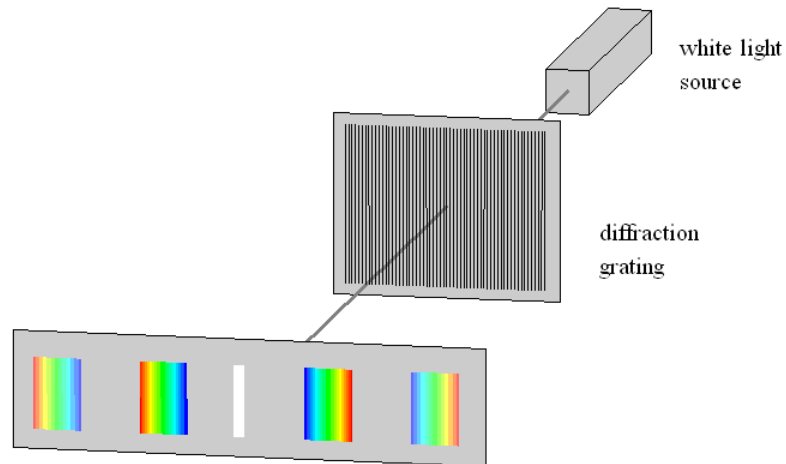
However, for paths 3 and 4, path 4 is  $\frac{1}{2}$  wavelength longer than path 3. Light taking path 4 is  $\frac{1}{2}$  wavelength out of phase with light from path 3. The waves add destructively (cancel), and there is no light:



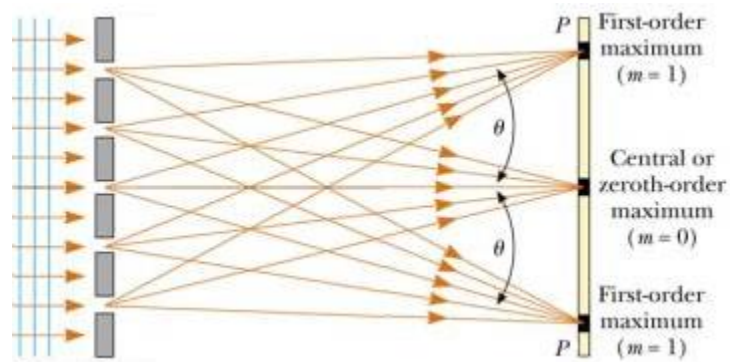
Farther up or down the right side will be alternating locations where the difference in path length results in waves that are in phase (constructive interference = bright spots), vs. out-of-phase (destructive interference = dark spots).

Use this space for summary and/or additional notes.

**diffraction grating:** a screen with a series of evenly-spaced slits that scatters light in a repeating, predictable pattern.



When light shines through a diffraction grating, the following happens:



The patterns of light surrounding the center are the points where the waves of light add constructively.

Notice that blue and violet light (with the shortest wavelengths) is diffracted the least and appears closest to the center, whereas red light is diffracted more and appears farther away. This is because shorter waves need to turn less far to fit through the slits than longer waves.

Use this space for summary and/or additional notes.

The equation for a diffraction grating is:

$$d \sin \theta_m = m \lambda$$

where:

- $m$  = the number of waves that equals the difference in the lengths of the two paths (integer)
- $\theta_m$  = the angle of emergence (or angle of deviation) in order for light from one slit to add constructively to light from a neighboring slit that is  $m$  wavelengths away.
- $d$  = the distance between the slits
- $\lambda$  = the wavelength of the light

Use this space for summary and/or additional notes.

**Sample Problem:**

Q: Mr. Bigler has three laser pointers. The red laser has a wavelength of 650 nm, the green laser has a wavelength of 532 nm, and the blue laser has a wavelength of 405 nm. If each of these is shone through a diffraction grating with 5 000 lines per cm, what will the angle of emergence be for each color?

A: For our diffraction grating, 5 000 lines per cm equals 500 000 lines per meter.

$$d = \frac{1}{500\,000} = 2 \times 10^{-6} \text{ m}^{-1}$$

For the red laser, 650 nm equals  $\lambda = 650 \text{ nm} = 6.50 \times 10^{-7} \text{ m}$

The equation is:

$$d \sin \theta_m = m \lambda$$

For the red laser at  $m = 1$ , this becomes:

$$(6.67 \times 10^{-6}) \sin \theta = (1)(6.50 \times 10^{-7})$$

$$\sin \theta = \frac{6.50 \times 10^{-7}}{2 \times 10^{-6}} = 0.325$$

$$\theta = \sin^{-1}(0.325) = 19.0^\circ$$

For the green laser ( $\lambda = 532 \text{ nm} = 5.32 \times 10^{-7} \text{ m}$ ) and the blue laser ( $\lambda = 405 \text{ nm} = 4.05 \times 10^{-7} \text{ m}$ ):

$$\sin \theta = \frac{5.32 \times 10^{-7}}{2 \times 10^{-6}} = 0.266$$

and

$$\sin \theta = \frac{4.05 \times 10^{-7}}{2 \times 10^{-6}} = 0.203$$

$$\theta = \sin^{-1}(0.266) = 15.4^\circ$$

$$\theta = \sin^{-1}(0.203) = 11.7^\circ$$

Use this space for summary and/or additional notes.

## Scattering

**Unit:** Light & Optics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

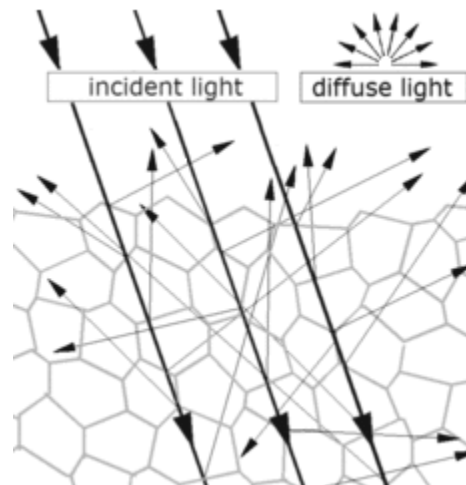
- why the sky is blue
- why the sun looks red at sunset

**Language Objectives:**

- Understand and correctly use the term “scattering.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

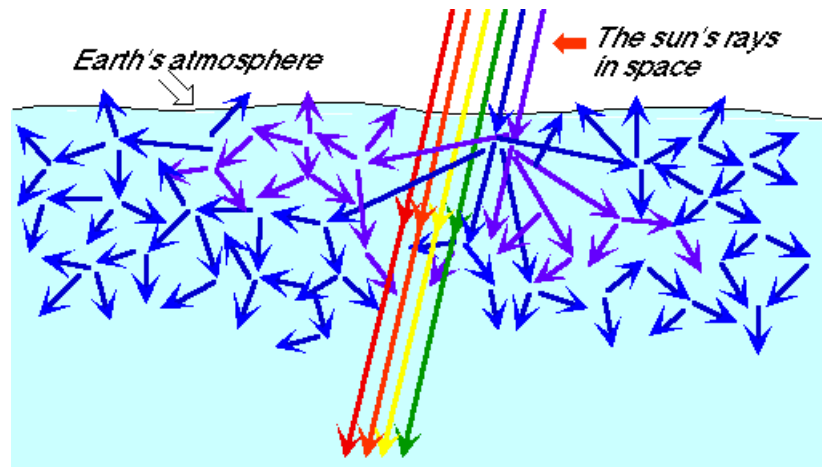
scattering: a change in the direction of rays of light caused by irregularities in the propagation medium, collisions with small particles, or at the interface between two media.



Rayleigh scattering: scattering of light because of collisions with small particles in the medium. Rayleigh scattering is named after the British physicist Lord Rayleigh.

Use this space for summary and/or additional notes:

Rayleigh scattering is responsible for the color of the sky. Small particles (0.5–1 micron) scatter visible light as it passes through Earth's atmosphere. Because light at the blue and violet end of the spectrum is about five times as likely to be scattered as light at the red end of the spectrum, the majority of the scattered light in our atmosphere is blue.



This is also why the sun appears yellow during the day—the combination of red, orange, yellow and green light appears yellow to us.

Water vapor molecules are much larger—ranging in size from 2–5 microns. For these larger particles, the probability of scattering is approximately the same for all wavelengths, which is why clouds appear white.

At sunset, because the angle of the sun is much lower, the light must pass through much more of the atmosphere before we see it. By the time the light gets to our eyes, all of the colors are removed by scattering except for the extreme red end of the spectrum, which is why the sun appears red when it sets.

Use this space for summary and/or additional notes.





## Introduction: Pressure & Fluid Mechanics

**Unit:** Pressure & Fluid Mechanics

### Topics covered in this chapter:

Pressure .....	422
Hydrostatics .....	426
Buoyancy.....	429
Gas Laws.....	434
Fluid Motion & Bernoulli's Law.....	442

Fluid mechanics is the study of behaviors that are specific to fluids (liquids and gases).

- *Pressure* is the property that is central to the topic of fluid mechanics.
- *Hydrostatics* and *Buoyancy* describe and give equations for the effects of gravity on pressure.
- *Gas Laws* describes behaviors and equations involving temperature, pressure and volume, as related to gases.
- *Fluid Motion & Bernoulli's Law* describes the effects of fluid motion on pressure.

This chapter focuses on real-world applications of fluids and pressure, including more demonstrations than most other topics. One of the challenges in this chapter is relating the equations to the behaviors seen in the demonstrations.

### Standards addressed in this chapter:

**Next Generation Science Standards (NGSS):**

No NGSS standards are addressed in this chapter.

**Massachusetts Curriculum Frameworks (2006):**

No MA curriculum standards are addressed in this chapter.

### Skills learned & applied in this chapter:

- Before & after problems.

Use this space for summary and/or additional notes:

## Pressure

**Unit:** Pressure & Fluid Mechanics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- pressure
- hydraulic pressure

**Skills:**

- calculate pressure as an applied force

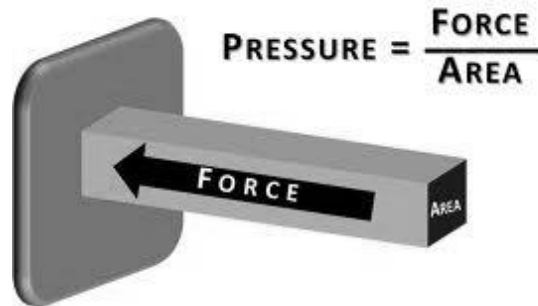
**Language Objectives:**

- Understand and correctly use the terms “pressure” and “hydraulic pressure.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to pressure and hydraulics.

**Notes:**

pressure: the exertion of force upon a surface by an object, fluid, *etc.* that is in contact with it.

Mathematically, pressure is defined as force divided by area:



$$P = \frac{F}{A}$$

Use this space for summary and/or additional notes:

The S.I. unit for pressure is the pascal (Pa).

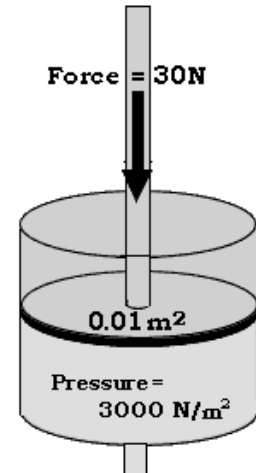
$$1 \text{ Pa} \equiv 1 \frac{\text{N}}{\text{m}^2} \equiv 1 \frac{\text{kg}}{\text{ms}^2}$$

(Note that Pa is a two-letter symbol.)

Some other common pressure units are:

- bar:  $1 \text{ bar} \equiv 100\,000 \text{ Pa} \equiv 10^5 \text{ Pa}$
- pound per square inch (psi)
- atmosphere (atm): the average atmospheric pressure on Earth at sea level.

$$1 \text{ atm} \equiv 101\,325 \text{ Pa} \equiv 101.325 \text{ kPa} \equiv 1.01325 \text{ bar} = 14.696 \text{ psi}$$



Air pressure can be described relative to a total vacuum (absolute pressure), but is more commonly described relative to atmospheric pressure (gauge pressure):

- absolute pressure: the total pressure on a surface. An absolute pressure of zero means there is zero force on the surface.
- gauge pressure: the difference between the pressure on a surface and atmospheric pressure. A gauge pressure of zero means the same as atmospheric pressure. The pressure in car tires is measured as gauge pressure. For example, a tire pressure of 30 psi (30 pounds per square inch, or  $30 \frac{\text{lb}}{\text{in}^2}$ ) would mean that the air inside the tires is pushing against the air outside the tires with a pressure of 30 psi. A flat tire would have a gauge pressure of zero.

### Sample Problem

Q: What is the pressure caused by a force of 25 N acting on a piston with an area of  $0.05 \text{ m}^2$ ?

A: 
$$P = \frac{F}{A} = \frac{25 \text{ N}}{0.05 \text{ m}^2} = 500 \text{ Pa}$$

Use this space for summary and/or additional notes.

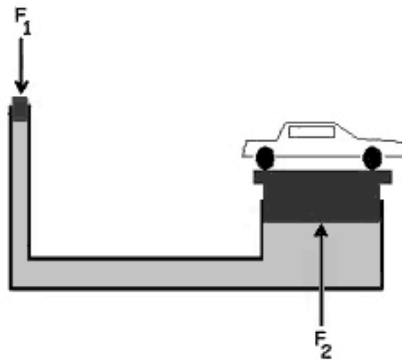
## Hydraulic Pressure

Pascal's Principle, which was discovered by the French mathematician Blaise Pascal, states that any pressure applied to a fluid is transmitted uniformly throughout the fluid.

Because  $P = \frac{F}{A}$ , if the pressure is constant, then the force can be increased by increasing the area where you want the force to be applied. Thus:

$$P_1 = P_2 \text{ which means } \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

This principle is called "hydraulics":



Conservation of energy tells us that the work done by  $F_1$  must equal the work done by  $F_2$ , which means  $F_1$  must act over a considerably larger distance than  $F_2$ .

This is how hydraulic brakes work in cars. When you step on the brake pedal, the hydraulic pressure is transmitted to the master cylinder and then to the slave cylinders. The master cylinder is much smaller in diameter than the slave cylinders, which means the force applied to the brake pads is considerably greater than the force from your foot.

Use this space for summary and/or additional notes.

**Sample Problem**

Q: In a hydraulic system, a force of 25 N will be applied to a piston with an area of  $0.5 \text{ m}^2$ . If the force needs to lift a weight of 500. N, what must be the area of the piston supporting the 500. N weight?

A: 
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{25 \text{ N}}{0.5 \text{ m}^2} = \frac{500. \text{ N}}{A_2}$$

$$25A_2 = (500)(0.5)$$

$$25A_2 = 250$$

$$A_2 = 10 \text{ m}^2$$

Use this space for summary and/or additional notes.

## Hydrostatics

**Unit:** Pressure & Fluid Mechanics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- hydrostatic pressure

**Skills:**

- calculate pressure exerted by a column of fluid

**Language Objectives:**

- Understand and correctly use the term “hydrostatic pressure.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to hydrostatic pressure.

**Notes:**

### Hydrostatic Pressure

hydrostatic pressure: the pressure caused by the weight of a column of fluid

The force of gravity pulling down on the molecules in a fluid creates pressure. The more fluid there is above a point, the higher the pressure at that point.

The atmospheric pressure that we measure at the surface of the Earth is caused by the air above us, all the way to the highest point in the atmosphere, as shown in the picture at right:



Use this space for summary and/or additional notes:

Assuming the density of the fluid is constant, the pressure in a column of fluid is caused by the weight (force of gravity) acting on an area. Because the force of gravity is  $mg$  (where  $g = 9.8 \frac{m}{s^2}$ ), this means:

$$P = \frac{F_g}{A} = \frac{mg}{A}$$

where:

$P$  = pressure

$g$  = acceleration due to gravity ( $9.8 \frac{m}{s^2}$  on Earth)

$A$  = area of the surface the fluid is pushing on

We can cleverly multiply and divide our equation by volume:

$$P = \frac{mg}{A} = \frac{mg \cdot V}{A \cdot V} = \frac{m}{V} \cdot \frac{gV}{A}$$

Then, recognizing that density,  $\rho^*$ , is mass divided by volume, we can substitute:

$$P = \rho \cdot \frac{gV}{A}$$

Finally, if the volume of an object is the area of the base times the height ( $h$ ), we can rewrite the equation as:

$$P = \frac{\rho g \cancel{A} h}{\cancel{A}} = \rho g h$$

---

\* Note that physicists use the Greek letter  $\rho$  for density. You need to pay careful attention to the difference between the Greek letter  $\rho$  ("rho") and the Roman letter "p".

Use this space for summary and/or additional notes.

Finally, if there is an external pressure,  $P_o$ , above the fluid, we have to add it to the pressure from the fluid itself, which gives us the equation:

$$P = P_o + \rho gh$$

where:

$P_o$  = pressure above the fluid (if relevant)

$\rho$  = density of the fluid (this is the Greek letter "rho")

$g$  = acceleration due to gravity ( $9.8 \frac{m}{s^2}$  on Earth)

$h$  = height of the fluid above the point of interest

### Sample Problem

Q: What is the water pressure in the ocean at a depth of 25 m? The density of sea water is  $1025 \frac{kg}{m^3}$ .

A:  $P = \rho gh = (1025 \frac{kg}{m^3})(9.8 \frac{m}{s^2})(25m)$

$$P = 251\,125 \text{ Pa} = \boxed{250\,000 \text{ Pa}} \text{ or } \boxed{2.50 \text{ bar}}.$$

Use this space for summary and/or additional notes.



## Buoyancy

**Unit:** Pressure & Fluid Mechanics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Buoyancy & Archimedes' Principle

**Skills:**

- Calculate the buoyant force on an object

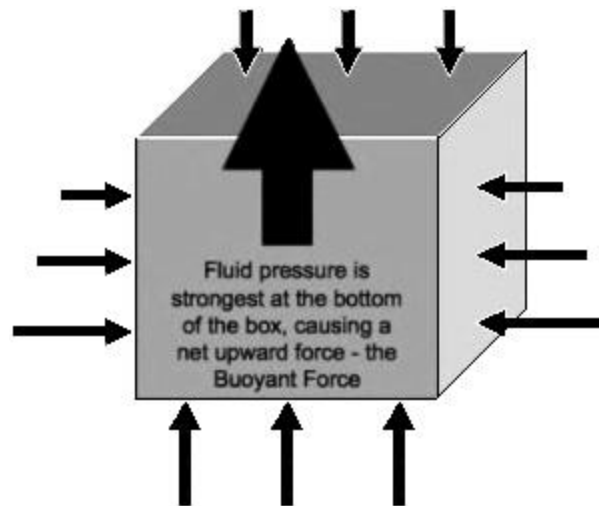
**Language Objectives:**

- Understand and correctly use the terms “displace” and “buoyant” or “buoyancy.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to buoyancy.

**Notes:**

displace: to push out of the way

buoyancy: a net upward force caused by the differences in hydrostatic pressure at different levels within a fluid:



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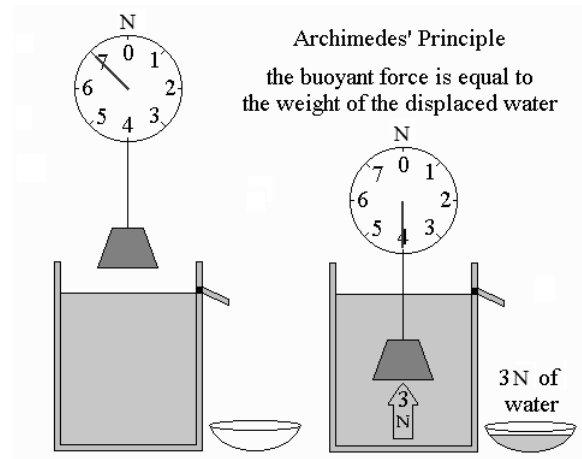
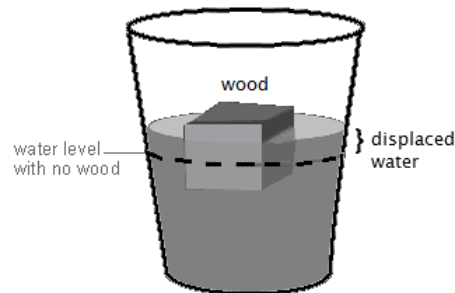
The hydrostatic pressure is stronger at the bottom of the object than at the top, which causes a net upward force on the object.

When an object displaces a fluid:

1. The volume of the fluid displaced equals the volume of the submerged part of the object.
2. The weight of the fluid displaced equals the buoyant force ( $F_B$ ).
3. The net force on the object  $F_{net} = F_g - F_B$ .

**If the object floats**, there is no net force, which means the weight of the object is equal to the buoyant force. This means  $F_{net} = F_g - F_B = 0$ ,  $F_g = F_B$ , and therefore  $mg = \rho V_d g$ . (Cancelling  $g$  from both sides gives  $m = \rho V_d$ .)

**If the object sinks**, the weight of the object is greater than the buoyant force. This means  $F_B = \rho V_d g$ ,  $F_g = mg$ , and the weight of the submerged object is  $F_{net} = F_g - F_B$ .



This concept is known as Archimedes' Principle, named for the ancient Greek scientist who discovered it.

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The buoyant force can be calculated from the following equation:

$$F_B = m_d g = \rho V_d g$$

where:

$F_B$  = buoyant force

$m_d$  = mass of fluid displaced by the object

$g$  = acceleration due to gravity ( $9.8 \frac{m}{s^2}$  on Earth)

$\rho$  = density of the fluid applying the buoyant force (e.g., water, air)

$V_d$  = volume of fluid displaced by the object

### Sample Problems:

Q: A cruise ship displaces 35 000 tonnes of water when it is floating.  
(1 tonne = 1 000 kg) If sea water has a density of  $1\,025 \frac{kg}{m^3}$ , what volume of water does the ship displace? What is the buoyant force on the ship?

A:  $\rho = \frac{m}{V}$

$$1\,025 \frac{kg}{m^3} = \frac{35\,000\,000 \text{ kg}}{V}$$

$$V = 34\,146 \text{ m}^3$$

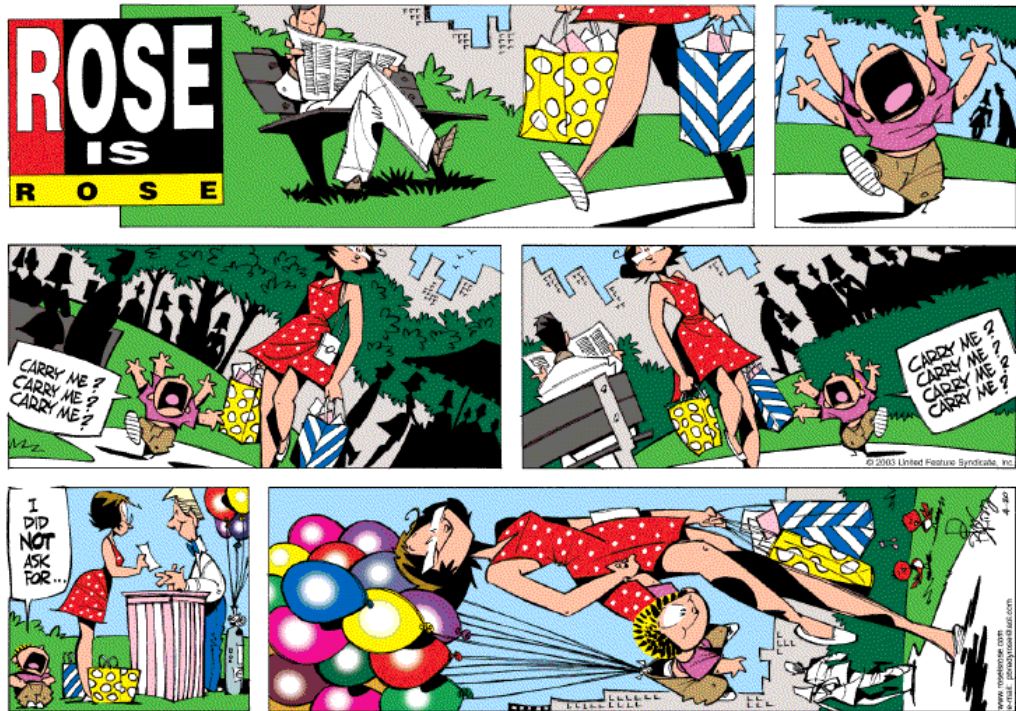
$$F_B = \rho V_d g$$

$$F_B = (1\,025 \frac{kg}{m^3})(34\,146 \text{ m}^3)(9.8 \frac{m}{s^2})$$

$$F_B = \boxed{3.43 \times 10^8 \text{ N}}$$

Use this space for summary and/or additional notes.

Q: Consider the following cartoon:



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Given the following assumptions:

- The balloons are standard 11" balloons, meaning that they have a diameter of 11 inches (28 cm).
- The temperature is 20°C. At this temperature, air has a density of  $1.200 \frac{\text{kg}}{\text{m}^3}$ , and helium has a density of  $0.166 \frac{\text{kg}}{\text{m}^3}$ .
- Pasquale is probably about four years old. The average mass a four-year-old boy is about 16 kg.
- The mass of an empty balloon plus string is 2.37 g = 0.00237 kg

How many balloons would it actually take to lift Pasquale?

Use this space for summary and/or additional notes.

A: In order to lift Pasquale,  $F_B = F_g$ .

$$F_g = mg = (16)(9.8) = 156.8 \text{ N}$$

$$F_B = \rho_{air} V_d g = (1.200)V_d(9.8)$$

Because  $F_B = F_g$ , this means:

$$156.8 = (1.200)V_d(9.8)$$

$$V_d = 13.33 \text{ m}^3$$

Assuming spherical balloons, the volume of one balloon is:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}(3.14)(0.14)^3 = 0.0115 \text{ m}^3$$

Therefore, we need  $\frac{13.33}{0.0115} = 1160$  balloon: to lift Pasquale.

However, the problem with this answer is that it doesn't account for the mass of the helium, the balloons or the strings.

Each balloon contains  $0.0115 \text{ m}^3 \times 0.166 \frac{\text{kg}}{\text{m}^3} = 0.00191 \text{ kg} = 1.91 \text{ g}$  of helium.

Each balloon (including the string) has a mass of  $2.37 \text{ g} = 0.00237 \text{ kg}$ , so the total mass of each balloon full of helium is  $0.00428 \text{ kg}$ .

This means if we have  $n$  balloons, the total mass of Pasquale plus the balloons is  $16 + 0.00428n$  kilograms. The total weight of Pasquale plus the balloons is therefore this number times  $9.8$ , which equals  $156.8 + 0.0419n$ .

The buoyant force of one balloon is:

$$F_B = \rho_{air} V_d g = (1.200)(0.0115)(9.8) = 0.135 \text{ N}$$

Therefore, the buoyant force of  $n$  balloons is  $0.135n$  newtons.

For Pasquale to be able to float,  $F_B = F_g$ , which means

$$0.135n = 0.0419n + 156.8$$

$$0.093n = 156.8$$

$$n = \boxed{1\ 685 \text{ balloons}}$$

Use this space for summary and/or additional notes.

## Gas Laws

**Unit:** Pressure & Fluid Mechanics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Boyle's Law (pressure vs. volume)
- Amontons' Law (pressure vs. temperature)
- Charles' Law (temperature vs. volume)
- Avogadro's Principle (number of particles vs. volume)
- ideal gas law
- combined gas law

**Skills:**

- Solve problems using the gas laws

**Language Objectives:**

- Understand and correctly use the terms "pressure," "volume," and "temperature," and "ideal gas."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using the combined gas law and the ideal gas law.

**Notes:**

ideal gas: a gas that behaves according to the Kinetic-Molecular Theory of Gases. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost. Most gases behave ideally when the temperature is high enough and the pressure is low enough that the molecules are not likely to form a liquid or solid, and when the molecules are made of substances that do not react chemically.

Use this space for summary and/or additional notes:

### Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional. If temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1 V_1 = P_2 V_2$$

(Note that by convention, gas laws use subscripts "1" and "2" instead of "i" and "f".)

### Amontons' Law

In 1702, French physicist Guillaume Amontons discovered that the pressure and temperature of a gas were directly proportional. If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(This law is often erroneously attributed to the French chemist Joseph Louis Gay-Lussac.)

### Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional. If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

### Avogadro's Principle

In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of molecules:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Use this space for summary and/or additional notes.

### The Combined Gas Law

If each of the above principles is combined, the following relationship is found for an ideal gas:

$$\frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \text{constant}$$

If pressure is in Pa, volume is in  $\text{m}^3$ , and temperature is in Kelvin, the value of the constant turns out to be  $1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$ . This number is called Boltzmann's constant, named for the German physicist Ludwig Boltzmann. Physicists usually use the variable  $k_B$  to represent Boltzmann's constant.

Note, however, that in most problems, the number of particles of gas doesn't change. This means  $N_1 = N_2$ , and we can cancel it from the equation, which gives:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

This equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

When using the combined gas law, any quantity that is not changing may therefore be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.) For example, suppose a problem doesn't mention anything about temperature. That means  $T$  is constant and you can cancel it. When you cancel  $T$  from both sides of the combined gas law, you get:

$$\frac{P_1 V_1}{\cancel{T_1}} = \frac{P_2 V_2}{\cancel{T_2}} \text{ which simplifies to } P_1 V_1 = P_2 V_2 \text{ (Boyle's Law)}$$

Use this space for summary and/or additional notes.



### Solving Problems Using the Combined Gas Law

You can use this method to solve any “before/after” gas law problem:

1. Determine which variables you have
2. Determine which values are initial (#1) vs. final (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures must be in Kelvin!)

Use this space for summary and/or additional notes.

### The Ideal Gas Law

The definition of Boltzmann's constant is called the ideal gas law:

$$\frac{PV}{NT} = k_B$$

Multiplying both sides of this equation by  $NT$  gives the ideal gas law in its most common form:

$$PV = Nk_B T$$

where:

$P$  = absolute pressure (Pa)

$V$  = volume ( $\text{m}^3$ )

$N$  = number of particles (or molecules)

$k_B$  = Boltzmann's constant ( $1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$ ).

$T$  = absolute temperature (K)

Note that the units on both sides of this equation are equivalent to Newton-meters, which are equivalent to joules (energy). This means that the ideal gas law is really just a mathematical statement of the work-energy theorem.

The left side of the equation,  $PV$ , represents the work that the gas can do on its surroundings. This is because:

$$P = \frac{F}{A}, \text{ which means } PV = \frac{FV}{A} = Fd = W$$

The right side of the equation,  $Nk_B T$ , represents the number of particles ( $N$ ) times the average kinetic energy of each particle ( $T$ ), which equals the total kinetic energy of the particles. (Boltzmann's constant,  $k_B$ , is just the number that makes the units work out correctly.)

In other words, the work that a gas can do equals the kinetic energy of its particles.

Use this space for summary and/or additional notes.

Chemists more commonly work with moles rather than particles. In chemistry, the ideal gas law is more commonly written:

$$PV = nRT$$

where:

$n$  = moles of gas

$R$  = gas constant ( $8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}$ )

$P$ ,  $V$ , and  $T$  are as above

The gas constant,  $R$ , is equal to Boltzmann's constant times the conversion from moles to particles (Avogadro's constant):

$$R = k_B N_A = (1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(6.02 \times 10^{23} \text{mol}^{-1}) = 8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}$$

### Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of particles (or number of moles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

Use this space for summary and/or additional notes.

**Sample problems:**

Q: A sample of an ideal gas contains  $2.10 \times 10^{24}$  particles, has a pressure of 1.20 bar, and a temperature of  $35^\circ\text{C}$ . What is its volume?

A: First, we need to declare our variables.

Note that for the units to work out correctly (because of the units of Boltzmann's constant), pressure must be in Pa, volume must be in  $\text{m}^3$ , and temperature must be in Kelvin.

$$P = 1.20 \text{ bar} \times \frac{100000 \text{ Pa}}{1 \text{ bar}} = 120000 \text{ Pa}$$

$V = V$  (because we don't know it yet)

$$N = 2.1 \times 10^{24}$$

$$k_B = 1.38 \times 10^{-23} \frac{\text{m}^2 \cdot \text{kg}}{\text{s}^2 \cdot \text{K}}$$

$$T = 35^\circ\text{C} + 273 = 308 \text{ K}$$

Then we substitute these numbers into the ideal gas law and solve:

$$PV = Nk_B T$$

$$(120000 \text{ Pa})V = (2.10 \times 10^{24})(1.38 \times 10^{-23} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K}})(308 \text{ K})$$

$$120000 V = 8926$$

$$V = 0.0744 \text{ m}^3$$

Use this space for summary and/or additional notes.

Q: A gas has a temperature of 25°C and a pressure of 1.5 bar. If the gas is heated to 35°C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25°C and 35°C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

**Time 1 ("before"):**

$$P_1 = 1.5 \text{ bar}$$

$$T_1 = 25^\circ\text{C} + 273 = 298 \text{ K}$$

**Time 2 ("after"):**

$$P_2 = P_2$$

$$T_2 = 35^\circ\text{C} + 273 = 308 \text{ K}$$

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V}_1}{T_1} = \frac{P_2 \cancel{V}_2}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Amontons' Law)}$$

4. Plug in our values and solve:

$$\frac{1.5 \text{ bar}}{298 \text{ K}} = \frac{P_2}{308 \text{ K}} \rightarrow \boxed{P_2 = 1.55 \text{ bar}}$$

Use this space for summary and/or additional notes.

## Fluid Motion & Bernoulli's Law

**Unit:** Pressure & Fluid Mechanics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Bernoulli's Law/Equation

**Language Objectives:**

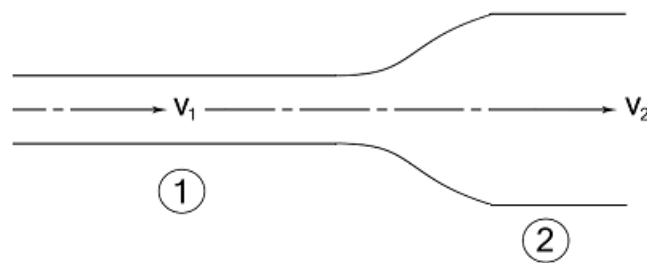
- Understand and correctly use the term "flow."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using Bernoulli's equation.

**Notes:**

flow: the net movement of a fluid

velocity of a fluid: the average velocity of a particle of fluid as the fluid flows past a reference point.

When a fluid flows through a pipe, the total volume of fluid per unit of time ( $\frac{m^3}{s}$ ) is equal to the velocity ( $\frac{m}{s}$ ) times the cross-sectional area ( $m^2$ ). If the total flow remains the same, but the, but the diameter of the pipe changes:



then the cross-sectional area ( $A$ ) times the fluid velocity ( $v$ ) at point 1 equals the cross-sectional area times the fluid velocity at point 2:

$$A_1 v_1 = A_2 v_2$$

Use this space for summary and/or additional notes:

According to the Dutch-Swiss mathematician Daniel Bernoulli, the pressures in a moving fluid are caused by:

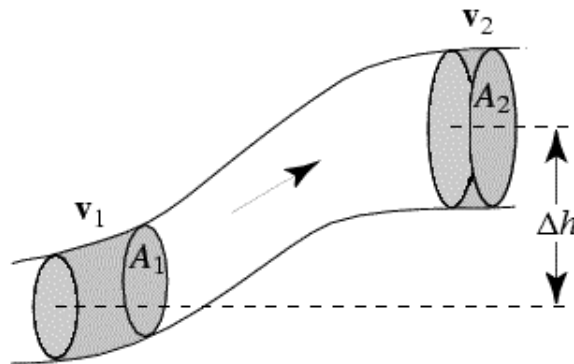
- The pressure exerted by the fluid,  $P$  (This is the pressure we would measure with a pressure gauge.)
- The hydrostatic pressure,  $\rho gh$ .
- The dynamic pressure,  $\frac{1}{2} \rho v^2$ , which results from the force that the moving fluid particles exert on the other fluid particles around them. (Note that  $\frac{1}{2} \rho v^2$  has units of pressure:  $\left(\frac{\text{kg}}{\text{m}^3}\right) \cdot \left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = \text{Pa}.$ )

A change in any of these pressures affects the others, which means:

$$P + \rho gh + \frac{1}{2} \rho v^2 = \text{constant}$$

This equation is Bernoulli's equation.

If the velocity and height of the fluid are changing, as in the following diagram, then the pressure must also change as a result:

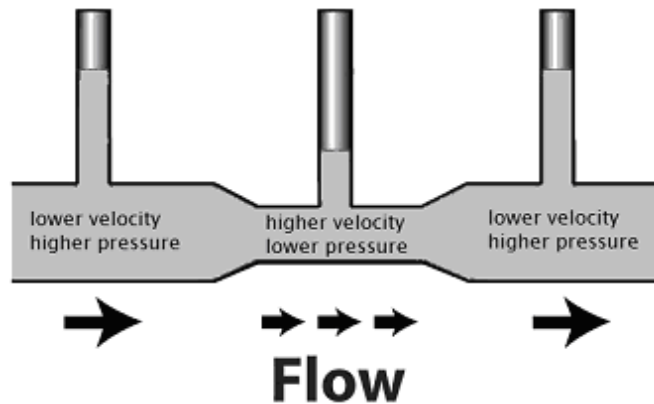


In this situation, Bernoulli's equation becomes:

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

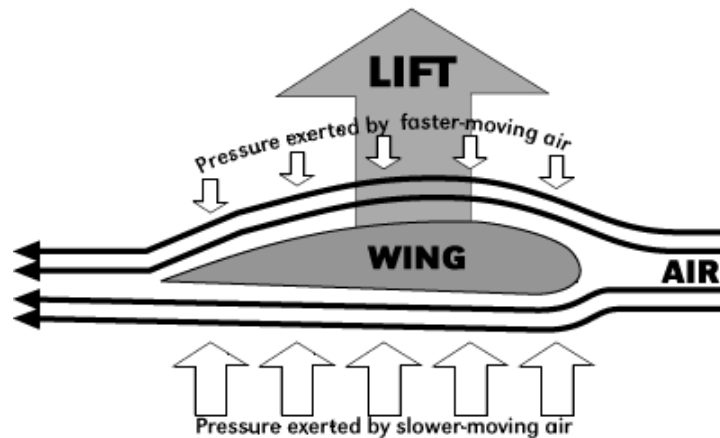
Use this space for summary and/or additional notes.

In Bernoulli's equation, increasing the fluid velocity ( $v$ ) increases the  $\frac{1}{2}\rho v^2$  term. Consider the following example:



This pipe is horizontal, which means  $h$  is constant; therefore  $\rho gh$  is constant. This means that if  $\frac{1}{2}\rho v^2$  increases, then pressure ( $P$ ) must decrease so that  $P + \rho gh + \frac{1}{2}\rho v^2$  remains constant.

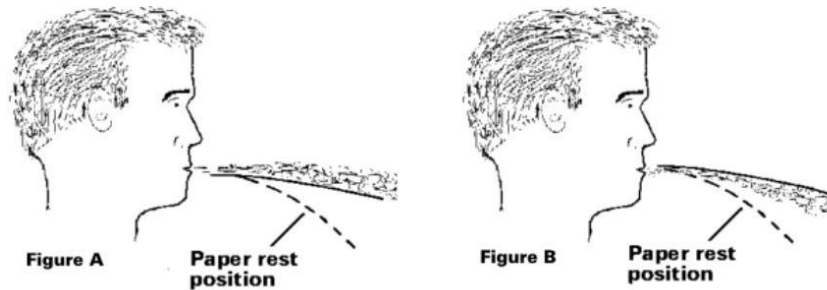
This decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:



Use this space for summary and/or additional notes.



A common demonstration of Bernoulli's Law is to blow across a piece of paper:



The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

### Sample Problems:

Q: A fluid in a pipe with a diameter of 0.40 m is moving with a velocity of  $0.30 \frac{\text{m}}{\text{s}}$ . If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: The cross-sectional area of the first pipe is:

$$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126 \text{ m}^2$$

The cross-sectional area of the second pipe is:

$$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314 \text{ m}^2$$

$$A_1 v_1 = A_2 v_2$$

$$(0.126)(0.30) = (0.0314)v_2$$

$$v_2 = \boxed{1.2 \frac{\text{m}}{\text{s}}}$$

Use this space for summary and/or additional notes.

Q: A fluid with a density of  $1250 \frac{\text{kg}}{\text{m}^3}$  has a pressure of 45 000 Pa as it flows at  $1.5 \frac{\text{m}}{\text{s}}$  through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of  $3.4 \frac{\text{m}}{\text{s}}$  through it?

A:

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$
$$45\,000 + (1250)(9.8)(0) + (\frac{1}{2})(1250)(1.5)^2 =$$
$$P_2 + (1250)(9.8)(2.5) + (\frac{1}{2})(1250)(3.4)^2$$
$$45\,000 + 1406 = P_2 + 30\,625 + 7225$$
$$46\,406 = P_2 + 37\,850$$
$$P_2 = 8\,556 \text{ Pa} = \boxed{8\,600 \text{ Pa}}$$

Use this space for summary and/or additional notes.

## Introduction: Atomic & Particle Physics

**Unit:** Atomic & Particle Physics

### Topics covered in this chapter:

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This chapter discusses the particles that atoms and other matter are made of, how those particles interact, and the process by which radioactive decay can change the composition of a substance from one element into another.

- *The Bohr Model of the Hydrogen Atom* describes the first attempts to use quantum mechanics to describe the behavior of the electrons in an atom. *The Quantum Mechanical Model of the Atom* describes the evolution of atomic theory from the Bohr model to the present day.
- *Fundamental Forces* describes the four natural forces that affect everything in the universe. The strong nuclear force and the weak nuclear force are particularly relevant to this chapter.
- *The Standard Model* and *Particle Interactions* describe properties of and interactions between the particles that all matter is made of.
- *Radioactive Decay*, *Nuclear Equations*, *Mass Defect & Binding Energy*, *Half-Life*, and *Nuclear Fission & Fusion* describe and give equations for the nuclear changes that radioactive elements undergo.

Use this space for summary and/or additional notes:

One of the challenging aspects of this chapter is that it describes process that happen on a scale that is much too small to observe directly. Another challenge is the fact that the Standard Model continues to evolve. Many of the connections between concepts that make other topics easier to understand have yet to be made in the realm of atomic & particle physics.

**Standards addressed in this chapter:**

**Next Generation Science Standards (NGSS):**

**HS-PS1-8:** Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the processes of fission, fusion, and radioactive decay.

**HS-PS4-3.** Evaluate the claims, evidence, and reasoning behind the idea that electromagnetic radiation can be described either by a wave model or a particle model, and that for some situations one model is more useful than the other.

**Massachusetts Curriculum Frameworks (2006):**

No MA physics standards are addressed in this chapter. However, this chapter addresses the following standards from the chemistry frameworks:

**2.5** Identify the three main types of radioactive decay (alpha, beta, and gamma) and compare their properties (composition, mass, charge, and penetrating power).

**2.6** Describe the process of radioactive decay by using nuclear equations, and explain the concept of half-life for an isotope (for example, C-14 is a powerful tool in determining the age of objects).

**2.7** Compare and contrast nuclear fission and nuclear fusion.

Use this space for summary and/or additional notes.

**Topics from this chapter assessed on the SAT Physics Subject Test:**

- **Quantum Phenomena**, such as photons and the photoelectric effect.
- **Atomic Physics**, such as the Rutherford and Bohr models, atomic energy levels, and atomic spectra.
- **Nuclear and Particle Physics**, such as radioactivity, nuclear reactions, and fundamental particles.
  1. The Discovery of the Atom
  2. Quantum Physics
  3. Nuclear Physics

Use this space for summary and/or additional notes.

## The Bohr Model of the Hydrogen Atom

**Unit:** Atomic & Particle Physics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

### Knowledge/Understanding Goals:

- developments leading to the Bohr model of the atom
- the Bohr model and its limitations
- how the modern quantum mechanical model of the atom grew out of the Bohr model

### Skills:

- calculate the frequency/wavelength of light emitted using the Rydberg equation
- calculate the energy associated with a quantum number using Bohr's equation

### Language Objectives:

- Understand and correctly use the terms "electron," "nucleus," and "spectral lines."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using the Rydbergh and Bohr equations.

### Notes:

#### Significant Developments Prior to 1913

Discovery of the Electron (1897): J.J. Thompson determined that cathode rays were actually particles emitted from atoms that the cathode was made of. These particles had an electrical charge, so they were named "electrons".

Discovery of the Nucleus (1909): Ernest Rutherford's famous "gold foil experiment" determined that atoms contained a dense, positively-charged region that comprised most of the atom's mass. This region was named the "nucleus", after the nucleus of a cell.

Use this space for summary and/or additional notes:

Rutherford (“Planetary”) Model of the Atom (1911): The atom was believed to be like a miniature solar system, with electrons orbiting the nucleus in much the same way as planets orbit the sun.

Balmer Formula (1885): Johann Balmer devised an empirical equation to relate the emission lines in the visible spectrum for the hydrogen atom.

Rydberg Formula (1888): Johannes Rydberg developed a generalized formula that could describe the wave numbers of all of the spectral lines in hydrogen (and similar elements).

There are several series of spectral lines for hydrogen, each of which converge at different wavelengths. Rydberg described the Balmer series in terms of a pair of integers ( $n_1$  and  $n_2$ , where  $n_1 < n_2$ ), and devised a single formula with a single constant (now called the Rydberg constant) that relates them.

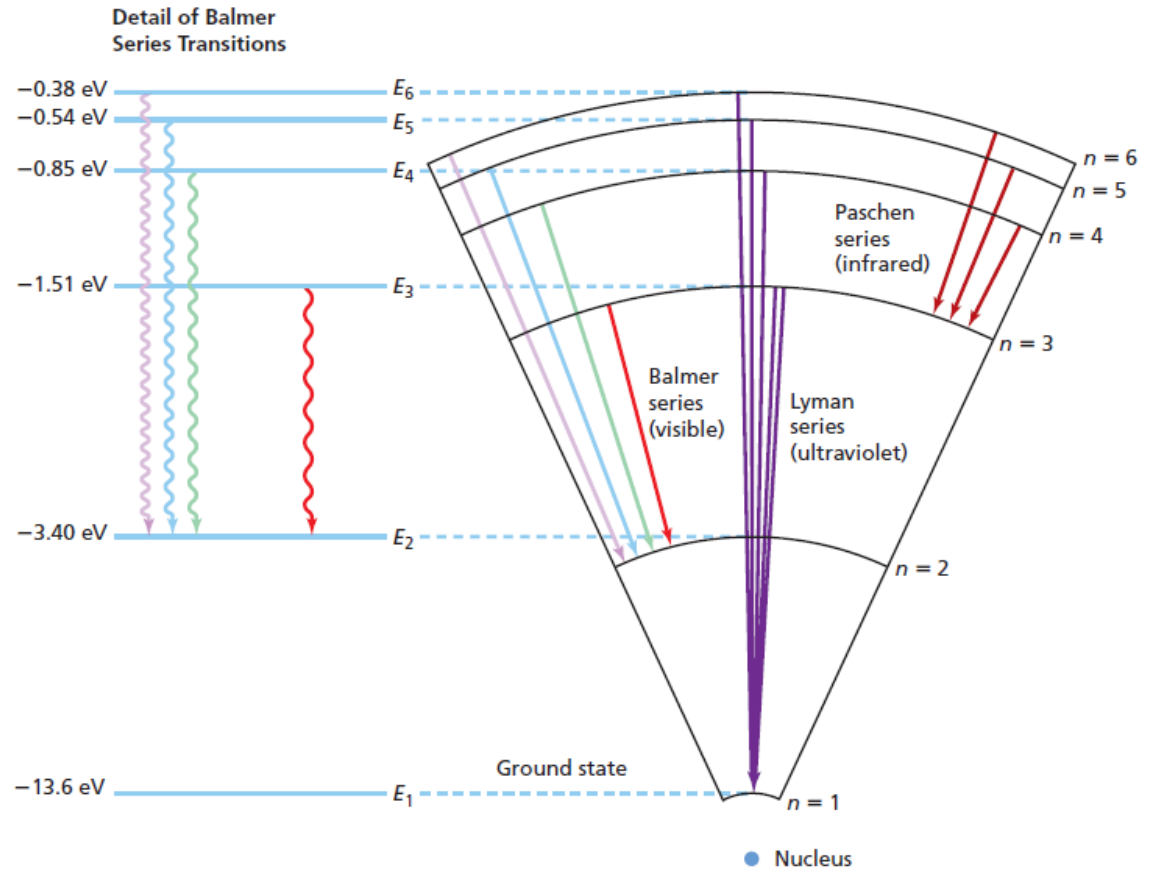
$$\frac{1}{\lambda_{vac}} = R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Rydberg’s equation was later found to be consistent with other series discovered later, including the Lyman series (in the ultraviolet region; first discovered in 1906) and the Paschen series (in the infrared region; first discovered in 1908).

Those series and their converging wavelengths are:

Series	Wavelength	$n_1$	$n_2$
Lyman	91 nm	1	$2 \rightarrow \infty$
Balmer	365 nm	2	$3 \rightarrow \infty$
Paschen	820 nm	3	$4 \rightarrow \infty$

Use this space for summary and/or additional notes.



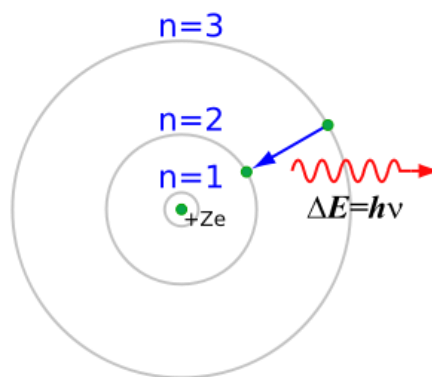
"Old" Quantum Theory (ca. 1900): sub-atomic particles obey the laws of classical mechanics, but that only certain "allowed" states are possible.

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### Bohr's Model of the Atom (1913)

In 1913, Niels Bohr combined atomic, spectroscopy, and quantum theories into a single theory. Bohr hypothesized that electrons moved around the nucleus as in Rutherford's model, but that these electrons had only certain allowed quantum values of energy, which could be described by a quantum number ( $n$ ). The value of that quantum number was the same  $n$  as in Rydberg's equation, and that using quantum numbers in Rydberg's equation could predict the wavelengths of light emitted when the electrons gained or lost energy by moved from one quantum level to another.



Bohr's model gained wide acceptance, because it related several prominent theories of the time. The theory worked well for hydrogen, giving a theoretical basis for Rydberg's equation. Bohr defined the energy associated with a quantum number ( $n$ ) in terms of Rydberg's constant:

$$E_n = -\frac{R_H}{n^2}$$

Although the Bohr model worked well for hydrogen, the equations could not be solved exactly for atoms with more than one electron, because of the additional effects that electrons exert on each other (such as the Coulomb force,  $F = \frac{kq_1q_2}{d^2}$ ).

Use this space for summary and/or additional notes.

## The Quantum Mechanical Model of the Atom

**Unit:** Atomic & Particle Physics

**NGSS Standards:** HS-PS4-3

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- the de Broglie model of the atom
- the wave-particle duality of nature
- the Schrödinger model of the atom

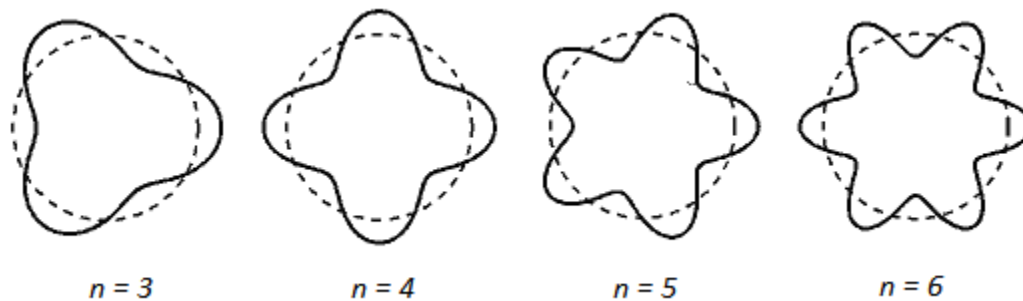
**Language Objectives:**

- Understand and correctly use the terms “orbit” and “orbital.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

### Louis de Broglie

In 1924, French physicist Louis de Broglie suggested that, like light, electrons can act as both a particle and a wave. He theorized that the reason that only integer values for quantum numbers were possible was because as the electron orbits the nucleus, its path must be an integer multiple of the wavelength:

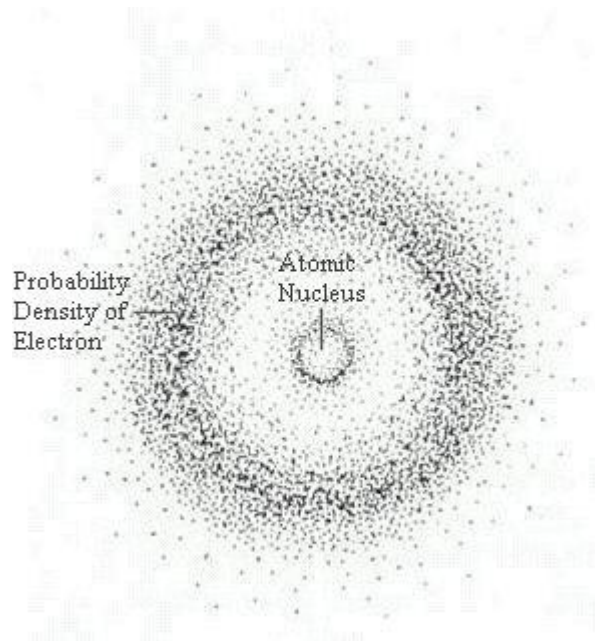


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## Erwin Schrödinger

In 1925, Austrian physicist Erwin Schrödinger found that by treating each electron as a unique wave function, the energies of the electrons could be predicted by the mathematical solutions to a wave equation. The use of Schrödinger's wave equation to construct a probability map for where the electrons can be found in an atom is the basis for the modern quantum-mechanical model of the atom.

To understand the probability map, it is important to realize that because the electron acts as a wave, it is detectable when the amplitude of the wave is nonzero, but not detectable when the amplitude is zero. This makes it appear as if the electron is teleporting from place to place around the atom. If you were somehow able to take a time-lapse picture of an electron as it moves around the nucleus, the picture might look something like this:



Notice that there is a region close to the nucleus where the electron is unlikely to be found, and a ring a little farther out where there is a much higher probability of finding the electron.

Use this space for summary and/or additional notes.

As you get farther and farther from the nucleus, Schrödinger's equation predicts different shapes for these probability distributions. These regions of high probability are called "orbitals," because of their relation to the orbits originally suggested by the planetary model.

Schrödinger was awarded the Nobel prize in physics in 1933 for this discovery.

Use this space for summary and/or additional notes.

## Fundamental Forces

**Unit:** Atomic & Particle Physics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

### Knowledge/Understanding Goals:

- fundamental forces

### Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

### Notes:

#### The Four Fundamental Forces

strong force (or “strong nuclear force”): an attractive force between quarks. The strong force holds the nuclei of atoms together. The energy comes from converting mass to energy.

weak force (or “weak nuclear force”): the force that causes protons and/or neutrons in the nucleus to become unstable and leads to beta nuclear decay. This happens because the weak force causes an up or down quark to change flavor. (This process is described in more detail in the Standard Model of particle physics.)

electromagnetic force: the force between electrical charges. If the charges are the same (“like charges”)—both positive or both negative—the particles repel each other. If the charges are different (“opposite charges”)—one positive and one negative—the particles attract each other.

gravity: the force that causes masses to attract each other. Usually only observable if one of the masses is very large (like a planet).

Use this space for summary and/or additional notes:

The following table shows the relative strengths of the fundamental forces:

Force	Relative Strength	Effective Distance
Strong	1	Diameter of the nucleus
Weak	$10^{-6}$	$\frac{1}{3}$ diameter of the nucleus
Electromagnetic	$\frac{1}{137}$	$\infty$ , but gets smaller as (distance) <sup>2</sup>
Gravity	$10^{-39}$	$\infty$ , but gets smaller as (distance) <sup>2</sup>

Use this space for summary and/or additional notes.

## The Standard Model

**Unit:** Atomic & Particle Physics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- particles of the standard model
- how particles interact

**Language Objectives:**

- Understand and correctly use the names of particles and their properties that are described in this section.
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

The Standard Model is a theory of particle physics that:

- identifies the particles that matter is comprised of
- describes properties of these particles, including their mass, charge, and “spin”
- describes interactions between these particles

The Standard Model dates to the mid-1970s, when the existence of quarks was first experimentally confirmed. Physicists are still discovering new particles and relationships between particles, so the model and the ways it is represented are evolving, much like its cousin the Periodic Table of the Elements was evolving at the turn of the twentieth century. The table and the model described in these notes represent our understanding, as of the early twenty-first century. By the middle of this century, the Standard Model may evolve to a form that is substantially different from the way we represent it today.

The Standard Model in its present form does not incorporate dark matter or gravitational attraction.

Use this space for summary and/or additional notes:

The Standard Model is often presented in a table, with rows, columns, and color-coded sections used to group subsets of particles according to their properties.

As of the early twenty-first century, the standard model is represented by a table similar to this one:

		Generation				
		I	II	III		
mass →		2.4 MeV/c <sup>2</sup>	1.27 GeV/c <sup>2</sup>	171.2 GeV/c <sup>2</sup>	0 MeV/c <sup>2</sup>	125.3 GeV/c <sup>2</sup>
charge →		+ $\frac{2}{3}$	+ $\frac{2}{3}$	+ $\frac{2}{3}$	0	0
spin →		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
		<b>u</b> up quark	<b>c</b> charm quark	<b>t</b> top quark	<b>γ</b> photon	<b>H<sup>0</sup></b> Higgs boson
	<b>quarks</b>	4.8 MeV/c <sup>2</sup>	104 MeV/c <sup>2</sup>	4.2 GeV/c <sup>2</sup>	0 MeV/c <sup>2</sup>	
		- $\frac{1}{3}$	- $\frac{1}{3}$	- $\frac{1}{3}$	0	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		<b>d</b> down quark	<b>s</b> strange quark	<b>b</b> bottom quark	<b>g</b> gluon	
		< 2.2 eV/c <sup>2</sup>	< 0.17 MeV/c <sup>2</sup>	< 15.5 MeV/c <sup>2</sup>	91.2 GeV/c <sup>2</sup>	
		0	0	0	0	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		<b>ν<sub>e</sub></b> electron neutrino	<b>ν<sub>μ</sub></b> muon neutrino	<b>ν<sub>τ</sub></b> tau neutrino	<b>Z<sup>0</sup></b> Z boson	
	<b>leptons</b>	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	80.4 GeV/c <sup>2</sup>	
		-1	-1	-1	±1	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		<b>e</b> electron	<b>μ</b> muon	<b>τ</b> tau	<b>W<sup>±</sup></b> W boson	
						<b>gauge bosons</b>

Properties shown in the table include mass, charge, and spin.

- **Mass** is shown in units of electron volts divided by the speed of light squared ( $\frac{eV}{c^2}$ ). An electron volt (eV) is the energy acquired by an electric potential difference of one volt applied to one electron. (Recall that the metric prefix “M” stands for mega ( $10^6$ ) and the metric prefix “G” stands for giga ( $10^9$ ).) The  $c^2$  in the denominator comes from Einstein’s famous equation,  $E = mc^2$ .
- **Charge** is the same property that we studied in the electricity and magnetism unit. The magnitude and sign of charge is relative to the charge of an electron, which is defined to be  $-1$ .
- **Spin** is the property that is believed to be responsible for magnetism. (The name is because originally, magnetism was thought to come from electrons spinning in their orbitals.)

Use this space for summary and/or additional notes.



The two fundamental classes of particles are fermions and bosons.

### Fermions

The three columns on the left side of the table are fermions. Fermions are described by Fermi-Dirac statistics and obey the Pauli exclusion principle. Fermions are the building blocks of matter. They have a spin of  $\frac{1}{2}$ , and each fermion has its own antiparticle. The antiparticle of a fermion is identical to its corresponding particle, but has a charge of the opposite sign. Antiparticles have the same name as the corresponding particle with the prefix “anti-”; for example, the antiparticle of a tau neutrino is a tau antineutrino. (Note, however, that for historical reasons an antielectron is usually called a positron.)

### Quarks

Quarks are fermions that carry what is known as color charge, which means they interact via the strong force (sometimes called the color force). The interaction of these color charges is a subfield of particle physics called quantum chromodynamics (QCD). The strong force is the force that holds quarks together in protons and neutrons. The strong force also holds the nucleus together in atoms.

Quarks that carry a positive charge ( $+\frac{2}{3}$ ) are called “up-type” quarks. These include the following flavors:

- up quark
- charm quark
- top quark

Quarks that carry a negative charge ( $-\frac{1}{3}$ ) are called “down-type” quarks. These include the following flavors:

- down quark
- strange quark
- bottom quark

Use this space for summary and/or additional notes.

### Leptons

Leptons are fermions that do not carry color charge (*i.e.*, do not interact via the strong force). Electron-like leptons carry an electric charge of  $-1$ :

- electron
- muon
- tau

Neutrinos are leptons that do not carry a charge:

- electron neutrino
- muon neutrino
- tau neutrino

Neutrinos oscillate, which causes their masses to be indefinite.

Use this space for summary and/or additional notes.

## Bosons

The four particles in the second column from the right are gauge bosons. (There are other types of bosons not shown in this table.) Bosons have integer spins and do not obey the Pauli Exclusion Principle.

The four fundamental forces of nature (the strong force, the weak force, the electromagnetic force, and the gravitational force) are mediated by gauge bosons. Mass is created by the Higgs boson.

Each of the fundamental bosons is its own antiparticle, except for the  $W^-$  boson (whose antiparticle is the  $W^+$  boson).

The elementary bosons are:

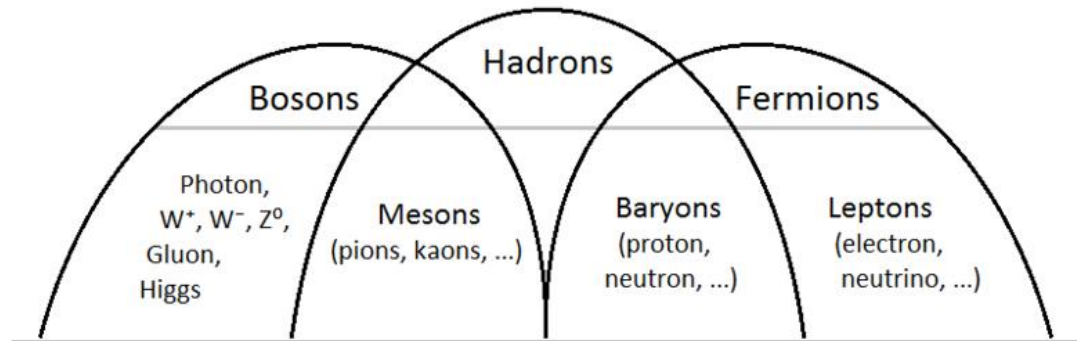
- photon ( $\gamma$ ): mediates electromagnetism
- W boson ( $W^-$  or  $W^+$ ): mediates the weak interaction
- Z boson (Z): mediates the weak interaction
- gluon (g): mediates the strong interaction
- Higgs boson: causes mass
- graviton (G): causes gravity

(Note that the graviton is not actually part of the Standard Model. It is included here for completeness. The existence of the graviton is theorized but has not yet, as of mid-2012, been confirmed.)

Use this space for summary and/or additional notes.

## Hadrons

Hadrons are a special class of strongly-interacting composite particles (meaning that they are comprised of multiple individual particles). Hadrons can be bosons or fermions. Hadrons composed of strongly-interacting fermions are called baryons; hadrons composed of strongly-interacting bosons are called mesons.



## Baryons

The most well-known baryons are protons and neutrons, each comprised of three quarks. Protons are made of two up quarks and one down quark, and carry a charge of +1. Neutrons are made of one up quark and two down quarks, and carry a charge of zero.

Some of the better-known baryons include:

- nucleons (protons & neutrons).
- hyperons, *e.g.*, the  $\Lambda$ ,  $\Sigma$ ,  $\Xi$ , and  $\Omega$  particles. These contain one or more strange quarks, and are much heavier than nucleons.
- various charmed and bottom baryons.
- pentaquarks, which contain four quarks and an antiquark.

Use this space for summary and/or additional notes.

### Mesons

Ordinary mesons are comprised of a quark and an antiquark. Examples include the pion, kaon, and the  $J/\psi$ . Mesons mediate the residual strong force between nucleons.

Some of the exotic mesons include:

- tetraquarks, which contain two quarks and two antiquarks.
- glueball, a bound set of gluons with no quarks.
- hybrid mesons, which contain one or more quark/antiquark pairs and one or more gluons.

Use this space for summary and/or additional notes.

## Particle Interactions (Feynman Diagrams)

**Unit:** Atomic & Particle Physics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

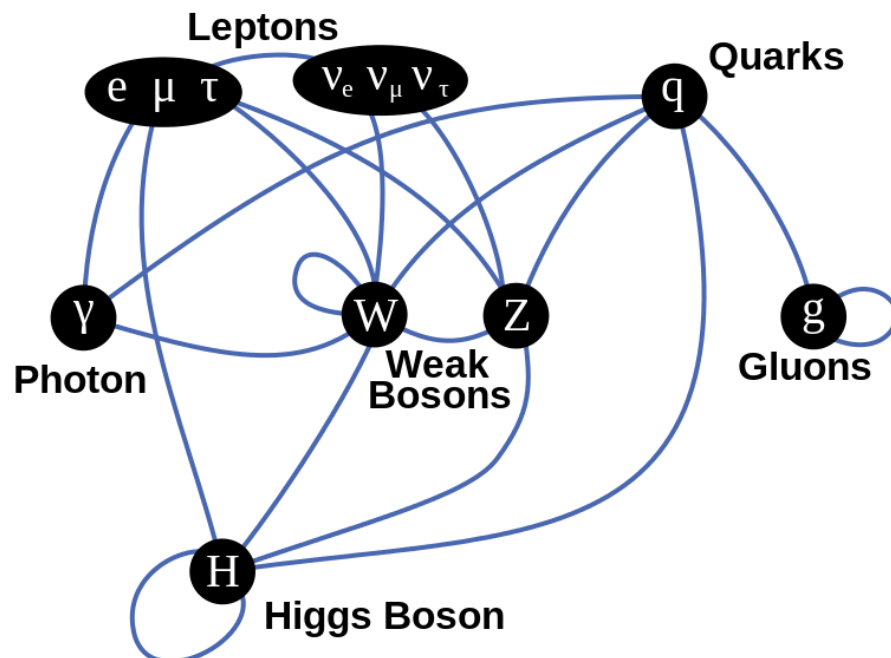
- how to read and draw a Feynman diagram

**Language Objectives:**

- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

In particle physics, the Standard Model describes the types of particles found in nature, their properties, and how they interact. The following diagram shows which types of particles can interact with which other types.



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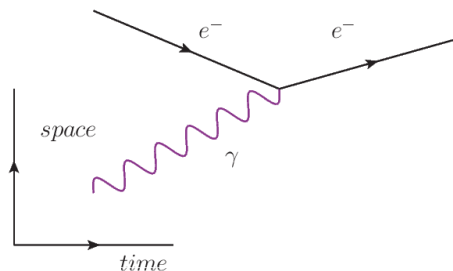
The interactions between particles can be shown pictorially in diagram called a Feynman diagram, named for the American physicist Richard Feynman. The Feynman diagram tells the “story” of the interaction.

The characteristics of a Feynman diagram are:

1. Straight lines represent the motion of a particle. The arrow points to the right for a negatively-charged particle, and to the left for a positively-charged particle.
2. A wavy line represents a photon.
3. A coiled line (like a spring) represents a gluon.
4. The x-axis is time. The interaction starts (in terms of time) on the left and proceeds from left to right.
5. The y-axis represents space. Lines coming together represent particles coming together. Lines moving apart represent particles moving away from each other.
6. Each vertex, where two or more lines come together, represents an interaction.

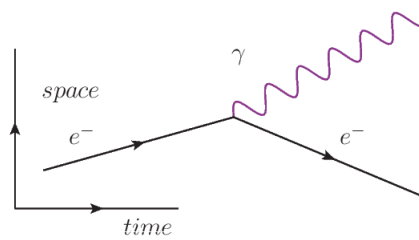
Probably the best way to explain the diagrams is by example.

In the following diagram, we start (at the left) with an electron ( $e^-$ ) and photon ( $\gamma$ ). The two come together, and the electron absorbs the photon.

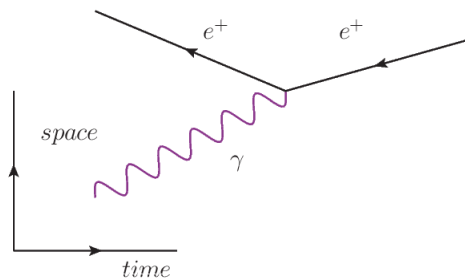


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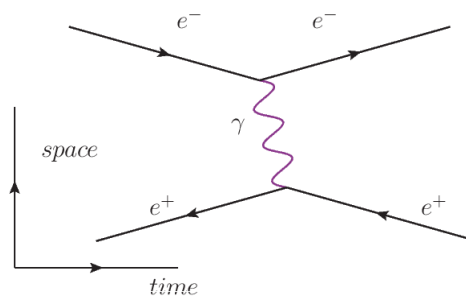
In the following diagram, we start (at the left) with an electron ( $e^-$ ) by itself. The electron emits a photon ( $\gamma$ ), but is otherwise unchanged.



In the following diagram, we start (at the left) with a positron ( $e^+$ ) and photon ( $\gamma$ ). (Note that the arrow pointing to the left indicates a positively-charged particle.) The two come together, and the positron absorbs the photon.



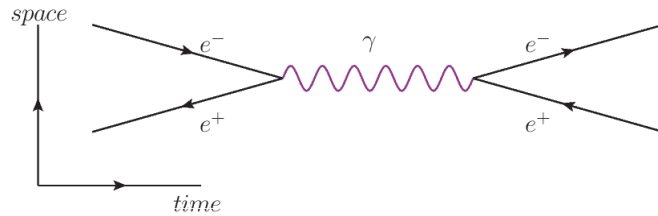
In the following image, we start with an electron ( $e^-$ ) and positron ( $e^+$ ) (coming in from the left). They exchange a photon ( $\gamma$ ) between them. (Note that the diagram does not make it clear which particle emits the photon and which one absorbs it.) Then the two particles exit.



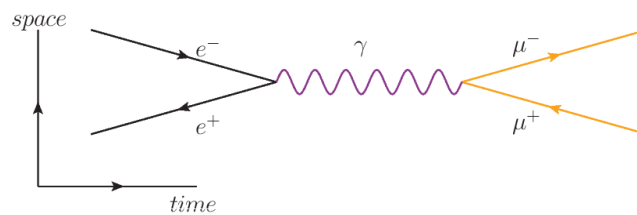
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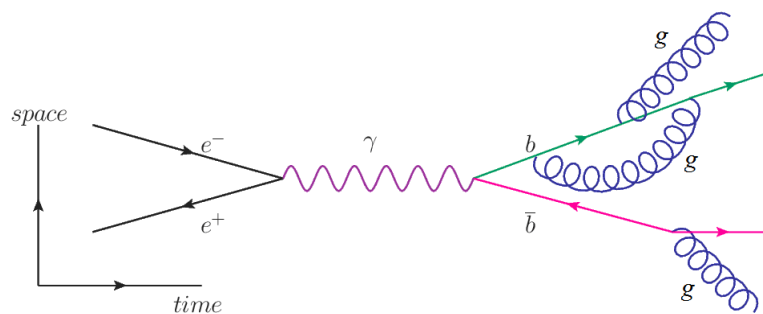
In the following diagram, we start with an electron ( $e^-$ ) and positron ( $e^+$ ). They come together and annihilate each other, producing a photon ( $\gamma$ ). (You can tell this because for a length of time, nothing else exists except for the photon.) Then the photon pair-produces a new electron/positron pair.



In the following diagram, an electron ( $e^-$ ) and positron ( $e^+$ ) annihilate each other as above, but this time the photon produces a muon ( $\mu^-$ )/antimuon ( $\mu^+$ ) pair. (Again, note that the muon, which has a negative charge, has the arrow pointing to the right. The antimuon, which has a positive charge, has the arrow pointing to the left.)



Finally, in the following diagram, an electron ( $e^-$ ) and positron ( $e^+$ ) annihilate each other, producing a photon ( $\gamma$ ). The photon pair-produces a bottom quark ( $b$ ) and an antibottom quark ( $\bar{b}$ ), which radiate gluons ( $g$ ).



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## Radioactive Decay

**Unit:** Atomic & Particle Physics

**NGSS Standards:** HS-PS1-8

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- types of radioactive decay
- types of particles & emissions created by radioactive decay

**Skills:**

- determining the products of radioactive decay

**Language Objectives:**

- Understand and correctly use the terms “radioactive decay,” “nuclear instability,” “alpha decay,” “beta decay,” “gamma rays,” and “penetrating power.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

nuclear instability: When something is unstable, it is likely to change. If the nucleus of an atom is unstable, changes can occur that affect the number of protons and neutrons in the atom.

Note that when this happens, the nucleus ends up with a different number of protons. This causes the atom to literally turn into an atom of a different element. When this happens, the physical and chemical properties instantaneously change into the properties of the new element!

radioactive decay: the process by which the nucleus of an atom changes, transforming the element into a different element or isotope.

nuclear equation: an equation describing (through chemical symbols) what happens to an atom as it undergoes radioactive decay.

Use this space for summary and/or additional notes:

### Causes of Nuclear Instability

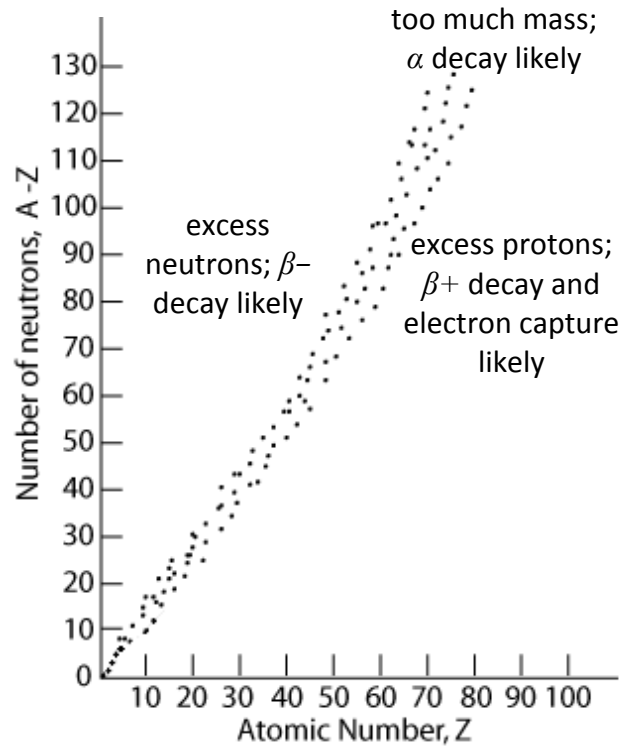
Two of the causes of nuclear instability are:

**size:** because the strong force acts over a limited distance, when nuclei get too large (more than 82 protons), it is no longer possible for the strong force to keep the nucleus together indefinitely. The form of decay that results from an atom exceeding its stable size is called alpha ( $\alpha$ ) decay.

**the weak nuclear force:** the weak force is caused by the exchange (absorption and/or emission) of W and Z bosons. This causes a down quark to change to an up quark or vice-versa. The change of quark flavor has the effect of changing a proton to a neutron, or a neutron to a proton. (Note that the action of the weak force is the only known way of changing the flavor of a quark.) The form of decay that results from the action of the weak force is called beta ( $\beta$ ) decay.

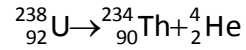
Use this space for summary and/or additional notes.

band of stability: isotopes with a ratio of protons to neutrons that results in a stable nucleus (one that does not spontaneously undergo radioactive decay). This observation suggests that the ratio of up to down quarks within the nucleus is somehow involved in preventing the weak force from causing quarks to change flavor.



Use this space for summary and/or additional notes.

alpha ( $\alpha$ ) decay: a type of radioactive decay in which the nucleus loses two protons and two neutrons (an alpha particle). An alpha particle is a  ${}^4_2\text{He}^{+2}$  ion (the nucleus of a helium-4 atom), with two protons, a mass of 4 amu, and a charge of +2. For example:



Atoms are most likely to undergo alpha decay if they have an otherwise stable proton/neutron ratio but a large atomic number.

Alpha decay has never been observed in atoms with an atomic number less than 52 (tellurium), and is rare in elements with an atomic number less than 73 (tantalum).

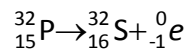
Net effects of  $\alpha$  decay:

- Atom loses 2 protons and 2 neutrons (atomic number goes down by 2 and mass number goes down by 4)
- An  $\alpha$  particle (a  ${}^4_2\text{He}^{+2}$  ion) is ejected from the nucleus at high speed.

Use this space for summary and/or additional notes.

beta minus ( $\beta^-$ ) decay: a type of radioactive decay in which a neutron is converted to a proton and the nucleus ejects a high speed electron ( ${}_{-1}^0e$ ).

Note that a neutron consists of one up quark and two down quarks (udd), and a proton consists of two up quarks and one down quark (uud). When  $\beta^-$  decay occurs, the weak force causes one of the quarks changes its flavor from down to up, which causes the neutron (udd) to change into a proton (uud). Because a proton was gained, the atomic number increases by one. However, because the proton used to be a neutron, the mass number does not change. For example:



Atoms are likely to undergo  $\beta^-$  decay if they have too many neutrons and not enough protons to achieve a stable neutron/proton ratio. Almost all isotopes that are heavier than isotopes of the same element within the band of stability (because of the “extra” neutrons) undergo  $\beta^-$  decay.

Net effects of  $\beta^-$  decay:

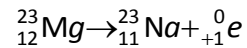
- Atom loses 1 neutron and gains 1 proton (atomic number goes up by 1; mass number does not change)
- A  $\beta^-$  particle (an electron) is ejected from the nucleus at high speed.

Note that a  $\beta^-$  particle is assigned an atomic number of  $-1$ . *This does not mean an electron is some sort of “anti-proton”*. The  $-1$  is just used to make the equation for the number of protons work out in the nuclear equation.

Use this space for summary and/or additional notes.

beta plus ( $\beta^+$ ) decay: a type of radioactive decay in which a proton is converted to a neutron and the nucleus ejects a high speed antielectron (positron,  ${}^0_{+1}e$ ).

$\beta^+$  decay is the opposite of  $\beta^-$  decay. When  $\beta^+$  decay occurs, one of the quarks changes its flavor from up to down, which changes the proton (uud) into a neutron (udd). Because a proton was lost, the atomic number decreases by one. However, because the neutron used to be a proton, the mass number does not change. For example:



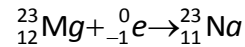
Atoms are likely to undergo  $\beta^+$  decay if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability ("not enough neutrons") undergo  $\beta^+$  decay.

Net effects of  $\beta^+$  decay:

- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)
- A  $\beta^+$  particle (an antielectron or positron) is ejected from the nucleus at high speed.

Use this space for summary and/or additional notes.

electron capture (sometimes called “K-capture”): when the nucleus of the atom “captures” an electron from the innermost shell (the K-shell) and incorporates it into the nucleus. This process is exactly the reverse of  $\beta^-$  decay; during electron capture, a quark changes flavor from up to down, which changes a proton (uud) into a neutron (udd):



Note that  $\beta^+$  decay and electron capture produce the same products. Electron capture can sometimes (but not often) occur without  $\beta^+$  decay. However,  $\beta^+$  decay is always accompanied by electron capture.

Atoms are likely to undergo electron capture (and usually also  $\beta^+$  decay) if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability undergo electron capture, and usually also  $\beta^+$  decay.

Net effects of electron capture:

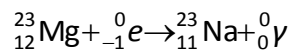
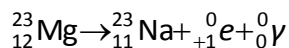
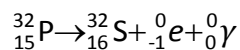
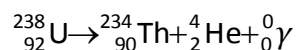
- An electron is absorbed by the nucleus.
- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)

Use this space for summary and/or additional notes.



gamma ( $\gamma$ ) rays: most radioactive decay produces energy. Some of that energy is emitted in the form of gamma rays, which are very high energy photons of electromagnetic radiation. (Radio waves, visible light, and X-rays are other types of electromagnetic radiation.) Because gamma rays are waves (which have no mass), they can penetrate far into substances and can do a lot of damage. Because gamma rays are not particles, emission of gamma rays does not change the composition of the nucleus.

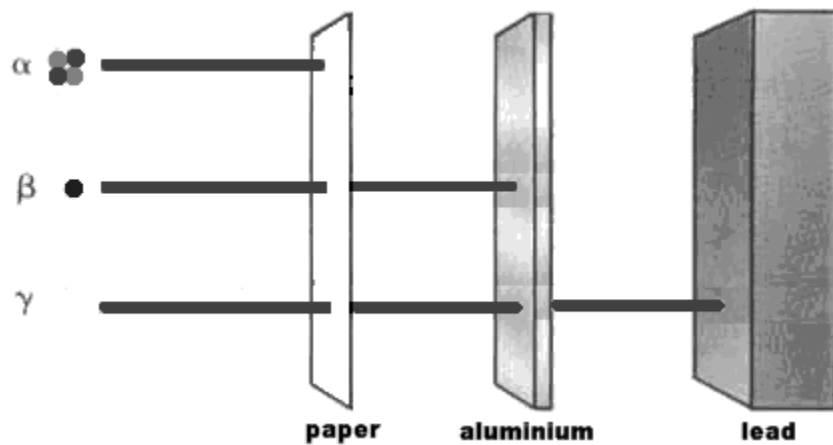
All of the types of radioactive decay mentioned in these notes also produce  $\gamma$  rays. This means to be complete, we would add gamma radiation to each of the radioactive decay equations described above:



Use this space for summary and/or additional notes.

penetrating power: the distance that radioactive particles can penetrate into/through another substance is directly related to the velocity of the emission (faster = more penetrating) and inversely related to the mass of the emission (heavier = less penetrating):

<b>gamma rays</b>	<b>beta particles</b>	<b>alpha particles</b>
lightest	←—————→	heaviest
fastest	←—————→	slowest
most penetrating power	←————→	least penetrating power



Note also that denser substances (such as lead) do a better job of blocking and absorbing radioactive emissions. This is why lead is commonly used as shielding for experiments involving radioactive substances.

Use this space for summary and/or additional notes.

## Nuclear Equations

**Unit:** Atomic & Particle Physics

**NGSS Standards:** HS-PS1-8

**MA Curriculum Frameworks (2006):** N/A

**Skills:**

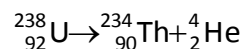
- write & solve nuclear equations

**Language Objectives:**

- Set up and solve word problems involving nuclear equations.

**Notes:**

nuclear equation: a chemical equation describing the process of an isotope undergoing radioactive decay. For example:



In a nuclear equation, the number of protons (atomic number) and the total mass (mass number) are conserved on both sides of the arrow. If you look at the bottom (atomic) numbers, and replace the arrow with an = sign, you would have the following:

$$92 = 90 + 2$$

Similarly, if you look at the top (mass) numbers, and replace the arrow with an = sign, you would have:

$$238 = 234 + 4$$

Use this space for summary and/or additional notes:

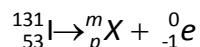
**Sample problems:**

1. What are the products of beta ( $\beta^-$ ) decay of  $^{131}\text{I}$ ?

**Answer:**

A  $\beta^-$  particle is an electron, which we write as  ${}_{-1}^0e$  in a nuclear equation.

This means  $^{131}\text{I}$  decays into some unknown particle plus  ${}_{-1}^0e$ . The equation is:



We can write the following equations for the atomic and mass numbers:

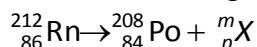
Atomic #s:  $53 = p + -1 \rightarrow p = 54$ ; therefore  $X$  is Xe

Mass #s:  $131 = m + 0 \rightarrow m = 131$

Therefore, particle  $X$  is  ${}_{54}^{131}\text{Xe}$ . So our final answer is:

The two products of decay in this reaction are  ${}_{54}^{131}\text{Xe}$  and  ${}_{-1}^0e$ .

2. Which particle was produced in the following radioactive decay reaction:

**Answer:**

The two equations are:

Atomic #s:  $86 = 84 + p \rightarrow p = 2$ ; therefore  $X$  is He

Mass #s:  $212 = 208 + m \rightarrow m = 4$

Therefore, particle  $X$  is  ${}_{2}^4\text{He}$ , which means it is an  $\alpha$  particle.

Use this space for summary and/or additional notes.

## Mass Defect & Binding Energy

**Unit:** Atomic & Particle Physics

**NGSS Standards:** HS-PS1-8

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding:**

- Understand where the energy behind the strong force (which holds the nucleus together) comes from.
- Be able to give a definition of mass defect.

**Skills:**

- Calculate the binding energy of an atom.
- Calculate the energy given off by a radioactive decay based on the binding energies before and after.

**Language Objectives:**

- Understand and correctly use the terms “mass defect” and “binding energy.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving calculating the mass defect and binding energy of an isotope.

**Notes:**

mass defect: the difference between the actual mass of an atom, and the sum of the masses of the protons, neutrons, and electrons that it contains.

binding energy: the energy that holds the nucleus of an atom together through the strong nuclear force

Use this space for summary and/or additional notes:

The binding energy comes from a small amount of mass (the mass defect) that was turned into a large amount of energy. The amount of energy is given by the equation:

$$E = mc^2$$

where  $E$  is the binding energy,  $m$  is the mass defect, and  $c$  is the speed of light ( $3 \times 10^8$  m/s, which means  $c^2$  is a very large number).

**For example:**

A proton has a mass of 1.00728 amu.

A neutron has a mass of 1.00867 amu.

An electron has a mass of 0.00055 amu.

${}_{92}^{238}\text{U}$  has 92 protons, 146 neutrons, and 92 electrons. This means the total mass should be:

92 protons  $\times$  1.00728 amu = 92.66976 amu

146 neutrons  $\times$  1.00867 amu = 147.26582 amu

92 electrons  $\times$  0.00055 amu = 0.05060 amu

9.266976 + 147.26582 + 0.05060 = 239.98618 amu

The actual observed mass of  ${}_{92}^{238}\text{U}$  is 238.0003 amu.

The mass defect is the “missing” 1.98588 amu per atom, or 1.98588 g/mol.

Because  $E = mc^2$ , that means the binding energy of one mole of  ${}_{92}^{238}\text{U}$  is  $1.98588 \text{ g/mol} \times (3.00 \times 10^8)^2 = 1.78729 \times 10^{17} \text{ J/mol}$ . In case you don't realize just how large that number is, the binding energy of just 238 g (1 mole) of  ${}_{92}^{238}\text{U}$  would be enough energy to heat every house on Earth for an entire winter!

You can figure out how much energy is produced by spontaneous radioactive decay by calculating the difference in the sum of the binding energies of the atoms before and after the decay.

Use this space for summary and/or additional notes.

## Half-Life

**Unit:** Atomic & Particle Physics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

### Knowledge/Comprehension:

- understand what half-life means

### Skills:

- solve problems having to do with:
  - amount of radioactive material remaining
  - length of time elapsed, based on amount of material remaining

### Language Objectives:

- Understand and correctly use the term “half-life”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the half-life of a radioactive substance.

### Notes:

The atoms of radioactive elements are unstable, and they spontaneously decay (change) into atoms of other elements.

For any given atom, there is a certain probability,  $P$ , that it will undergo radioactive decay in a given amount of time. The half-life,  $\tau_{1/2}$ , is how much time it would take to have a 50% probability of the atom decaying. If you start with  $n$  atoms, after one half-life, half of them ( $0.5n$ ) will have decayed.

If we start with 32 g of  $^{53}\text{Fe}$ , which has a half-life ( $\tau_{1/2}$ ) of 8.5 minutes, we would observe the following:

<b># minutes</b>	0	8.5	17	25.5	34
<b># half lives</b>	0	1	2	3	4
<b>amount left</b>	32 g	16 g	8 g	4 g	2 g

Use this space for summary and/or additional notes:

### Amount of Material Remaining

Most half-life problems in a first-year high school chemistry course involve a whole number of half-lives and can be solved by making a table like the one above. For problems that do not involve a whole number of half-lives, you need to use the exponential decay equation.

Because  $n$  is decreasing, the number of atoms (and consequently also the mass) remaining after any specific period of time follows the exponential decay function:

$$A = A_0 \left(\frac{1}{2}\right)^n$$

where  $A$  is the amount you have now,  $A_0$  is the amount you started with, and  $n$  is the number of half-lives that have elapsed.

Because the number of half-lives equals the total time elapsed ( $t$ ) divided by the half-life ( $\tau_{1/2}$ ), we can replace  $n = \frac{t}{\tau_{1/2}}$  and rewrite the equation as:

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} \quad \text{or} \quad \frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$

If you want to find either  $A$  or  $A_0$ , you will be given the values for  $t$  and  $\tau_{1/2}$ , and you can just plug them into the above equation.

Use this space for summary and/or additional notes.



**Sample Problem:**

If you start with 228 g of  $^{90}\text{Sr}$ , how much would remain after 112.4 years?

Solution:

$$A_0 = 228 \text{ g}$$

$$A = A$$

$\tau_{1/2} = 28.1$  years (from the "Selected Radioisotopes" table in your reference tables)

$$t = 112.4 \text{ years}$$

$$A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{\tau_{1/2}}}$$

$$A = (228) \left( \frac{1}{2} \right)^{\frac{112.4}{28.1}}$$

$$A = (228) \left( \frac{1}{2} \right)^4$$

$$A = (228) \left( \frac{1}{16} \right)$$

$$A = 14.25 \text{ g}$$

Or use a chart:

<b># years</b>	0	28.1	56.2	84.3	112.4
<b># half lives</b>	0	1	2	3	4
<b>amount left</b>	228 g	114 g	57 g	28.5 g	14.25 g

Use this space for summary and/or additional notes.

### Finding the Time that has Passed

If the amount you started with divided by the amount left is an exact power of two, you have an integer number of half-lives and you can just make a table.

#### Sample problem:

If you started with 64 g of  $^{131}\text{I}$ , how long would it take until there was only 4 g remaining? The half-life ( $\tau_{1/2}$ ) of  $^{131}\text{I}$  is 8.07 days.

#### Solution:

$\frac{64}{4} = 16$  which is a power of 2, so we can simply make a table:

<b>half lives</b>	0	1	2	3	4
<b>amount remaining</b>	64 g	32 g	16 g	8 g	4 g

From the table, after 4 half-lives, we have 4 g remaining.

The half-life ( $\tau_{1/2}$ ) of  $^{131}\text{I}$  is 8.07 days.

$$8.07 \times 4 = 32.3 \text{ days}$$

Use this space for summary and/or additional notes.

If you need to find the elapsed time and it is not an exact half-life, you need to use logarithms.

In mathematics, *the only reason you ever need to use logarithms is when you need to solve for a variable that's in an exponent.* For example, suppose we have the expression of the form  $a^b = c$ .

If  $b$  is a constant, we can solve for either  $a$  or  $c$ , as in the expressions:

$$a^3 = 21 \quad (\sqrt[3]{a^3} = \sqrt[3]{21} = 2.76)$$
$$6^2 = c \quad (6^2 = 36)$$

However, we can't do this if  $a$  and  $c$  are constants and we need to solve for  $b$ , as in the expression:

$$3^b = 17$$

To solve for  $b$ , we need to get  $b$  out of the exponent. We do this by taking the logarithm of both sides:

$$b \log(3) = \log(17)$$
$$b = \frac{\log(17)}{\log(3)}$$
$$b = \frac{1.23}{0.477} = 2.58$$

It doesn't matter which base you use; using  $\ln$  instead of  $\log$  gives the same result:

$$b \ln(3) = \ln(17)$$
$$b = \frac{\ln(17)}{\ln(3)}$$
$$b = \frac{2.83}{1.10} = 2.58$$

Use this space for summary and/or additional notes.

We can apply this same logic to the half-life equation:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$
$$\log A - \log A_0 = \frac{t}{\tau_{1/2}} \log\left(\frac{1}{2}\right)$$

**Sample problem:**

If you started with 64 g of  $^{131}\text{I}$ , how long would it take until there was only 5.75 g remaining? The half-life ( $\tau_{1/2}$ ) of  $^{131}\text{I}$  is 8.07 days.

**Solution:**

We have 5.75 g remaining. However,  $\frac{64}{5.75} = 11.13$ , which is not a power of two.

This means we don't have an integer number of half-lives, and we need to use logarithms:

$$\log A - \log A_0 = \frac{t}{\tau_{1/2}} \log\left(\frac{1}{2}\right)$$
$$\log 5.75 - \log 64 = \frac{t}{8.07} \log\left(\frac{1}{2}\right)$$
$$0.7597 - 1.8062 = \frac{t}{8.07} (-0.3010)$$
$$-1.0465 = -0.03730 t$$
$$28.1 \text{ days} = t$$

Use this space for summary and/or additional notes.

## Nuclear Fission & Fusion

Unit: Atomic & Particle Physics

NGSS Standards: HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- fission and fusion

Language Objectives:

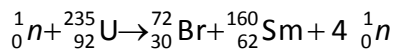
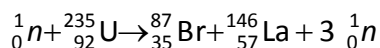
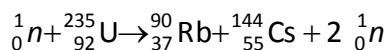
- Understand and correctly use the terms “fission” and “fusion.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

### Fission

fission: the intentional splitting of the nucleus of an atom, usually by bombarding it with a high-speed neutron.

The process of nuclear fission is accomplished by smashing the nucleus of an atom, usually with a high-speed neutron. When this happens, atoms can split in hundreds of ways. For example, when  $^{235}\text{U}$  is hit by a neutron, it can split more than 200 ways. Three examples that have been observed are:



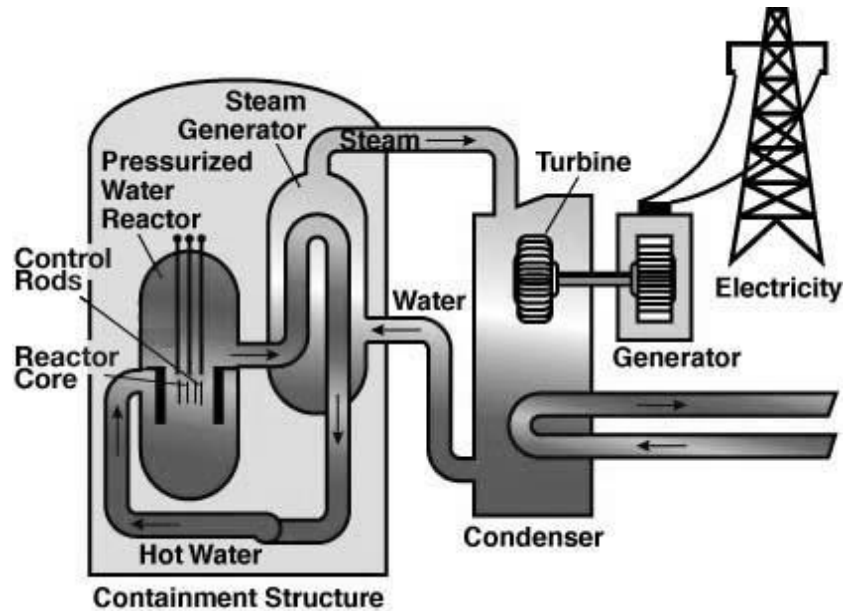
Note that each of these bombardments produces more neutrons. A reaction that produces more fuel (in this case, neutrons) than it consumes will accelerate. This self-propagation is called a chain reaction.

Note also that the neutron/proton ratio of  $^{235}\text{U}$  is about 1.5. The stable neutron/proton ratio of each of the products would be approximately 1.2. This means that almost all of the products of fission reactions have too many neutrons to be stable, which means they undergo  $\beta^-$  decay.

Use this space for summary and/or additional notes:

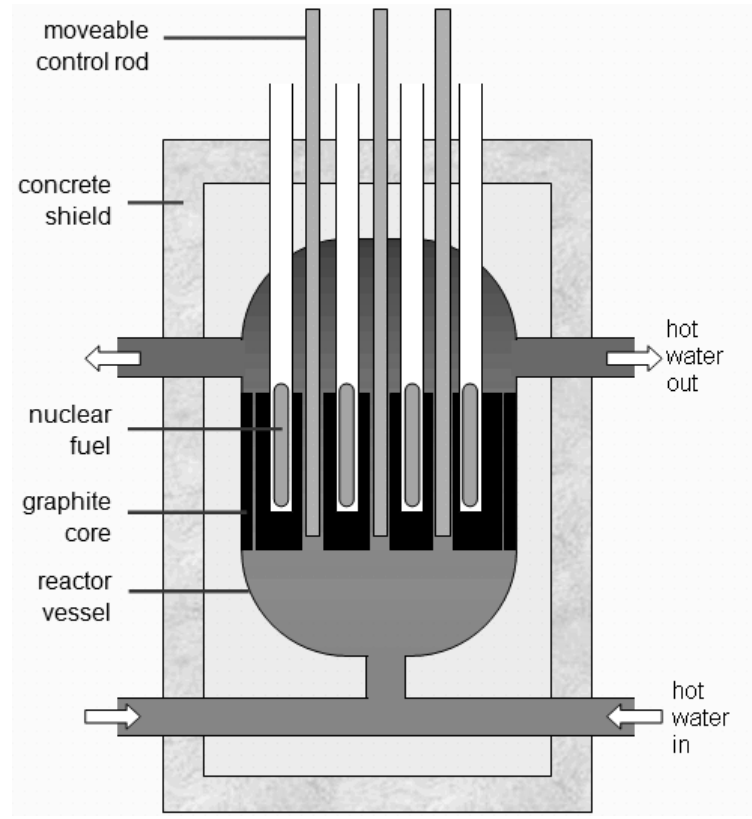
### Nuclear Reactors

In a nuclear reactor, the heat from a fission reaction is used to heat water. The radioactive hot water from the reactor (under pressure, so it can be heated well above 100°C without boiling) is used to boil clean (non-radioactive) water. The clean steam is used to turn a turbine, which generates electricity.



Use this space for summary and/or additional notes.

The inside of the reactor looks like this:



The fuel is the radioactive material (such as  $^{235}\text{U}$ ) that is undergoing fission. The graphite in the core of the reactor is used to absorb some of the neutrons. The moveable control rods are adjusted so they can absorb some or all of the remaining neutrons as desired. If the control rods are all the way down, all of the neutrons are absorbed and no heating occurs. When the reactor is in operation, the control rods are raised just enough to make the reaction proceed at the desired rate.

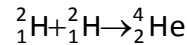
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## Fusion

fusion: the joining together of the nuclei of two atoms, accomplished by colliding them at high speeds.

Nuclear fusion reactions occur naturally on stars (such as the sun), and are the source of the heat and energy that stars produce.

On the sun, fusion occurs between atoms of deuterium ( $^2\text{H}$ ) to produce helium:

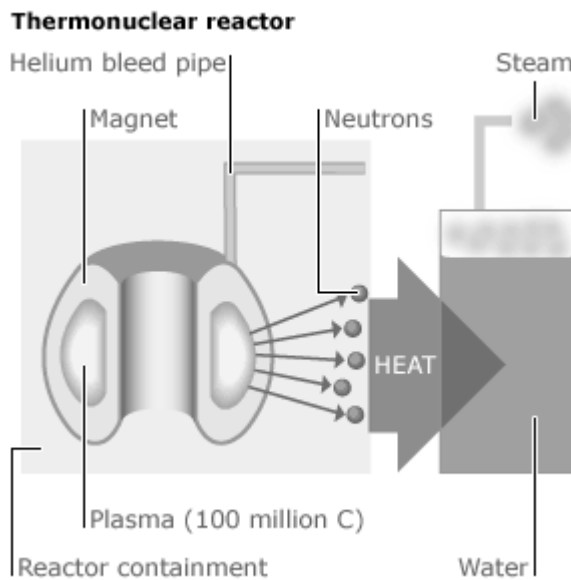


The major challenge in building nuclear fusion reactors is the high temperatures produced—on the order of  $10^6$ – $10^9$  °C. In a tokamak fusion reactor, the starting materials are heated until they become plasma—a sea of highly charged ions and electrons. The highly charged plasma is kept away from the sides by extremely powerful electromagnets.

Use this space for summary and/or additional notes.



Below is a schematic of the ITER tokamak reactor currently under construction in southern France.



The MIT Plasma Science & Fusion Center has a smaller tokamak reactor that they use for research. The MIT reactor is able to conduct fusion reactions lasting for only a few seconds; if the reaction continued beyond this point, the current in the electromagnets would become hot enough to melt the copper wire and fuse the coils of the electromagnet together.

After each "burst" (short fusion reaction), the electromagnets in the MIT reactor need to be cooled in a liquid nitrogen bath ( $-196^{\circ}\text{C}$ ) for fifteen minutes before the reactor is ready for the next burst.

Use this space for summary and/or additional notes.

## Practical Uses for Nuclear Radiation

**Unit:** Atomic & Particle Physics

**NGSS Standards:** N/A

**MA Curriculum Frameworks (2006):** N/A

**Knowledge/Understanding Goals:**

- recognize practical uses for nuclear radiation

**Language Objectives:**

- Accurately describe and apply the concepts described in this section using appropriate academic language.

**Notes:**

While most people think of the dangers and destructive power of nuclear radiation, there are a lot of other uses of radioactive materials:

**Power Plants:** nuclear reactors can generate electricity in a manner that does not produce CO<sub>2</sub> and other greenhouse gases.

**Cancer Therapy:** nuclear radiation can be focused in order to kill cancer cells in patients with certain forms of cancer. Radioprotective drugs are now available that can help shield non-cancerous cells from the high-energy gamma rays.

**Radioactive Tracers:** chemicals made with radioactive isotopes can be easily detected in complex mixtures or even in humans. This enables doctors to give a patient a chemical with a small amount of radioactive material and track the progress of the material through the body and determine where it ends up. It also enables biologists to grow bacteria with radioactive isotopes and follow where those isotopes end up in subsequent experiments.

Use this space for summary and/or additional notes:

**Irradiation of Food:** food can be exposed to high-energy gamma rays in order to kill germs. These gamma rays kill all of the bacteria in the food, but do not make the food itself radioactive. (Gamma rays cannot build up inside a substance.) This provides a way to create food that will not spoil for months on a shelf in a store. There is a lot of irrational fear of irradiated food in the United States, but irradiation is commonly used in Europe. For example, irradiated milk will keep for months on a shelf at room temperature without spoiling.

**Carbon Dating:** Because  $^{14}\text{C}$  is a long-lived isotope (with a half-life of 5,700 years), the amount of  $^{14}\text{C}$  in archeological samples can give an accurate estimate of their age. One famous use of carbon dating was its use to prove that the Shroud of Turin (the supposed burial shroud of Jesus Christ) was actually made between 1260 A.D. and 1390 A.D.

**Smoke Detectors:** In a smoke detector,  $^{241}\text{Am}$  emits positively-charged alpha particles, which are directed towards a metal plate. This steady flow of positive charges completes an electrical circuit. If there is a fire, smoke particles neutralize positive charges. This makes the flow of charges through the electrical circuit stop, which is used to trigger the alarm.

Use this space for summary and/or additional notes.

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Table A. Metric Prefixes			
Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000	$10^{24}$	yotta	Y
1 000 000 000 000 000 000 000	$10^{21}$	zeta	Z
1 000 000 000 000 000 000	$10^{18}$	exa	E
1 000 000 000 000 000	$10^{15}$	peta	P
1 000 000 000 000	$10^{12}$	tera	T
1 000 000 000	$10^9$	giga	G
1 000 000	$10^6$	mega	M
1 000	$10^3$	kilo	k
100	$10^2$	hecto	h
10	$10^1$	deca	da
1	$10^0$	—	—
0.1	$10^{-1}$	deci	d
0.01	$10^{-2}$	centi	c
0.001	$10^{-3}$	milli	m
0.000 001	$10^{-6}$	micro	μ
0.000 000 001	$10^{-9}$	nano	n
0.000 000 000 001	$10^{-12}$	pico	p
0.000 000 000 000 001	$10^{-15}$	femto	f
0.000 000 000 000 000 001	$10^{-18}$	atto	a
0.000 000 000 000 000 000 001	$10^{-21}$	zepto	z
0.000 000 000 000 000 000 000 001	$10^{-24}$	yocto	y

Description	Symbol	Precise Value	Common Approximation
universal gravitational constant	$G$	$6.67384(80) \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$	$6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$
acceleration due to gravity at sea level	$g$	$9.80665 \frac{\text{m}}{\text{s}^2} *$	$9.8 \frac{\text{m}}{\text{s}^2}$
acceleration due to gravity on Earth's surface	$g$	$9.7639 \frac{\text{m}}{\text{s}^2}$ to $9.8337 \frac{\text{m}}{\text{s}^2}$	$9.8 \frac{\text{m}}{\text{s}^2}$
speed of light in a vacuum	$c$	$299792458 \frac{\text{m}}{\text{s}} *$	$3.00 \times 10^8 \frac{\text{m}}{\text{s}}$
electrostatic constant	$k$	$8.9875517873681764 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} *$	$8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$
charge of a proton or electron (elementary charge)	$e$	$\pm 1.602176565(35) \times 10^{-19} \text{ C}$	$\pm 1.6 \times 10^{-19} \text{ C}$
1 coulomb (C)		$6.24150965(16) \times 10^{18}$ elementary charges	$6.24 \times 10^{18}$ elementary charges
(electric) permittivity of a vacuum	$\epsilon_0$	$8.85418782 \times 10^{-12} \frac{\text{A}^2 \cdot \text{s}^4}{\text{kg} \cdot \text{m}^3}$	$8.85 \times 10^{-12} \frac{\text{A}^2 \cdot \text{s}^4}{\text{kg} \cdot \text{m}^3}$
(magnetic) permeability of a vacuum	$\mu_0$	$1.25663706 \times 10^{-6} \frac{\text{mkg}}{\text{A}^2 \cdot \text{s}^2}$	$1.26 \times 10^{-6} \frac{\text{mkg}}{\text{A}^2 \cdot \text{s}^2}$
1 electron volt (eV)		$1.602176565(35) \times 10^{-19} \text{ J}$	$1.6 \times 10^{-19} \text{ J}$
Planck's constant	$h$	$6.62606957(29) \times 10^{-34} \text{ J} \cdot \text{s}$	$6.6 \times 10^{-34} \text{ J} \cdot \text{s}$
1 universal mass unit (u)		931.494061(21) MeV	931 MeV
Avogadro's constant	$N_A$	$6.02214129(27) \times 10^{23} \text{ mol}^{-1}$	$6.02 \times 10^{23} \text{ mol}^{-1}$
Boltzmann constant	$k_B$	$1.3806488(13) \times 10^{-23} \frac{\text{J}}{\text{K}}$	$1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$
gas constant	$R$	$8.314462(75) \frac{\text{J}}{\text{molK}}$	$8.31 \frac{\text{J}}{\text{molK}}$
standard atmospheric pressure at sea level		101 325 Pa $\equiv$ 1.01325 bar*	100 000 Pa $\equiv$ 1.0 bar
rest mass of an electron	$m_e$	$9.10938215(45) \times 10^{-31} \text{ kg}$	$9.11 \times 10^{-31} \text{ kg}$
mass of a proton	$m_p$	$1.67262177(74) \times 10^{-27} \text{ kg}$	$1.67 \times 10^{-27} \text{ kg}$
mass of a neutron	$m_n$	$1.67492735(74) \times 10^{-27} \text{ kg}$	$1.67 \times 10^{-27} \text{ kg}$

\* denotes an exact value (by definition)

Substance	Static ( $\mu_s$ )	Kinetic ( $\mu_k$ )	Substance	Static ( $\mu_s$ )	Kinetic ( $\mu_k$ )
rubber on concrete (dry)	0.90	0.68	wood on wood (dry)	0.42	0.30
rubber on concrete (wet)		0.58	wood on wood (wet)	0.2	
rubber on asphalt (dry)	0.85	0.67	wood on metal	0.3	
rubber on asphalt (wet)		0.53	wood on brick	0.6	
rubber on ice		0.15	wood on concrete	0.62	
steel on ice	0.03	0.01	Teflon on Teflon	0.04	0.04
waxed ski on snow	0.14	0.05	Teflon on steel	0.04	0.04
aluminum on aluminum	1.2	1.4	graphite on steel	0.1	
cast iron on cast iron	1.1	0.15	leather on wood	0.3–0.4	
steel on steel	0.74	0.57	leather on metal (dry)	0.6	
copper on steel	0.53	0.36	leather on metal (wet)	0.4	
diamond on diamond	0.1		glass on glass	0.9–1.0	0.4
diamond on metal	0.1–0.15		metal on glass	0.5–0.7	

Table D. Quantities, Variables and Units				
Quantity	Variable	MKS Unit Name	MKS Unit Symbol	S.I. Base Unit
distance/displacement, (length, height)	$d, \vec{d}, (\ell, h)$	meter*	m	m
angle	$\vartheta$	radian, degree	—, °	—
area	$A$	square meter	$m^2$	$m^2$
volume	$V$	cubic meter, liter	$m^3, \ell, L$	$m^3$
time	$t$	second*	s	s
velocity	$\vec{v}$	meter/second	$\frac{m}{s}$	$\frac{m}{s}$
speed of light	$c$			
angular velocity	$\vec{\omega}$	radians/second	$\frac{1}{s}$	$\frac{1}{s}$
acceleration	$\vec{a}$	meter/second <sup>2</sup>	$\frac{m}{s^2}$	$\frac{m}{s^2}$
acceleration due to gravity	$\vec{g}$			
mass	$m$	kilogram*	kg	kg
force	$\vec{F}$	newton	N	$\frac{kg \cdot m}{s^2}$
pressure	$P$	pascal	Pa	$\frac{kg}{m \cdot s^2}$
energy	$E$	joule	J	$\frac{kg \cdot m^2}{s^2}$
potential energy	$U$			
heat	$Q$			
work	$W$	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
torque	$\vec{\tau}$	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
power	$P$	watt	W	$\frac{kg \cdot m^2}{s^3}$
momentum	$\vec{p}$	newton-second	N·s	$\frac{kg \cdot m}{s}$
impulse	$\vec{J}$			
moment of inertia	$I$	kilogram-meter <sup>2</sup>	$kg \cdot m^2$	$kg \cdot m^2$
angular momentum	$\vec{L}$	newton-meter-second	N·m·s	$\frac{kg \cdot m^2}{s}$
frequency	$f$	hertz	Hz	$s^{-1}$
wavelength	$\lambda$	meter	m	m
period	$T$	second	s	s
index of refraction	$n$	—	—	—
electric current	$\vec{I}$	ampere*	A	A
electric charge	$q$	coulomb	C	A·s
potential difference (voltage) electromotive force (emf)	$V$ $\varepsilon$	volt	V	$\frac{kg \cdot m^2}{A \cdot s^3}$
electrical resistance	$R$	ohm	$\Omega$	$\frac{kg \cdot m^2}{A^2 \cdot s^3}$
capacitance	$C$	farad	F	$\frac{A^2 \cdot s^4}{m^2 \cdot kg}$
electric field	$\vec{E}$	newton/coulomb volt/meter	$\frac{N}{C}, \frac{V}{m}$	$\frac{kg \cdot m}{A \cdot s^3}$
magnetic field	$\vec{B}$	tesla	T	$\frac{kg}{A \cdot s^2}$
temperature	$T$	kelvin*	K	K
amount of substance	$n$	mole*	mol	mol
luminous intensity	$I_v$	candela*	cd	cd

\* S.I. base unit

Table E. Mechanics Formulas and Equations		
<b>Kinematics (Distance, Velocity &amp; Acceleration)</b>	$\vec{d} = \Delta s = s - s_o$ $\bar{v} = \frac{\vec{d}}{t} = \frac{\Delta s}{t} = \frac{\vec{v}_o + \vec{v}}{2}$ $\Delta \vec{v} = \vec{v} - \vec{v}_o = \vec{a}t$ $s - s_o = \vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a}t^2$ $\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}d$	$\Delta$ = change, difference $\Sigma$ = sum $d$ = distance (m) $\vec{d}$ = displacement (m) $s$ = position (m) $t$ = time (s)
<b>Forces &amp; Dynamics</b>	$\vec{F} = m\vec{a} \quad \vec{F}_g = m\vec{g}$ $F_g = \frac{Gm_1m_2}{d^2}$ $F_f \leq \mu_s F_N \quad F_f = \mu_k F_N$	$\vec{v}$ = velocity $\left(\frac{m}{s}\right)$ $\bar{v}$ = average velocity $\left(\frac{m}{s}\right)$ $\vec{a}$ = acceleration $\left(\frac{m}{s^2}\right)$ $a_c$ = centripetal acceleration $\left(\frac{m}{s^2}\right)$
<b>Circular and Simple Harmonic Motion</b>	$\vec{F}_s = -k\vec{x}$ $T_{spring} = 2\pi\sqrt{\frac{m}{k}}$ $U_{spring} = \frac{1}{2}kx^2$ $T_{pendulum} = 2\pi\sqrt{\frac{L}{g}}$ $a_c = \frac{v^2}{r} = \omega^2 r$ $\theta = \theta_o + \omega_o t + \frac{1}{2}at^2$ $F_c = ma_c = \frac{mv^2}{r}$ $\vec{\tau} = \vec{r} \times \vec{F} \quad \tau = rF \sin\theta$	$\vec{F}$ = force (N) $F_f$ = force due to friction (N) $\vec{F}_g$ = force due to gravity (N) $F_N$ = normal force (N) $F_c$ = centripetal force (N) $m$ = mass (kg) $\vec{g}$ = acceleration due to gravity $\left(\frac{m}{s^2}\right)$ $G$ = universal gravitational constant $\left(\frac{N \cdot m^2}{kg^2}\right)$ $r$ = radius (m) $\vec{r}$ = radius (vector) $\mu$ = coefficient of friction ( <i>dimensionless</i> ) $\theta$ = angle ( $^\circ$ , rad) $\omega$ = angular velocity $\left(\frac{rad}{s}\right)$
<b>Momentum</b>	$\vec{p} = m\vec{v}$ $\Sigma m_i \vec{v}_i = \Sigma m_f \vec{v}_f$ $\vec{J} = \Delta \vec{p} = \vec{F}_{net} \Delta t$ $\vec{L} = \vec{r} \times \vec{p} \quad L = rp \sin\theta$	$k$ = spring constant $\left(\frac{N}{m}\right)$ $\vec{x}$ = displacement of spring (m) $L$ = length of pendulum (m) $\vec{\tau}$ = torque (N·m) $E_k$ = kinetic energy (J) $U$ = potential energy (J)
<b>Energy, Work &amp; Power</b>	$W = \vec{F} \cdot \vec{d} = Fd \cos\theta$ $U_g = mgh$ $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $E_{total} = U + E_k + Q$ $W = \Delta E_k = -\Delta U$ $P = \frac{W}{t}$	$h$ = height (m) $Q$ = heat (J) $P$ = power (W) $W$ = work (N·m) $T$ = (time) period (Hz) $\vec{p}$ = momentum (N·s) $\vec{J}$ = impulse (N·s) $\vec{L}$ = angular momentum (N·m·s)

Table F. Heat and Thermal Physics Formulas and Equations		
Temperature	$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32$ $\text{K} = ^{\circ}\text{C} + 273.15$	$\Delta$ = change $^{\circ}\text{F}$ = Fahrenheit temperature ( $^{\circ}\text{F}$ ) $^{\circ}\text{C}$ = Celsius temperature ( $^{\circ}\text{C}$ ) $\text{K}$ = Kelvin temperature (K)
	$Q = mC\Delta T$ $Q_{\text{melt}} = m\Delta H_{\text{fus}}$ $Q_{\text{boil}} = m\Delta H_{\text{vap}}$ $C_p - C_v = R$ $\Delta L = \alpha L_i \Delta T$ $\Delta V = \beta V_i \Delta T$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\frac{Q}{t} = -kA \frac{\Delta T}{L}$ $\frac{Q}{t} = -\frac{1}{R_i} A \Delta T$	$Q$ = heat (J, kJ) $m$ = mass (kg) $C$ = specific heat capacity ( $\frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$ ) ( $C_p$ = const. pressure; $C_v$ = const. volume) $T$ = temperature (K) $t$ = time (s) $L$ = length (m) $k$ = coefficient of thermal conductivity ( $\frac{\text{J}}{\text{m}\cdot\text{s}\cdot^{\circ}\text{C}}$ , $\frac{\text{W}}{\text{m}\cdot^{\circ}\text{C}}$ ) $V$ = volume ( $\text{m}^3$ ) $\alpha$ = linear coefficient of thermal expansion ( $^{\circ}\text{C}^{-1}$ ) $\beta$ = volumetric coefficient of thermal expansion ( $^{\circ}\text{C}^{-1}$ ) $R_i$ = "R" value of insulation $R$ = gas constant ( $\frac{\text{J}}{\text{mol}\cdot\text{K}}$ ) $U$ = internal energy (J) $W$ = work (N·m)
Heat		
Thermodynamics	$\Delta U = \Delta Q + \Delta W$	

Table G. Thermal Properties of Selected Materials								
Substance	Melting Point ( $^{\circ}\text{C}$ )	Boiling Point ( $^{\circ}\text{C}$ )	Heat of Fusion $\Delta H_{\text{fus}}$ ( $\frac{\text{kJ}}{\text{kg}}$ )	Heat of Vaporization $\Delta H_{\text{vap}}$ ( $\frac{\text{kJ}}{\text{kg}}$ )	Specific Heat Capacity $C_p$ ( $\frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$ ) at 25 $^{\circ}\text{C}$	Thermal Conductivity $k$ ( $\frac{\text{J}}{\text{m}\cdot\text{s}\cdot^{\circ}\text{C}}$ ) at 25 $^{\circ}\text{C}$	Coefficients of Expansion at 20 $^{\circ}\text{C}$	
							Linear $\alpha$ ( $^{\circ}\text{C}^{-1}$ )	Volumetric $\beta$ ( $^{\circ}\text{C}^{-1}$ )
air (gas)	—	—	—	—	1.012	0.024	—	—
aluminum (solid)	659	2467	395	10460	0.897	250	$2.3 \times 10^{-5}$	$6.9 \times 10^{-5}$
ammonia (gas)	-75	-33.3	339	1369	4.7	0.024	—	—
argon (gas)	-189	-186	29.5	161	0.520	0.016	—	—
carbon dioxide (gas)	-78		574		0.839	0.0146	—	—
copper (solid)	1086	1187	134	5063	0.385	401	$1.7 \times 10^{-5}$	$5.1 \times 10^{-5}$
brass (solid)	—	—	—	—	0.380	120	$1.9 \times 10^{-5}$	$5.6 \times 10^{-5}$
diamond (solid)	3550	4827	10 000	30 000	0.509	2200	$1 \times 10^{-6}$	$3 \times 10^{-6}$
ethanol (liquid)	-117	78	104	858	2.44	0.171	$2.5 \times 10^{-4}$	$7.5 \times 10^{-4}$
glass (solid)	—	—	—	—	0.84	0.96–1.05	$8.5 \times 10^{-6}$	$2.55 \times 10^{-5}$
gold (solid)	1063	2660	64.4	1577	0.129	310	$1.4 \times 10^{-5}$	$4.2 \times 10^{-5}$
granite (solid)	1240	—	—	—	0.790	1.7–4.0	—	—
helium (gas)	—	-269	—	21	5.193	0.142	—	—
hydrogen (gas)	-259	-253	58.6	452	14.30	0.168	—	—
iron (solid)	1535	2750	289	6360	0.450	80	$1.18 \times 10^{-5}$	$3.33 \times 10^{-5}$
lead (solid)	327	1750	24.7	870	0.160	35	$2.9 \times 10^{-5}$	$8.7 \times 10^{-5}$
mercury (liquid)	-39	357	11.3	293	0.140	8	$6.1 \times 10^{-5}$	$1.82 \times 10^{-4}$
paraffin wax (solid)	46–68	~300	~210	—	2.5	0.25	—	—
silver (solid)	962	2212	111	2360	0.233	429	$1.8 \times 10^{-5}$	$5.4 \times 10^{-5}$
steam (gas) @ 100 $^{\circ}\text{C}$	—	—	—	2260	2.080	0.016	—	—
water (liq.) @ 25 $^{\circ}\text{C}$	0	100	334	—	4.181	0.58	$6.9 \times 10^{-5}$	$2.07 \times 10^{-4}$
ice (solid) @ -10 $^{\circ}\text{C}$	—	—	—	—	2.11	2.18	—	—



Table H. Electricity & Magnetism Formulas & Equations

<p><b>Electrostatic Charges &amp; Electric Fields</b></p>	$F_e = \frac{kq_1q_2}{d^2}$ $\vec{F}_e = q\vec{E}$ $W = q\vec{E} \cdot \vec{d} = qEd \cos \theta$ $V = \frac{W}{q} = \vec{E} \cdot \vec{d}$ $U_E = qV = \frac{kq_1q_2}{d}$ $I = \frac{\Delta q}{t}$	
<p><b>Circuits</b></p>	$V = IR$ $P = VI = I^2R = \frac{V^2}{R}$ $W = Q = Pt = VIt = I^2Rt = \frac{V^2t}{R}$ $R = \frac{\rho \ell}{A}$ $C = \frac{q}{V}$ $U_{\text{capacitor}} = \frac{1}{2}qV = \frac{1}{2}CV^2$	<p><math>\Delta</math> = change</p> <p><math>\vec{F}_e</math> = force due to electric field (N)</p> <p><math>k</math> = electrostatic constant <math>\left(\frac{N \cdot m^2}{C^2}\right)</math></p> <p><math>q</math> = charge (C)</p> <p><math>\vec{E}</math> = electric field <math>\left(\frac{N}{C}, \frac{V}{m}\right)</math></p> <p><math>V</math> = voltage = electric potential difference (V)</p> <p><math>W</math> = work (N·m)</p> <p><math>d</math> = distance (m)</p> <p><math>\vec{I}</math> = current (A)</p> <p><math>t</math> = time (s)</p>
<p><b>Series Circuits</b></p>	$I = I_1 = I_2 = I_3 = \dots$ $V = V_1 + V_2 + V_3 + \dots$ $R_{eq} = R_1 + R_2 + R_3 + \dots$ $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$ $P_{total} = P_1 + P_2 + P_3 + \dots$	<p><math>R</math> = resistance (<math>\Omega</math>)</p> <p><math>P</math> = power (W)</p> <p><math>Q</math> = heat (J)</p> <p><math>\rho</math> = resistivity (<math>\Omega \cdot m</math>)</p> <p><math>\ell</math> = length (m)</p> <p><math>A</math> = cross-sectional area (<math>m^2</math>)</p> <p><math>U</math> = potential energy (J)</p> <p><math>C</math> = capacitance (F)</p>
<p><b>Parallel Circuits</b></p>	$I = I_1 + I_2 + I_3 + \dots$ $V = V_1 = V_2 = V_3 = \dots$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$ $C_{total} = C_1 + C_2 + C_3 + \dots$ $P_{total} = P_1 + P_2 + P_3 + \dots$	<p><math>\vec{v}</math> = velocity (of moving charge or wire) <math>\left(\frac{m}{s}\right)</math></p> <p><math>\vec{B}</math> = magnetic field (T)</p> <p><math>\mu_0</math> = magnetic permeability of free space</p> <p><math>r</math> = radius (distance) from wire</p>
<p><b>Magnetism</b></p>	$\vec{F} = q(\vec{v} \times \vec{B}) \quad F = qvB \sin \theta$ $\vec{F} = \ell(\vec{I} \times \vec{B}) \quad F = \ell IB \sin \theta$ $V = \ell(\vec{v} \times \vec{B}) \quad V = \ell vB \sin \theta$ $B = \frac{\mu_0 I}{2\pi r}$	
<p><b>Electromagnetic Induction</b></p>	$\frac{\#turns_{in}}{\#turns_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$ $P_{in} = P_{out}$	

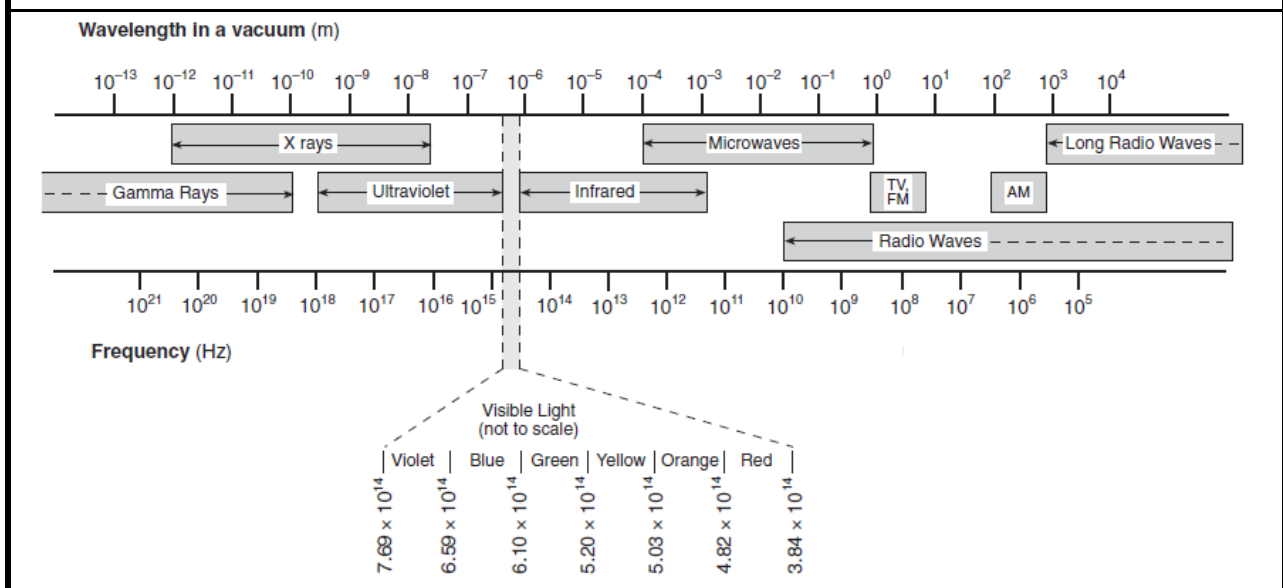
Component	Symbol	Component	Symbol
wire	—	battery	
switch		ground	
fuse		resistor	
voltmeter		variable resistor (potentiometer, rheostat, dimmer)	
ammeter		lamp (light bulb)	
ohmmeter		capacitor	
		diode	

Color	Digit	Multiplier
black	0	$\times 10^0$
brown	1	$\times 10^1$
red	2	$\times 10^2$
orange	3	$\times 10^3$
yellow	4	$\times 10^4$
green	5	$\times 10^5$
blue	6	$\times 10^6$
violet	7	$\times 10^7$
gray	8	$\times 10^8$
white	9	$\times 10^9$
gold		$\pm 5\%$
silver		$\pm 10\%$

Substance	Resistivity ( $\Omega \cdot m$ )	
silver	$1.59 \times 10^{-8}$	conductors
copper	$1.72 \times 10^{-8}$	
gold	$2.44 \times 10^{-8}$	
aluminum	$2.82 \times 10^{-8}$	
tungsten	$5.60 \times 10^{-8}$	
iron	$9.71 \times 10^{-8}$	
nichrome	$1.50 \times 10^{-6}$	
graphite	$3 \times 10^{-5}$ to $6 \times 10^{-4}$	semi-conductors
germanium	0.001 to 0.5	
silicon	0.1 to 60	
sea water	0.2	conductors
drinking water	20 to 2 000	
deionized water	$1.8 \times 10^5$	insulators
glass	$1 \times 10^9$ to $1 \times 10^{13}$	
rubber, hard	$1 \times 10^{13}$ to $1 \times 10^{13}$	
paraffin (wax)	$1 \times 10^{13}$ to $1 \times 10^{17}$	
air	$1.3 \times 10^{16}$ to $3.3 \times 10^{16}$	
quartz, fused	$7.5 \times 10^{17}$	

Table L. Waves & Optics		
<b>Waves</b>	$v = f\lambda$ $f = \frac{1}{T}$ $v_{\text{wave on a string}} = \sqrt{\frac{F_T}{\mu}}$ $f_{\text{dopplershifted}} = f \left( \frac{v_{\text{wave}} + v_{\text{detector}}}{v_{\text{wave}} + v_{\text{source}}} \right)$	$v$ = velocity of wave ( $\frac{\text{m}}{\text{s}}$ ) $f$ = frequency (Hz) $\lambda$ = wavelength (m) $T$ = period (of time) (s) $F_T$ = tension (force) on string (N) $\mu$ = elastic modulus of string ( $\frac{\text{kg}}{\text{m}}$ )
<b>Reflection &amp; Refraction</b>	$\theta_i = \theta_r$ $n = \frac{c}{v}$ $n_1 \sin \theta_1 = n_2 \sin \theta_2$ $\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$ $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$	$\theta_i$ = angle of incidence ( $^\circ, \text{rad}$ ) $\theta_r$ = angle of reflection ( $^\circ, \text{rad}$ ) $\theta_c$ = critical angle ( $^\circ, \text{rad}$ ) $n$ = index of refraction ( <i>dimensionless</i> ) $c$ = speed of light in a vacuum ( $\frac{\text{m}}{\text{s}}$ )
<b>Mirrors &amp; Lenses</b>	$d_f = \frac{r_c}{2}$ $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{d_f}$ $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$	$d_f$ = distance to the focus of a mirror or lens (m) $r_c$ = radius of curvature of a spherical mirror (m) $d_i$ = distance from the mirror or lens to the image (m) $d_o$ = distance from the mirror or lens to the object (m) $h_i$ = height of the image (m) $h_o$ = height of the object (m) $m$ = magnification ( <i>dimensionless</i> )

Figure M. The Electromagnetic Spectrum



Temp. (°C)	Water			Air	
	Density (kg/m <sup>3</sup> )	Speed of Sound (m/s)	Vapor Pressure (Pa)	Density (kg/m <sup>3</sup> )	Speed of Sound (m/s)
0	999.78	1 403	611.73	1.288	331.30
5	999.94	1 427	872.60	1.265	334.32
10	999.69	1 447	1 228.1	1.243	337.31
20	998.19	1 481	2 338.8	1.200	343.22
25	997.02	1 496	3 169.1	1.180	346.13
30	995.61	1 507	4 245.5	1.161	349.02
40	992.17	1 526	7 381.4	1.124	354.73
50	990.17	1 541	9 589.8	1.089	360.35
60	983.16	1 552	19 932	1.056	365.88
70	980.53	1 555	25 022	1.025	371.33
80	971.79	1 555	47 373	0.996	376.71
90	965.33	1 550	70 117	0.969	382.00
100	954.75	1 543	101 325	0.943	387.23

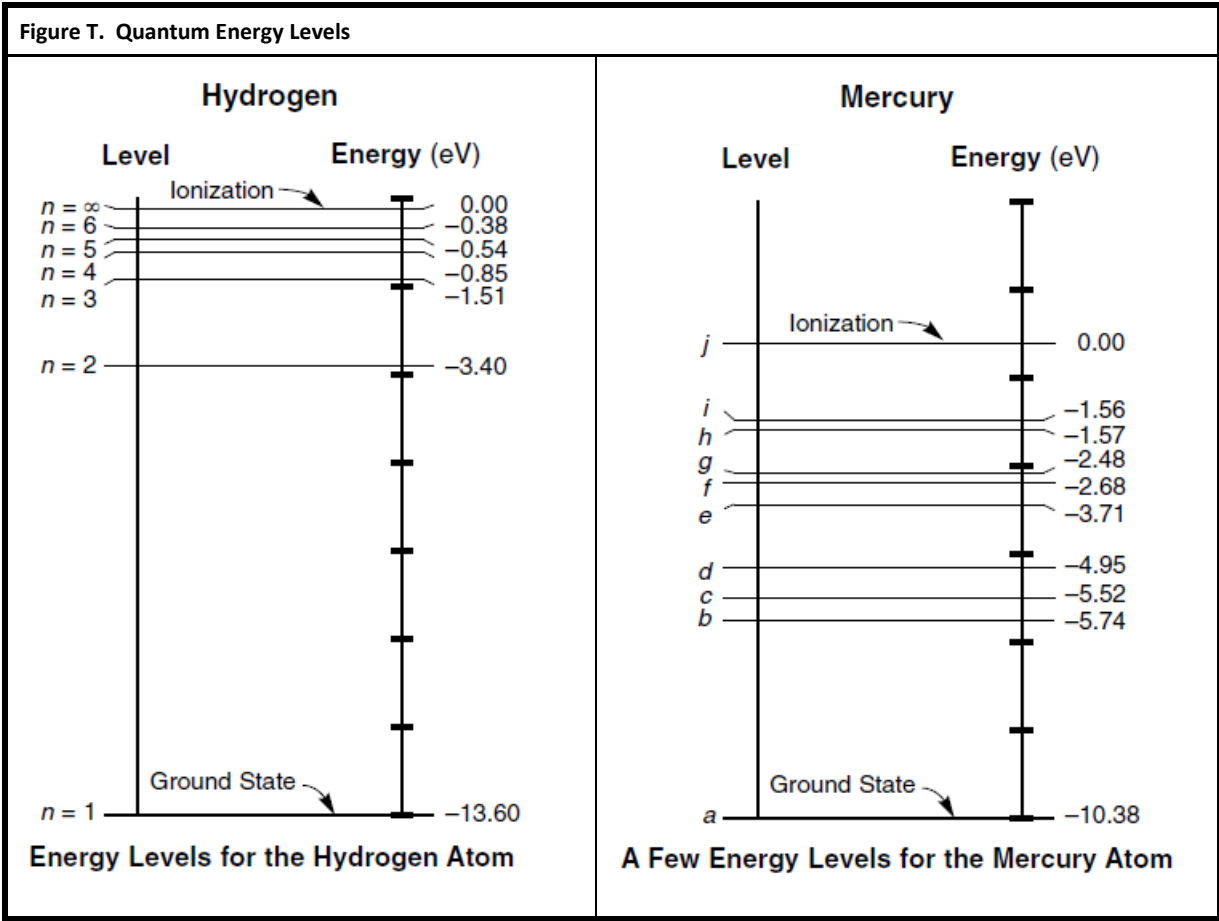
Substance	Index of Refraction	Substance	Index of Refraction
air	1.000293	silica (quartz), fused	1.459
ice	1.309	Plexiglas	1.488
water	1.3330	Lucite	1.495
ethyl alcohol	1.36	glass, borosilicate (Pyrex)	1.474
human eye, cornea	1.38	glass, crown	1.50–1.54
human eye, lens	1.41	glass, flint	1.569–1.805
safflower oil	1.466	sodium chloride, solid	1.516
corn oil	1.47	PET (#1 plastic)	1.575
glycerol	1.473	zircon	1.777–1.987
honey	1.484–1.504	cubic zirconia	2.173–2.21
silicone oil	1.52	diamond	2.417
carbon disulfide	1.628	silicon	3.96

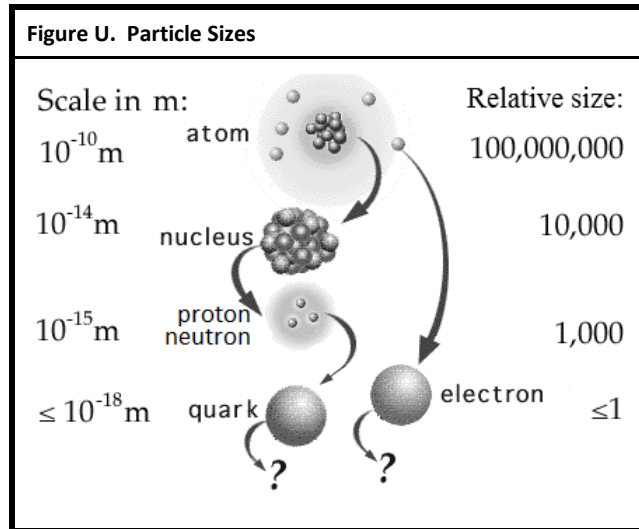
Table P. Fluid Mechanics Formulas and Equations		
<b>Density &amp; Pressure</b>	$P = \frac{F}{A}$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ $\rho = \frac{m}{V}$ $P = P_o + \rho gh$ $A_1 v_1 = A_2 v_2$ $P + \rho gh + \frac{1}{2} \rho v^2 = \text{constant}$	$\Delta$ = change $\rho$ = density ( $\frac{\text{kg}}{\text{m}^3}$ ) $m$ = mass (kg) $V$ = volume ( $\text{m}^3$ ) $P$ = pressure (Pa) $g$ = acceleration due to gravity ( $\frac{\text{m}}{\text{s}^2}$ ) $h$ = height or depth (m) $A$ = area ( $\text{m}^2$ ) $v$ = velocity (of fluid) ( $\frac{\text{m}}{\text{s}}$ )
<b>Forces, Work &amp; Energy</b>	$F_B = \rho V_d g$ $PV = Nk_B T = nRT$ $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ $E_{k(\text{molecular})} = \frac{3}{2} k_B T$ $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{\mu}}$ $W = -P\Delta V$	$F$ = force (N) $n$ = number of moles (mol) $R$ = gas constant ( $\frac{\text{J}}{\text{molK}}$ ) $N$ = number of molecules $k_B$ = Boltzmann's constant ( $\frac{\text{J}}{\text{K}}$ ) $T$ = temperature (K) $M$ = molar mass ( $\frac{\text{g}}{\text{mol}}$ ) $\mu$ = molecular mass (kg) $E_k$ = kinetic energy (J) $W$ = work (N·m)

Table Q. Planetary Data								
	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from Sun (m)	$5.79 \times 10^{10}$	$1.08 \times 10^{11}$	$1.50 \times 10^{11}$	$2.28 \times 10^{11}$	$7.78 \times 10^{11}$	$1.43 \times 10^{12}$	$2.87 \times 10^{12}$	$4.50 \times 10^{12}$
Radius (m)	$2.44 \times 10^6$	$6.05 \times 10^6$	$6.37 \times 10^6$	$3.39 \times 10^6$	$6.99 \times 10^7$	$5.82 \times 10^7$	$2.54 \times 10^7$	$2.46 \times 10^7$
Mass (kg)	$3.30 \times 10^{23}$	$4.87 \times 10^{24}$	$5.97 \times 10^{24}$	$6.42 \times 10^{23}$	$1.90 \times 10^{27}$	$5.68 \times 10^{26}$	$8.68 \times 10^{25}$	$1.02 \times 10^{26}$
Density ( $\frac{\text{kg}}{\text{m}^3}$ )	5430	5250	5520	3950	1330	690	1290	1640
Orbit (years)	0.24	0.62	1.00	1.88	11.86	84.01	164.79	248.54
Rotation Period (hours)	1408	5832	23.9	24.6	9.9	10.7	17.2	16.1
Tilt of axis	2°	177.3°	23.5°	25.2°	3.1°	26.7°	97.9°	29.6°
# observed satellites	0	0	1	2	67	62	27	13

Table R. Additional Solar System Data	
Radius of the sun (m)	$6.96 \times 10^8$
Mass of the sun (kg)	$1.99 \times 10^{30}$
Radius of the moon (m)	$1.74 \times 10^6$
Mass of the moon (kg)	$7.35 \times 10^{22}$
Distance of moon from Earth (m)	$3.84 \times 10^8$

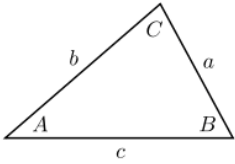
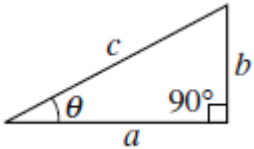
Table S. Atomic & Particle Physics (Modern Physics)		
Energy	$E_{\text{photon}} = hf = \frac{hc}{\lambda} = pc$ $\lambda = \frac{h}{p}$ $E_{\text{photon}} = E_i - E_f$ $E = mc^2$	<p><math>E</math> = energy (J)</p> <p><math>h</math> = Planck's constant (J·s)</p> <p><math>f</math> = frequency (Hz)</p> <p><math>c</math> = speed of light (<math>\frac{m}{s}</math>)</p> <p><math>\lambda</math> = wavelength (m)</p> <p><math>p</math> = momentum (N·s)</p> <p><math>m</math> = mass (kg)</p> <p><math>\gamma</math> = Lorentz factor (<i>dimensionless</i>)</p>
Special Relativity	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L_o}{L} = \frac{\Delta t'}{\Delta t} = \frac{m_{rel}}{m_o}$	<p><math>L</math> = length in moving reference frame (m)</p> <p><math>L_o</math> = length in stationary reference frame (m)</p> <p><math>\Delta t'</math> = time in stationary reference frame (s)</p> <p><math>\Delta t</math> = time in moving reference frame (s)</p> <p><math>m_o</math> = mass in stationary reference frame (kg)</p> <p><math>m_{rel}</math> = apparent mass in moving reference frame (kg)</p> <p><math>v</math> = velocity (<math>\frac{m}{s}</math>)</p>





**Table V. The Standard Model**

		Generation				
		I	II	III		
mass →		2.4 MeV/c <sup>2</sup>	1.27 GeV/c <sup>2</sup>	171.2 GeV/c <sup>2</sup>	0 MeV/c <sup>2</sup>	125.3 GeV/c <sup>2</sup>
charge →		$+\frac{2}{3}$	$+\frac{2}{3}$	$+\frac{2}{3}$	0	0
spin →		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
		<b>u</b> up quark	<b>c</b> charm quark	<b>t</b> top quark	<b>γ</b> photon	<b>H<sup>0</sup></b> Higgs boson
	<b>quarks</b>	4.8 MeV/c <sup>2</sup> $-\frac{1}{3}$ <b>d</b> down quark	104 MeV/c <sup>2</sup> $-\frac{1}{3}$ <b>s</b> strange quark	4.2 GeV/c <sup>2</sup> $-\frac{1}{3}$ <b>b</b> bottom quark	0 MeV/c <sup>2</sup> 0 <b>g</b> gluon	
		< 2.2 eV/c <sup>2</sup> 0 $\frac{1}{2}$ <b>ν<sub>e</sub></b> electron neutrino	< 0.17 MeV/c <sup>2</sup> 0 $\frac{1}{2}$ <b>ν<sub>μ</sub></b> muon neutrino	< 15.5 MeV/c <sup>2</sup> 0 $\frac{1}{2}$ <b>ν<sub>τ</sub></b> tau neutrino	91.2 GeV/c <sup>2</sup> 0 1 <b>Z<sup>0</sup></b> Z boson	
	<b>leptons</b>	0.511 MeV/c <sup>2</sup> -1 $\frac{1}{2}$ <b>e</b> electron	105.7 MeV/c <sup>2</sup> -1 $\frac{1}{2}$ <b>μ</b> muon	1.777 GeV/c <sup>2</sup> -1 $\frac{1}{2}$ <b>τ</b> tau	80.4 GeV/c <sup>2</sup> ±1 1 <b>W<sup>±</sup></b> W boson	<b>gauge bosons</b>

Table W. Geometry & Trigonometry Formulas		
Triangles	$A = \frac{1}{2}bh$ $c^2 = a^2 + b^2 - 2ab \cos C$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	
Right Triangles	$c^2 = a^2 + b^2$ $\sin \theta = \frac{b}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos \theta = \frac{a}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}}$ $a = c \cos \theta$ $b = c \sin \theta$	
Rectangles, Parallelograms and Trapezoids	$A = \bar{b}h$	<p><math>a, b, c</math> = length of a side of a triangle  <math>\theta</math> = angle  <math>A</math> = area  <math>C</math> = circumference  <math>S</math> = surface area  <math>V</math> = volume  <math>b</math> = base  <math>h</math> = height  <math>\ell</math> = length  <math>w</math> = width  <math>r</math> = radius</p>
Rectangular Solids	$V = \ell wh$	
Circles	$C = 2\pi r$ $A = \pi r^2$	
Cylinders	$S = 2\pi r \ell + 2\pi r^2 = 2\pi r(\ell + r)$ $V = \pi r^2 \ell$	
Spheres	$S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	



degree	radian	sine	cosine	tangent	degree	radian	sine	cosine	tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.747
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.331
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.430
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.301
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.081
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.636
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.290
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	∞

**Table Y. Some Exact and Approximate Conversions**

Length	1 cm	≈	width of a small paper clip	
	1 inch (in.)	≡	2.54 cm	
	length of a US dollar bill	=	6.14 in.	≈ 15.6 cm
	12 in.	≡	1 foot (ft.)	≈ 30 cm
	3 ft.	≡	1 yard (yd.)	≈ 1 m
	1 m	=	0.3048 ft.	≈ 39.37 in.
	1 km	≈	0.6 mi.	
	5,280 ft.	≡	1 mile (mi.)	≈ 1.6 km
Mass/ Weight	1 small paper clip	≈	0.5 gram (g)	
	1 US 1¢ coin (1983–present)	=	2.5 g	
	1 US 5¢ coin	=	5 g	
	1 oz.	≈	30 g	
	one medium-sized apple	≈	1 N	≈ 3.6 oz.
	1 pound (lb.)	≡	16 oz.	≈ 454 g
	1 pound (lb.)	≈	4.45 N	
	1 ton	≡	2000 lb.	≈ 0.9 tonne
1 tonne	≡	1000 kg	≈ 1.1 ton	
Volume	1 pinch	=	$\leq \frac{1}{8}$ teaspoon (tsp.)	
	1 mL	≈	10 drops	
	1 tsp.	≈	5 mL	≈ 60 drops
	3 tsp.	≡	1 tablespoon (Tbsp.)	≈ 15 mL
	2 Tbsp.	≡	1 fluid ounce (fl. oz.)	≈ 30 mL
	8 fl. oz.	≡	1 cup (C)	≈ 250 mL
	16 fl. oz.	≡	1 U.S. pint (pt.)	≈ 500 mL
	20 fl. oz.	≡	1 Imperial pint (UK)	≈ 600 mL
	2 pt.	≡	1 U.S. quart (qt.)	≈ 1 L
	4 qt. (U.S.)	≡	1 U.S. gallon (gal.)	≈ 3.8 L
5 qt. (U.S.)	≡	1 Imperial gal. (UK)	≈ 4.7 L	
Speed	$1 \frac{\text{m}}{\text{s}}$	≈	$2.24 \frac{\text{mi.}}{\text{h}}$	
	$60 \frac{\text{mi.}}{\text{h}}$	≈	$100 \frac{\text{km}}{\text{h}}$	≈ $27 \frac{\text{m}}{\text{s}}$

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